Study on the Virtual Test for Reliability for Contact Stress of Gear Tooth Face

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Abstract. Based on the technology of virtual test for reliability for the study of small sample sizes by using Box-Behnken sampling method and FEA method, the paper presents a method for studying the randomness of tooth face contact stress by using technology of virtual test for reliability. According to that the tooth face contact stress normally has the characteristic of skewed distribution, the paper presents the experimental data handing method that fitting virtual experimental data by using three-parameter weibull distribution, and then estimating the distribution parameters by using Matrix Analysis and bilinear regression analysis method together.

Introduction

Reliability test is an effective means to obtain statistic data and the reliability information of product [1]. The original mismachining tolerance of accessory gear and load fluctuation of gear in working process immediately influence the contact accuracy and stress of gear, thus they influence the strength and life of gear. So it is necessary to do the research about reliability test on the tooth face contact stress. Because of the high cost and difficulty in the conventional test of reliability test for studying tooth face contact stress, it has practical significance to do the research about virtual test for reliability for gear meshing problem.

This paper uses APDL to create a virtual experimental prototype for gear meshing, and this make the prototype has parametric feature, so it can simulate the original mismachining tolerance of accessory gear and load fluctuation of gear in working process accurately. Besides, the paper uses Box-Behnken sampling method to sample from random variables, so as to realize the study on virtual test of small sample sizes. Because that the tooth face contact stress normally has the characteristic of skewed distribution, the paper fits virtual experimental data for reliability by using three-parameter weibull distribution which has multiple forms of distribution [2], and then estimate the three distribution parameters by using matrix analysis method and bilinear regression analysis method together.

Box-Behnken sampling method

Box-Behnken sampling method is one kind of effective design of experiments. This method gets three probability bench marks from each random variable, and the marks segment into central point and central point of edges by some rules, which are the sample sets [3]. Fig. 1 shows the sample set based on Box-Behnken of three variables (O_1, O_2, O_3).

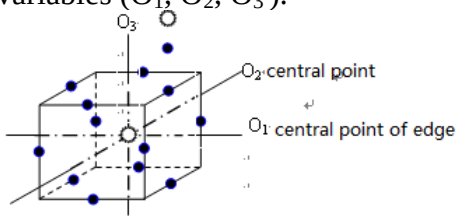

Fig. 1 sample set based on Box-Behnken

For random variable of free distribution, use equation (1) to define probability bench mark of random variable

$$\int_{-\infty}^{r_i} g(r)\mathrm{d}r = p_i \quad i = 1, 2, 3 \tag{1}$$

In the equation, $g(r)$ is the distribution density function of random variable; p_i means probability level, normally set $p_1 = 0.01$, $p_2 = 0.5$, $p_3 = 0.99$.

For random variable of normal distribution

$$r_i = \mu + \sigma\Phi^{-1}(p_i) \tag{2}$$

Based on Box-Behnken sampling method, the paper samples from random variables that influence the structural reliability, and the sampling value of each group of random variable makes up a sample set. Conduct one finite element analysis with each sample set, which is one virtual test, and all the virtual tests with all the Box-Behnken sample sets make up the whole test procedure of virtual test for reliability.

Virtual test for reliability for gear meshing

Establishment of solid model of prototype. In the process of transferring data between CAD software and FEA software, the parametric feature of a model will be lost, and the study on the randomness of original mismachining tolerance of gear can't be realized. So, the paper uses APDL to create enough key points on the tooth curve in ANSYS according to the parametric equation of tooth curve and fillet curve, and then it fits these key points with B-splint curve to create the tooth curve, so as to create the solid model of gear tooth. In order to reduce the calculated amount of finite elements, it only creates three neighboring teeth on gear by rotating and copying, and others are replaced by graduated arc, thus creating the solid model.

Establishment of finite element model of prototype. After creating contact pair, applying load and boundary conditions, the finite element model of this virtual test prototype is presented in Fig. 2.

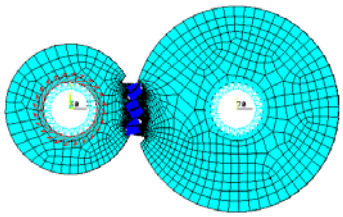

Fig. 2 FEM model of virtual test prototype of gear meshing

Result analysis of virtual test. According to the Stress-Strength interference, the gear will lose efficacy once the maximum contact stress on the tooth face is larger than the allowable contact stress, and the probability of not losing efficacy like that is defined as the contact strength reliability of gear tooth face [5].In order to analyze the result of virtual test for reliability for gear meshing, it needs to get the maximum contact stress on contact surface.

Experimental data Handing method

Because that the tooth face contact stress normally has the characteristic of skewed distribution, the experimental data handing method of virtual test for reliability for gear meshing uses Weibull distribution, whose distribution function is presented in equation (3).

$$F(x) = 1 - \exp\left[-\left(\frac{x - a_3}{a_2}\right)^{a_1}\right] \tag{3}$$

In the equation, $a_1\ (a_1 > 0)$ is shape parameter; $a_2\ (a_2 > 0)$ is scale parameter; a_3 is positional parameter; $x\ (x \geq a_3)$ is random variable.

