

# Image Up-Sampling Using Discrete Wavelet Transform

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## Abstract

Image up-sampling is found to be a very effective technique useful in today's digital image processing applications or rendering devices. In image up-sampling, an image is enhanced from a lower resolution to a higher resolution with the degree of enhancement depending upon application requirements. It is known that the traditional interpolation based approaches for up-sampling, such as Bilinear or Bicubic interpolation, blur the resultant images [1, 2]. Furthermore; in color imagery, these interpolation based up-sampling methods may have color infringing artifacts in the areas where the images contain sharp edges and fine textures. In this paper, we present an interesting up-sampling algorithm using 9/7 bi-orthogonal Spline filters based Discrete Wavelet Transform (DWT). The proposed method preserves much of the sharp edge features in the image, and lessens the amount of color artifacts. Effectiveness of the proposed algorithm has been demonstrated based on evaluation of PSNR and  $\Delta E_{ab}^*$  quality metrics of the original image and the reconstructed image.

**Keywords:** DWT, up-sampling, interpolation.

## 1. Introduction

After Mallat [3] proposed the multi-resolution representation of signals based on wavelet decomposition, the discrete wavelet transform (DWT) became a versatile signal processing tool. The DWT can be implemented by filtering operations with well-defined filter coefficients [4, 5]. Unlike the Fourier transforms or average filters, the wavelet coefficients can be selected to suit particular application criteria. For example, the 9/7 bi-orthogonal Spline filters based DWT was found effective for image coding [6], and has been adapted as one of the two default filter kernels in the new JPEG2000 image compression standard [7, 8].

Two-dimensional DWT can be implemented by applying the one-dimensional DWT in each row of the image to produce an intermediate result and then

applying the same 1-D DWT column-wise on the intermediate result to produce the final result (known as wavelet coefficients)[8]. As shown in Fig. 1(a), four subbands  $LL$ ,  $HL$ ,  $LH$ , and  $HH$  are generated after applying one level of DWT in the input image  $I$ . It is also known that the low-pass subband ( $LL$ ) represents a 4:1 sub-sampled version of the original signals. The basic idea of using 2-D DWT for image up-sampling [9] is to modify the wavelet coefficient of the image to a higher resolution by a suitable method and then inverse transforming (IDWT) it to obtain the up-sampled image as shown schematically in Fig. 1.

The rest of this paper is organized as follows. Section 2 describes the details of the proposed algorithm. Discussions on how to select the key scaling factor and different implementations are also provided. The experimental results are presented in section 3, and the paper is concluded in section 4.

## 2. Proposed Methodology

We explain the proposed up-sampling technique with an example, as shown in Fig. 1. We like to up-sample an input image ( $I$ ) of resolution  $m \times n$  (say  $4 \times 4$  as in Fig. 1) to an image ( $I'$ ) of resolution  $2m \times 2n$ .

In the first step, we transform the image using the forward discrete wavelet transform (FDWT) in order to decompose it into four subbands – one low frequency subband ( $LL$ ) and three high frequency subbands ( $HL$ ,  $LH$  and  $HH$ ), resulting in the wavelet coefficient image  $I_{DWT}$  of size  $m \times n$ . These four subbands contain different information about the image. The  $HL$  and  $LH$  subbands contain edge information in different directions, which will be used for the purpose of enhancement in the next step. One may apply the same principle as in LH and HL subbands in the HH subband as well. However based on our experimental results, we have noticed that dropping the HH subband does not impact the perceptual quality of the up-sampled image at all and the difference in PSNR quality is also non-significant. As a result, we proposed to drop the HH subband in order to reduce the computational requirement.

The second step is to form a new wavelet coefficient image  $I'_{DWT}$  of size  $2m \times 2n$ . We call it a virtual DWT image, whose  $LL$  subband is nothing but the original input image  $I$  with each pixel multiplied by a *scaling factor*  $s$ . This scaling factor is determined by the DC gain of the low pass filter coefficients used in the DWT as explained in section 2.2. Hence the dimension of the new  $LL$  subband is the same as the resolution  $m \times n$  of the original image  $I$ . The  $HH$  subband of the virtual DWT image ( $I'_{DWT}$ ) is considered to contain all zeros and hence it is nothing but a matrix of zeros with dimension  $m \times n$ . The new  $HL$  and  $LH$  subbands of the virtual DWT image are generated from the original  $HL$  and  $LH$  subbands (computed in the first step) by inserting zeros in alternate rows and columns as shown in Fig. 1(b).

