

## Calculation of slab-column middle connection under combined shear and unbalanced moment

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**Abstract.** The purpose: The method to calculate slab-column middle connections under combined interaction of vertical shear and unbalanced moment is given to perfect the design method of slab-column structure; Method: The calculation method is proposed based on the calculation method of slab-column connection and the experimental data made by domestic and foreign scholars is used to verify. Result: The results verified by the formula are shown to be in good agreement with actual results. Conclusion: The interaction of shear and unbalanced moment are taken into account in the formula, including the transfer mode of unbalanced moment and the beneficial effects of shear reinforcement, to make the calculation more reasonable.

### Introduction

Flat-slab system, in which slab is directly supported by column without beams. And it is intensively used due to its a series of outstanding advantages, such as decreasing floor-to-floor height, increasing number of structural stories and effective area when maintain the overall height unchanged, easy to decoration and good lightning and ventilation effects. Bending punching compound failure is caused by the vertical shear and unbalanced moment borne by the slab-column connection at the same time in the process of earthquake, which makes the flat-slab connections become the weak link in the whole flat-slab structure, then leads to flat-slab structure's serious damage or even collapse. But, only anti-shear checking method of slab-column connection is given in the Chinese concrete structure design code (GB50010-2010), while the slab-column connection's failure characteristics under combined interaction of vertical shear and unbalanced moment are not taken into account, which makes hard to design. Despite this, the flat-slab system is widely applied in practical engineering, and the basic structural form contains multilayer slab-column structure and middle-high slab-column and shear wall structure, and also it is widely adopted in framed-tube structure. Therefore, the transfer mode of unbalanced moments, the interaction of shear and unbalanced moment and the beneficial effects of shear reinforcement are considered simultaneously in the flat-slab connection formula, which promotes flat-slab system widely used in seismic design.

At present, the experimental study and theoretical analysis are mainly included based on the slab-column connection research, while the experimental study includes slab-column connections' the punching resisting performance and experiment<sup>[1]</sup>, the flexural performance and experiment<sup>[2~4]</sup>, and the analysis and experiment of slab-column connections under combined shear and unbalanced moment<sup>[5~7]</sup>. The punching resisting element was used in the specimen conducted by Southeast University in the several experiments, in which the vertical load and horizontal cyclic load are considered simultaneously. Because of stud shear reinforcement was used in only one specimen<sup>[7]</sup>, which cannot effective response the actual stress of slab-column connection and the beneficial effects of stud shear reinforcement in earthquake. In the theoretical research, the calculation method of slab-column connections under combined shear and unbalanced moment mainly includes ACI code method and the method proposed by Yunchang Ma, however, the transfer mode of unbalanced moments, the interaction of shear and unbalanced moment and the beneficial effects of shear reinforcement are not considered simultaneously in both methods.

Therefore, the calculation formula proposed in this paper is based on the slab-column connection design method, ACI318M-11 code and the formula of slab-column connections under combined the vertical shear and unbalanced moment proposed in the Chinese concrete structure design code. Then, the formula is verified by the related experiments.

### The current formula

At present, there are only two design methods that slab-column connection under combined shear and unbalanced moment are considered. One is ACI code method, the other is proposed by Yuchang Ma and Xilin Lv<sup>[5,8]</sup>.

**Slab-column structure design method based on ACI code.** The ACI code method's basic thought is some unbalanced moment ( $M - \alpha_0 M$ ) of slab-column connection is transferred by bending of the hidden beams arranged near the axis, the other ( $\alpha_0 M$ ) is transferred by the eccentricity

of connection shear. The coefficient  $\alpha_0$  can be expressed as  $\alpha_0 = 1 - \frac{1}{1 + 2/3\sqrt{a_t/a_m}}$ , and  $a_t, a_m$  —

the length of connection failure section along the direction of moment and perpendicular to the moment, and the concentrated reaction caused by the vertical load is replaced by the equivalent concentrated reaction design values caused by the unbalanced moment. This method only considers the transfer mode of the unbalanced moment without considering the interaction of shear and unbalanced moment.

**The method proposed by Yuchang Ma and XilinLv.** This method is based on Johansen's superposition theory and the expression of the critical combined load is  $\frac{M}{M_u} + \frac{V}{V_u} = 1$ , when under two independent loads.

$V_u$  — slab-column connection's punching shear strength (only considering the strength provided by concrete);

$M$  — connection's unbalanced moment design values;

$M_u$  — slab-column connection's flexural capacity,  $M_u = 6.89(m_u' + m_u)$ ;

$V$  — connection's shear design values;

$m_u, m_u'$  — slab's positive and negative ultimate resistant moment in unit width respectively,

$m_u = f_y A_s h_0 (1 - 0.5 \rho f_y / \alpha_1 f_c)$   $m_u' = f_y' A_s' h_0 (1 - 0.5 \rho' f_y' / \alpha_1 f_c)$

Though the interaction of shear and unbalanced moment is taken into account in this method, the transfer mode of the unbalanced moment is not considered. Moreover, the beneficial effects of shear reinforcement are not considered in the calculation of slab-column connection's punching shear strength.

