The Prediction for Index Futures Returns and the Relational Analysis of Spillover Effect among American and Eurasian Markets with the Grey Theorem

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Abstract
The grey theory is mainly uncertainty directed against the systematic model and fit for incomplete information. This paper adopts the grey prediction methods, GM(1,1) and GM(1,1 | optimal α), to investigate the return and volatility of major index futures among American and Eurasian markets. The grey relational theory and GM(1,N) model are further used to observe the volatility spillover effect and find the main influence factor in the volatility relatedness about the rate of returns of index futures. In terms of the prediction error, it is exhibited that GM(1,1) is not good for prediction. It also reveals by GM(1,N) that the rate of daily return of Dow Jones is the main influence factor on other index futures rate of returns. In conclusion, the American market firmly dominates global stock markets and forward markets.

Keywords: Spillover Effect; Grey Relational Theory; Grey Prediction

1. Introduction
Solinik[1] proposed the international assets pricing model, which insists that the lower international stock market correlation coefficient appears, the higher the possibility of utilizing cross-national investment to evade the risk even to carry on the arbitrage reveals. As a result, the studies of the dependence of mutual stock markets in various countries have become an important research theme.

The magnitude of literatures concerning about the prediction of stock prices, the rate of returns of stock index, and the volatility spillover effect of the rate of returns of stock price are beyond our imagination[1-3][7-8][10-13].However, the past researches are mostly aimed at the fluctuation of the stock market and price transmission effect between spot and forward market etc. In addition, the research about time series, although Genetic Algorithm[13], Neural Networks[2][13], and GARCH[8][11]are adopted and also obtain a good result, it is rare to apply with grey model theory of artificial intelligence in big range step concerning about the intercontinental rate of returns of stock price index futures analysis. The proposed aims of this research might appear in two dimensions:(1), utilizing GM(1,1) and GM(1,N) model to predict the cross-continental / cross-national volatility of rate of returns of the index futures and the relational analysis of the fluctuation among them; and (2), giving consideration to the global three major markets and regional rate of returns of index futures. In American markets, we cover the Dow-Jones Industrial Average (DJ) index futures, Nasdaq (NDQ) index futures, and S&P500 (SP) index futures for studies. In European market, we pull in the 100 kinds of indexes of Financial Times of London (FTSE) and France CAC40 (CAC) indexes futures. In Asian market, Nikkei225 (NK) index futures, the Taiwan Weighted Stock Index futures (TX), the financial insurance type stock index futures of Taiwan (TF), and the electronic type stock index futures of Taiwan (TE) are analytic targets.

2. Research Methods
From January 1, 2002 to December 22, 2004, and after deleting the data on different bargain day among three major markets, we obtain 628 daily closing prices of major stock price index futures of America, Europe and Asia that offered by Polaris Refco Futures. The research procedures are as follows: at first, to transform the daily closing price of stock price index futures into the rate of returns per day (the index futures closing price of the calculated day divided by that day before calculated), and then make a unit root test to prove whether the major daily rate of return of index futures are stable enough. After the above-mentioned test, we adopt GM(1,1) including residual check, rolling check and optimal GM(1,1 | ) models for prediction on the volatility of rate of returns of index futures, and compare the prediction performance among them during Dec 1 and Dec 22, 2004 (out-of-sample). At the same time, we observe the above-mentioned volatility spillover of the rate of return of nine stock prices index of three continents with grey relational theory. Finally, with GM(1,N)[4] model, we try to find out the influence factors among them, and offer a brief analysis and conclusion.

2.1. Unit Root Test
Before analyzing the time series, it’s necessary to examine the daily rate of returns of index futures appear to be with the nature of stationary. The statistical hypotheses are described as follows: null hypothesis H0:there are some root results existed; alternative hypothesis H1: there are no root results

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2.2. Grey Prediction Theory

The grey theory can deal with the system relational analysis and the model construction. Moreover, it is good for both prediction and decision making to find out about systematic situation (Kun-Li Wen[6] et al.). This research will separately utilize grey GM(1,1) to carry on the volatility prediction and GM(1,N) model, which is a commonly used method for grey model construction, and for the related analysis among major rate of returns of index futures.