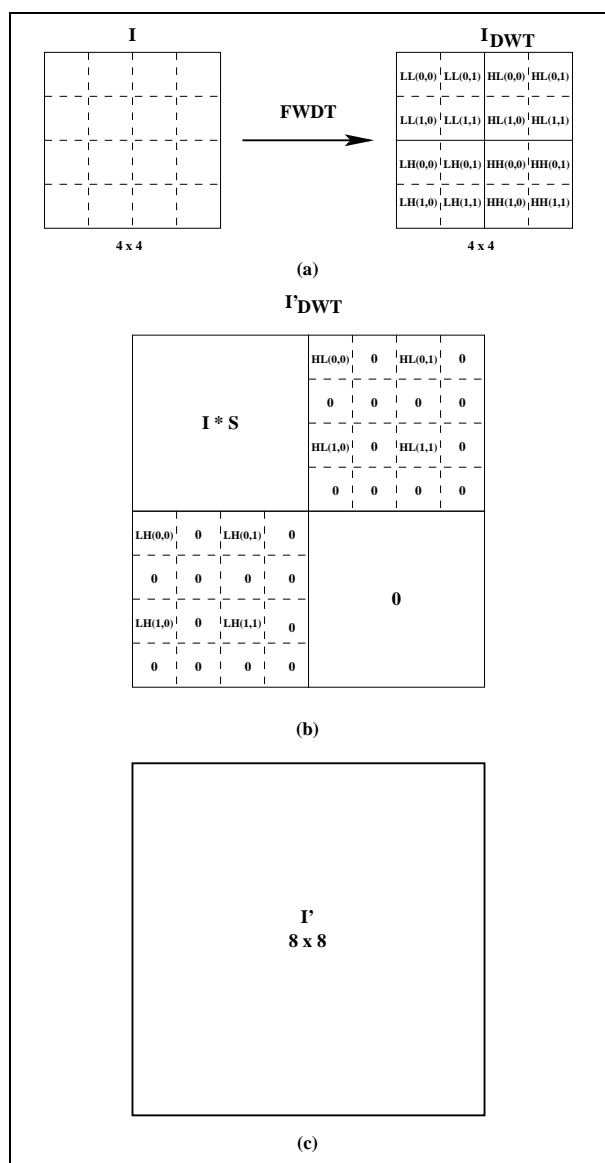


Fig. 1: DWT based image up-sampling.

In the last step, we inverse transform (IDWT) this virtual DWT image ( $I'_{DWT}$ ) that we formed in the second step as explained above. The resulting image after IDWT is the desired up-sampled image  $I'$  of resolution  $2m \times 2n$ .

To up-sample color images, the proposed technique can be applied independently in each color plane. The results of the up-sampling technique on color images have been shown in Fig. 2 and Fig. 3.

## 2.1. Implementations

The proposed algorithm can be implemented in different ways. Since the 2-D discrete wavelet transform is computed by applying the 1-D low-pass and high-pass filters in succession to the rows and the columns of the input image, and the filtering operation is simply a multiply-and-accumulate operation, the proposed algorithm can be easily implemented in any DWT based hardware architecture or by using a DSP (digital signal processor). The  $HL$  subband of the  $I'_{DWT}$  can be obtained by applying the 1-D high-pass filter row-wise, followed by the 1-D low-pass filter column wise on the input image  $I$ , and then setting the alternate row and column coefficients to zeros. The  $LH$  subband of the  $I'_{DWT}$  can be obtained in the similar fashion by switching the order of applying high- and low-pass filters.

Alternatively, for software implementation, the proposed up-sampling method can be summarized in the following steps:

1. Initialize a matrix  $I'_{DWT}$  of dimensions  $2m \times 2n$  with all its elements as zeros.
2. Set the original image to a matrix  $I$  of dimension  $m \times n$ .
3. Multiply  $I$  by the scale factor  $s$  to produce a matrix  $I_{LL}$ . Replace the top-left quadrant of  $I'_{DWT}$  by  $I_{LL}$ .
4. Apply the high-pass wavelet filter (without down-sampling) in each row of  $I$  followed by the low-pass filter in each column. Set alternate rows and columns of the resulting matrix to zeros to produce a matrix  $I_{HL}$ . Replace the top-right quadrant of the matrix  $I'_{DWT}$  by this  $I_{HL}$ .
5. Apply the low-pass wavelet filter (without down-sampling) in each row of  $I$  followed by the high-pass filter in each column. Set alternate rows and columns of the resulting matrix to zeros to produce a matrix  $I_{LH}$ . Replace the bottom-left quadrant of matrix  $I'_{DWT}$  by this  $I_{LH}$ .
6. Apply the inverse DWT on matrix  $I'_{DWT}$  to produce  $I'$ .