### The formula proposed in the paper

**The expression of formula.** The design suggestion of slab-column middle connection under combined shear and unbalanced moment:

$$\frac{M(1-\alpha_0)}{M_u} + \frac{F_{le}}{V_u} = 1 \quad (2-1)$$

$F_{le}$  — the equivalent concentrated reaction design values under consideration of the unbalanced moment according to the Chinese concrete structure design code;

$V_u$  — slab-column connection's punching shear strength;

$M$  — connection's unbalanced moment design values;

$M_u$  — slab-column connection's flexural capacity;

$V$  — connection's shear design values;

**A suggestion on  $V_u$  value.** The formula of slab that without resisting shear element in the current Chinese concrete structure design code:  $V_u=0.7\beta_{hf}\eta u_m h_0$  (2-2)

The coefficient  $\eta$ , can be calculated by the following formulas, and take the smaller value:

$$\eta_1=0.4+\frac{1.2}{\beta_s}, \eta_2=0.5+\frac{\alpha_s h_0}{4u_m};$$

$\eta_1$ ——the shape influence coefficient under local loads or concentrated reaction;

$\eta_2$ ——the ratio of calculation section perimeter and plate section effective height influence coefficient;

$\beta_s$ ——the ratio of long side and short side of rectangle under local loads or concentrated reaction, and  $\beta_s$  should not exceed 4,  $\beta_s$  takes 2 when it less than 2; to circular punching surface,  $\beta_s$  takes 2;

$\beta_h$ ——section height influence coefficient,  $\beta_h$  takes 1 when h is not more than 800mm;  $\beta_h$  takes 0.9, when h is not less than 2000mm; meantime the values are taken by interpolation;

$h_0$ ——section effective height, takes the average value of the two section effective heights;

$u_m$ ——the calculation section perimeter, takes the distance between local loads or concentrated reaction's action area peripheral  $h_0/2$  perpendicular to the section's most unfavorable perimeter.

The Chinese 2002 edition code thinks the safety degree is too high of the resisting punching loads formula in the 89 edition, and the coefficient is adjusted from 0.6 to 0.7 and has introduced plate section height influence coefficient  $\beta_h$ . The formula of 2002 edition code is preserved in the 2010 edition, and the formula is simple and the calculation is convenient. But compared to the foreign codes, the formula is more conservative in most situation.

The plate resisting punching shear takes the minimum in the following three formulas in ACI code:

$$\begin{cases} V_c = 0.17 \left(1 + \frac{2}{\beta}\right) \sqrt{f'_c} b_0 d \\ V_c = 0.083 \left(\frac{\alpha_s d}{b_0} + 2\right) \sqrt{f'_c} b_0 d \quad (2-4) \\ V_c = 0.33 \sqrt{f'_c} b_0 d \end{cases}$$

$\beta$ ——the ratio of long side and short side of action area of column section under concentrated loads or the reactions;

$\alpha_s$ ——inner column, side column, corner column take 40, 30, 20 respectively;

$b_0$ ——the critical perimeter; d is the plate effective height.

The experimental value and the design value of resisting punching formula in the Chinese code ratio's average is 2.476 after Xiaobin Xu<sup>[9]</sup> of HeBei United University concluded 67 slab-column punching experiments, while the experimental value and the design value of the American ACI code ratio's average is 2.285, and the ratio of them is 1.08. Though the expressions are different in the Chinese code and the American code, the difference of calculation result is only 8%. And compared to experimental value, the design values given in both codes are much more conservative, meanwhile, the Chinese code is more conservative than the American code<sup>[10]</sup>.

The result of slab-column connection plastic limit analysis under vertical shear in paper[2,3]:

$$V_u=(K+1)\mu_f u_m h_0 \quad (2-3) \quad K=\frac{1}{4} \left[ m + 2 \left( 1 - \sqrt{m+1} \right) \right], \quad m = \frac{\mu_c f_c}{\mu_t f_t};$$

$f_c, f_t$ ——concrete ultimate compressive strength and tensile strength;

$\mu_c, \mu_t$ ——concrete compressive strength and tensile strength reduction coefficient when materials inhomogeneity and complex stress, can approximately take 0.35.

The range of  $K$  value is between 1.12 and 1.75, and the range of  $(K+1)\mu_t$  is between 0.74 and 0.96 through the calculation of concrete C20~C50. Compared to the Chinese code, the coefficient is larger in the formula. And compared to ACI code and the Chinese code conservative design value, the  $V_u$  value is more reasonable calculated in formula (2-3).