In the process of handing experimental data, it first uses matrix analysis method to estimate the positional parameter a_3 of Weibull distribution, and then uses bilinear regression analysis method to

estimate the shape parameter a_1 and the scale parameter a_2, which has high analysis efficiency. Simple linear regression equation is presented in equation (4).

$$Y = \eta_1 X + \eta_2 \tag{4}$$

In the equation, η_1 is regression coefficient; η_2 is constant term; X、Y are coordinate values after linear transformation. Transpose with equation (1), and take the logarithm twice

$$\ln\ln\frac{1}{1-F(x)} = a_1 \times \ln(x-a_3) + (-a_1 \ln a_2) \tag{5}$$

Contrasting with equation (4) and (5), it shows that

$$\eta_1 = a_1 \tag{6}$$

$$\eta_2 = -a_1 \ln a_2 \tag{7}$$

$$X = \ln(x-a_3) \tag{8}$$

$$Y = \ln\ln\frac{1}{1-F(x)} \tag{9}$$

In the equation, distribution function $F(x)$ is unknown. As to small sample sizes, distribution function $F(x)$ can be replaced by medium rank $\hat{F}(x_i)$ of empirical distribution function.

$$\hat{F}(x_i) \approx \frac{i-0.3}{n+0.4} \quad i = 1, 2, \cdots, n \tag{10}$$

For sample x_i $(i = 1, 2, \cdots, n)$, calculate the x-coordinate value X_i after linear transformation with equation (6), and then calculate y-coordinate value Y_i after linear transformation with equation (9), (10). Use $\overline{X}$、$\overline{Y}$ to represent the mean value of x-coordinate and y-coordinate value after linear transformation, then the correlation coefficient ρ of each sample set is

$$\rho = \frac{\sum_{i=1}^{n} X_i Y_i - n\overline{X}\overline{Y}}{\left(\sum_{i=1}^{n} X_i^2 - n\overline{X}^2\right)^{\frac{1}{2}} \left(\sum_{i=1}^{n} Y_i^2 - n\overline{Y}^2\right)^{\frac{1}{2}}} \tag{11}$$

Given that DOF $\nu = n-2$, significance level $\alpha = 0.02$, then the minimum value of correlation coefficient ρ_α is [1]

$$\rho_\alpha = \frac{2.326}{\sqrt{\nu+1}} \tag{12}$$

If $|\rho| > \rho_\alpha$, it is considered that *X, Y* is linearly dependent, and the hypothesis that the samples obeys the Weibull distribution is acceptable; Otherwise, it is not acceptable. Regression coefficient η_1 and constant term η_2 are presented in equation (13), (14).

$$\eta_1 = \frac{\sum_{i=1}^{n} X_i Y_i - n\overline{X}\overline{Y}}{\sum_{i=1}^{n} X_i^2 - n\overline{X}^2} \tag{13}$$

$$\eta_2 = \overline{Y} - \zeta \overline{X} \tag{14}$$

Substitute the regression coefficient η_1 and constant term η_2 in equation (9) and (10), then the shape parameter a_1 and the scale parameter a_2 of Weibull distribution can be figured out.

Example of calculation

Given that the transmitted power *P* of a pair of gears and the revolving speed n_1 of driving gear have random fluctuation, and the tooth width *d* and the pressure angle α_0 of gears have original mismachining tolerance. Suppose that the random variables above all obey the normal distribution,

and the distribution parameters are presented in Table 1. Choose probability bench marks p_1 = 0.01, p_2 = 0.5,p_3 = 0.99, take samples by using Box-Behnken sampling method [3], conduct virtual test for gear meshing with each group of sample sets, and use Weibull distribution to fit the experimental results.

According to equation (12), (13), calculate the regression coefficient of regression line η_1=2.5804 and the constant term η_2 =10.2036. After linear transformation, the scatter diagram of X、Y and their regression line are presented in Fig. 3.

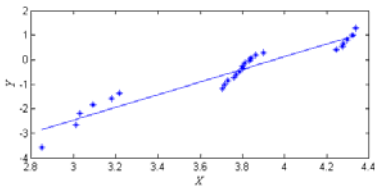

Fig. 3 scatter diagram and regression line of X、Y

Obviously, the distributions of sample sets naturally form three regions of relative concentration according to the contact stress value by size. Contrasting the value of each random variable in sample sets, it can be found that the transmitted nominal torque of driving gear in one region is close to each other, while it varies much from region to region, so the load fluctuation is the main factor that influences the tooth face contact stress.

By using matrix analysis method it can get the point estimation of position parameter $\hat{a}_3 = 1017.2291$; According to equation(7), the point estimation of shape parameter $\hat{a}_1 = \eta_1$ =2.5804; According to equation (8), the point estimation of scale parameter $\hat{a}_2 = \exp(-\frac{\eta_2}{\hat{a}_1})$ =52.157.

Conclusion

This paper studies the technology of virtual test for reliability for small sample sizes based on Box-Behnken sampling method and FEA method, and it presents a method of using technology of virtual test for reliability for the study of randomness of tooth face contact stress. According to that the tooth face contact stress normally has the characteristic of skewed distribution, it comes up with a experimental data handing method that fitting virtual experimental data by using three-parameter weibull distribution, and then estimating the distribution parameters by using matrix analysis method and bilinear regression analysis method together.

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