## 2.2. Scaling Factor

Depending upon the implementation of the discrete wavelet transform, one may choose a different DC gain for the analysis low-pass filter and Nyquist gain for the analysis high-pass filter, or even choose different filters. In this paper, we choose the 9/7 bi-orthogonal Spline filters due to the fact that it is commonly used in the literature and it is one of two default filters used in the JPEG2000 image compression standard. It is known to produce better perceptual quality. We also normalized the filters with both DC and Nyquist gains equal to the square root of 2. This will maintain the same dynamic range for all four subbands. Consequently, the scaling factor,  $s$ , used in the proposed algorithm should be  $\sqrt{2}$ , since that is the square of the DC gain.

The same theory applies to all other wavelet filters and users can use any other wavelet filter for the proposed image up-sampling technique.

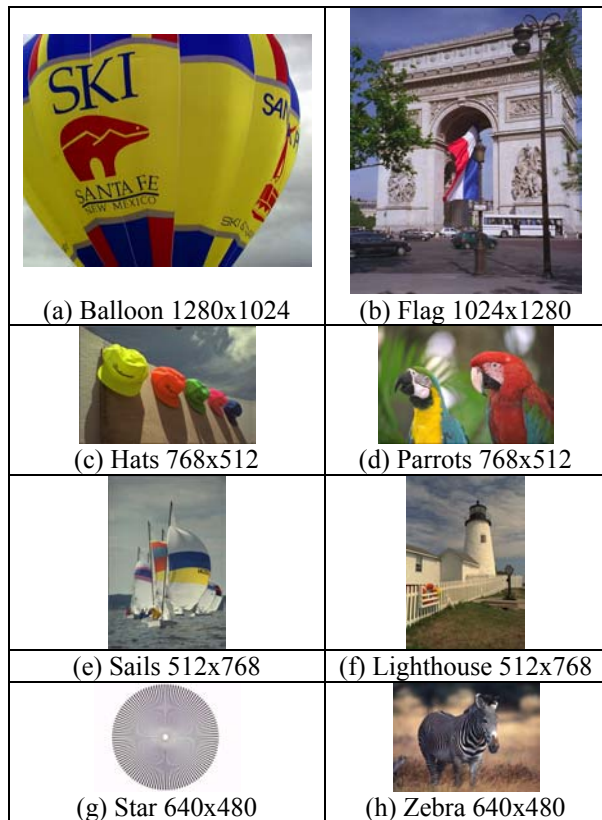


Fig. 2: Test Images used in the experiment.

## 3. Experimental Results

In order to demonstrate the proposed up-sampling method, we have used a large set of color images with different image size and content. Some of the representative results are shown in Figs. 2 and 3. The test set here includes two large size images (Balloon

and Flag), four natural mid-size images from Kodak Photo CD [10] (Hat, Parrots, Sails and Lighthouse), and two small-size images (Star and Zebra) with a lot of edges.

The test images are first down sampled by a factor of two in both vertical and horizontal directions. A simple sub-sampling method is used to down-sample these test images. Hence we intentionally dropped three fourths of the data from the original image. The resultant down-sampled images were then used as inputs for the proposed algorithm which up-sampled them to their original size. These up-sampled images are then compared with the original images. We also up-sampled the same image using bilinear and bicubic interpolation, to use for comparison with our results. As we can see in Fig. 3, the proposed DWT based up-sampled Zebra image, Fig. 3 (d), has much better visual quality with sharper edges as compared to the bilinear and bicubic interpolation results. We observed similar behavior with many other challenging images as well.

Table I: PSNR and  $\Delta E_{ab}^*$  comparisons

		PSNR (dB)			$\Delta E_{ab}^*$
		R	G	B	
Balloon	BL	30.14	30.22	33.14	2.67
	BC	30.09	30.29	33.05	2.70
	DWT	<b>32.07</b>	<b>33.02</b>	<b>34.53</b>	<b>2.27</b>
Flag	BL	26.95	26.68	26.70	2.27
	BC	26.83	26.56	26.59	2.33
	DWT	<b>28.11</b>	<b>27.88</b>	<b>28.04</b>	<b>2.07</b>
Hats	BL	30.05	<b>30.04</b>	<b>30.38</b>	<b>1.56</b>
	BC	29.91	29.86	30.06	1.65
	DWT	<b>30.07</b>	29.97	29.89	1.62
Parrots	BL	29.96	29.95	30.83	1.51
	BC	29.91	29.92	30.74	1.58
	DWT	<b>30.63</b>	<b>30.60</b>	<b>31.68</b>	<b>1.42</b>
Sails	BL	28.26	28.09	28.52	1.59
	BC	28.10	27.98	28.34	1.66
	DWT	<b>28.87</b>	<b>28.80</b>	<b>28.93</b>	<b>1.55</b>
Lighthouse	BL	25.26	<b>25.30</b>	<b>25.67</b>	<b>2.43</b>
	BC	24.99	24.98	25.31	2.56
	DWT	<b>25.38</b>	25.25	25.48	2.52
Star	BL	18.15	17.67	18.23	4.68
	BC	18.19	17.72	18.28	4.72
	DWT	<b>20.27</b>	<b>19.98</b>	<b>20.42</b>	<b>4.17</b>
Zebra	BL	22.87	22.89	22.92	<b>3.26</b>
	BC	22.60	22.63	22.63	3.61
	DWT	<b>24.15</b>	<b>24.17</b>	<b>24.06</b>	3.38