**The  $V_u$  calculation of resisting punching element plate.** The formula is given by Jie Tian, Kebin Xu<sup>[11]</sup>, Zefeng Pan<sup>[12]</sup> of Nanjing University of Aeronautics and Astronautics and the Chinese

unbonded prestressed concrete design code for prestressed concrete structures JGJ92-2004 and the Chinese concrete structure design code for the slab-column connection resisting punching calculation of resisting punching element respectively. According to the experimental researches, considering the aggregate occlusion after concrete cracking, the beneficial effect of reinforcement shear friction, through analysis and comparison, the  $V_u$  calculated in the current code is more safe and reasonable.  $V_u = 0.5f_t\eta\mu_m h_0 + 0.8f_{yv}A_{syu}$  (2-5)

$f_{yv}$ —anchor bolt tensile strength design value;

$A_{syu}$ —all the steel section area that intersected with the 45° punching failure pyramidal oblique section.

**A value suggestion on  $M_u$ .** The yield line theory is adopted in the analysis of middle column slab-column connection under the unbalanced moment. And according to the Principle of Virtual Work, the flexural capacity minimal upper value is given as follow:  $M_u = 7.92(m_u' + m_u)$ ; (2-6)

$m_u, m_u'$ —slab's positive and negative ultimate resistant moment in unit width respectively,

$$m_u = f_y A_s h_0 (1 - 0.5\rho f_y / \alpha_1 f_c) \quad m_u' = f_y' A_s' h_0 (1 - 0.5\rho' f_y' / \alpha_1 f_c)$$

**The verification of formula**

The formula (2-1) proposed in the paper is verified by the arrangement of experiments of resisting punching anchor bolt and unresisting punching element slab-column connection under combined shear and unbalanced moment. The current Chinese code is used to calculate  $F_{le}$ , and  $V_u$  of unresisting punching element slab-column connection is according to (2-3) to calculate, while  $V_u$  of resisting punching element slab-column connection is according to (2-5) and  $M_u$  is according to (2-6). The results is saw the following Table1 and Table2:

Table1 The calculation of unresisting punching element slab-column connection

Source	Slab	$h_0$ (mm)	$c_1$ (mm)	$c_2$ (mm)	$f_t$ (Mpa)	$f_y$ (Mpa)	$M$ (KN·m)	$V$ (KN)	$M_u$ (kN·m)	$F_{le}$ (KN)	$V_u$ (KN)	Results
[6]	IC2	75	400	400	2.06	378.5	75.6	18.8	128	209.8	243.6	1.216
	IC3	85	300	300	2.36	215.0	30.6	28.8	42.6	106.9	275.0	0.820
[13]	1	75	400	400	2.06	378.5	71.4	15.2	127.9	195.6	243.6	1.138
[14]	S1	63.5	305	305	2.92	322.7	34.24	5.83	34.5	151.9	280.3	1.137
	S2	63.5	305	305	2.52	330.3	38.76	5.83	50.6	171.2	219.6	1.240
	S3	63.5	305	305	2.47	335.1	41.13	5.83	67.3	181.3	212.8	1.219
	S4	63.5	305	305	2.51	319.9	35.48	11.79	49.1	163.2	218.7	1.180
	S5	63.5	305	305	2.53	339.9	37.52	26.02	52.0	186.1	219.6	1.280

Table 2 The calculation of resisting punching element slab-column connection

Source	Slab	$h_0$ (mm)	$c_1$ (mm)	$c_2$ (mm)	$f_t$ (Mpa)	$f_y$ (Mpa)	$A_s$ (mm <sup>2</sup> /m)	$M$ (KN·m)	$V$ (KN)	$M_u$ (kN·m)	$F_{le}$ (KN)	$V_u$ (KN)	Results
[13]	SC1	90	400	400	2.38	235	1159	108.9	57.7	146.4	322.9	555.7	1.047
	SC2	90	400	400	2.38	235	1159	105.1	57.7	146.4	313.6	555.7	0.996
	SC3	90	400	400	2.38	235	1159	104.8	57.7	146.4	312.9	378.5	1.257
	SC4	90	400	400	2.38	235	1159	110.2	57.7	146.4	326.0	555.7	1.039
	SC5	90	400	400	2.38	235	1159	111.3	57.7	146.4	328.7	378.5	1.324
[14]	2	108	250	250	2.71	452	1556(555)	162	150	226.7	693	582.5	1.619
	3	108	250	250	2.99	452	1556(555)	142	300	231.3	776	604.1	1.653
	4	108	250	250	3.08	446	1556(555)	150	300	229.4	803	691.8	1.553
	5	108	250	250	3.32	446	1556(555)	105	450	232.0	802	710.4	1.400
[7]	SSR5	90	300	300	2.17	279(364)	1260(900)	64	325	133.3	521.9	503.6	1.324

The relevant experiment data is used to verify the formula given in the paper, the average of unresisting punching element slab-column connection calculation result is 1.154, while the average of resisting punching element slab-column connection is 1.321. So the formula is more safe and reasonable.

## Summary

1) The interaction of shear and unbalanced moment, the transfer mode of unbalanced moment and the beneficial effects of shear reinforcement are taken into account in the formula, which perfects the calculation. 2) The results verified by the formula are shown to be in good agreement with actual results, and have a high accuracy calculation and safety. If you follow the “checklist” your paper will conform to the requirements of the publisher and facilitate a problem-free publication process.

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