Two commonly used objective measures, peak signal-to-noise ratio (PSNR) and CIELAB  $\Delta E_{ab}^*$  [11], are used to evaluate the quality of the up-sampled images. Table I shows the PSNR and  $\Delta E_{ab}^*$  comparisons between the proposed DWT based up-

sampling, bilinear, and bicubic interpolation techniques. It is evident from Table I that the proposed method outperforms the bilinear and bicubic interpolations in most cases. The four nature images from Kodak Photo CD (Hat, Parrots, Sails and Lighthouse) have more compatible results between the three different up-sampling methods due to the fact that there is less sharp edge content in those images.

#### 4. Conclusions and Future Works

In this paper, we proposed an interesting up-sampling algorithm based on the Discrete Wavelet Transform method. The proposed up-sampling method yields much better visual quality as compared to the current state of the art in the literature. We would like to further explore the impact of different scaling factors and wavelet filters for possible image sharpening and enhancement in our future studies.

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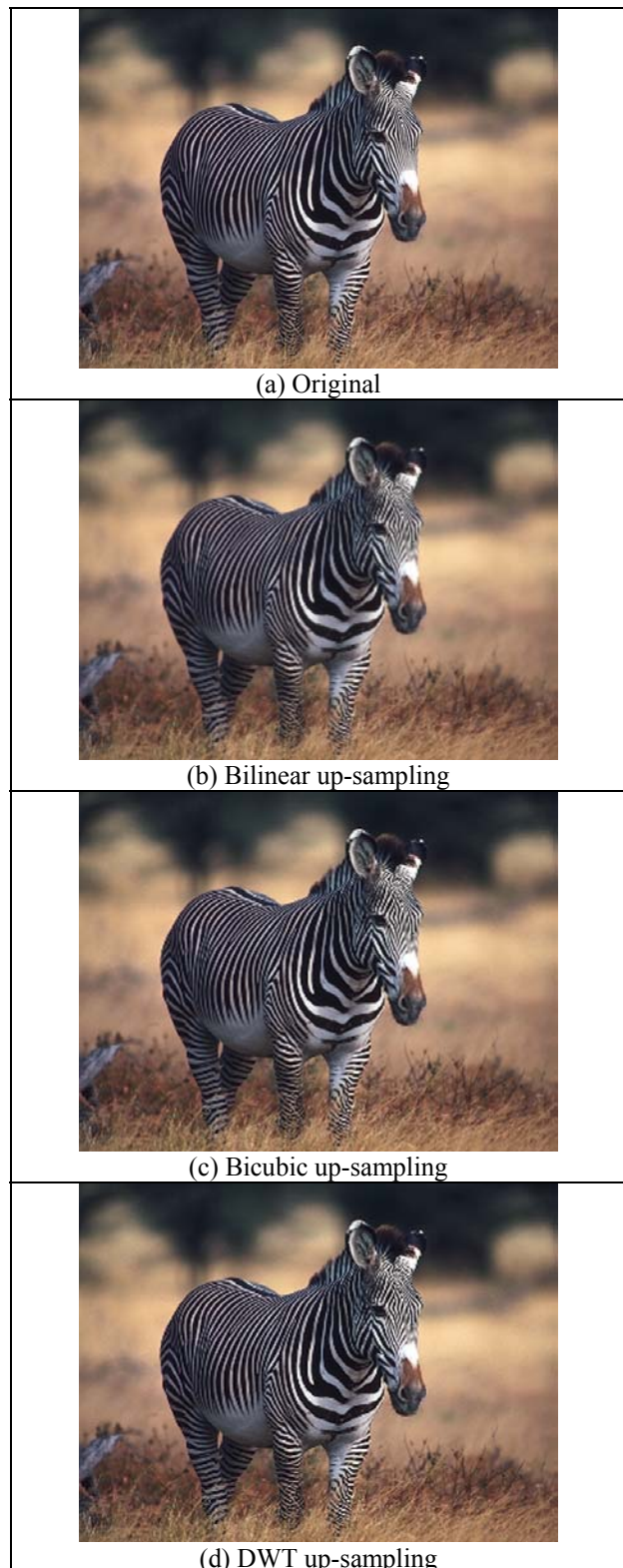


Fig. 3: Experimental results for ZEBRA image.