2.2.1. GM(1,1) Model

GM(1,1) model means first differential and one variable, which is generally used in making prediction.

\[ x^{(0)}(k)+ax^{(1)}(k)=b, \quad k=1,2,3,\ldots,n \]

a: Development coefficient;  
b: Grey function coefficient; Generally we use the least squares method to estimate parameter a, b;  
z: Background value.

\[ x^{(1)}(k)=\frac{1}{2}[x^{(1)}(k)+x^{(1)}(k-1)], \quad k=2,3,\ldots,n \]

The prediction formula common in use is

\[ x^{(0)}(k+1)=(1-e^a)(x^{(0)}(1)-\frac{b}{a})e^{ak} \]

2.2.2. GM(1,N) Model

GM(1,N) model means first differential and N variables and is generally used in multi-variable analysis field. In addition, the GM(h,N) model does not exist when h>=2.

\[ x_i^{(0)}(k)+az_i^{(1)}(k) = \sum_{j=1}^{N} b_{ij} x_j^{(1)}(k), \quad k=1,2,3,\ldots,n \]

a: Development coefficient; b: Gray function coefficient; Generally we use the least squares method to estimate parameter a, b.

The model of GM(1,N) is analyzed as follows: (Kung-Li, Wen)[5]

1. Set up original sequences

\[ x_i^{(0)}(0)=x_i^{(0)}(1), x_i^{(0)}(2), x_i^{(0)}(3),\ldots,x_i^{(0)}(k) \];

\[ x_i^{(0)} \] is a non-negative time series of n periods.

Where: i=1,2,3,...,N; k=1,2,3,...,n

2. Set up accumulated generating operation (AGO)

\[ AGO_1(x^{(0)}(k))=x^{(1)}(k) \]

\[ =\left[ \sum_{h=1}^{k} x^{(0)}(h) \right] \left[ \sum_{h=2}^{k} x^{(0)}(h) \right] \ldots \left[ \sum_{h=n}^{k} x^{(0)}(h) \right] \quad (2.a) \]

\[ x_{ij}^{(1)}=x_{ij}^{(1)}(1), x_{ij}^{(1)}(2), x_{ij}^{(1)}(3),\ldots,x_{ij}^{(1)}(k) \] -----(2.b)

where: j=1,2,3,...,N; k=1,2,3,...,n

3. Combining the AGO sequences with the major sequence

\[ x_{i}^{(0)}(k)+az_{i}^{(1)}(k) = \sum_{j=1}^{N} b_{ij} x_{j}^{(1)}(k) \] -----(3.a)

where: z_{i}^{(1)}(k)=0.5x_{i}^{(1)}(k)+0.5x_{i}^{(1)}(k-1), k>=2

4. Substituting all AGO values into equation (2.b), we have

\[ x_{i}^{(0)}(k)+az_{i}^{(1)}(k)=b_{2}x_{2}^{(1)}(k)+\ldots+b_{n}x_{n}^{(1)}(k) \]

where: k=1,2,3,...,n

5. Using the inverse and matrix method to find the values of b_{N} with the method as follows:

\[ a=(B'B)^{-1}B'Y_{N} \]

6. The relationship between the major sequence and the influencing sequences can be found by comparing the value of b_{N}

2.2.3. Error Analysis of GM(1,1) Prediction Model

(1) Traditional formula

The GM(1,1) model is defined as

\[ e(k)=| X^{(0)}(k)-\hat{x}^{(0)}(k) | x^{(0)}(k) \| \times 100\% \]

\[ \hat{x}^{(0)}(k) \] absolute value

\[ x^{(0)}(k) \] true value; \( \hat{x}^{(0)}(k) \) predicted value.

(2) Optimal a (The Error Analysis Toolbox)

To reduce the calculation time for the purpose of decreasing error analysis, this research uses Optimal a toolbox ushered by Wen[5].

2.2.4. GM(1,1) Rolling Model

In Liu[12] and Tsai’s[9] research, the grey rolling model was proposed as follows: Each time as the prediction model is built up, it is important to replace and re-arrange backward information with forward data into sequence.

2.3. Grey Relational Analysis

There are some commonly used quantitative methods in system analysis, including regression analysis, relevance analysis, principal component analysis, and factor analysis. The common characteristics in fore-cited methods require large sample size, and have a typical probability distribution, but sometimes there are some difficulties to be encountered in real problem. Moreover, the grey relational analysis is not restricted by preceding paragraphs, it can get the relatedness at random factor sequences under the incomplete information through data processing among factors. In this research, the localization grey relational analysis has been employed.

The steps of grey relational analysis are as
follows:

1. Define the inspected sequences and the reference sequence.

Let \( x_0 = (x_0(k); k=1,2,\ldots,n) \) reference sequence; and \( x_i = (x_i(k); k=1,2,\ldots,n) \) inspected sequences.

2. Data preprocessing: the process method includes initial value, maximization, minimization, average, and intervalization.

3. To find the difference sequences \( \Delta_0(k) \) among the inspected sequences and the reference sequence.

\[
\Delta_0(k) = |x_0(k) - x_i(k)|; i=1,2,\ldots,m; k=1,2,\ldots,n
\]

4. To find the max. and min. \( \Delta_0(k) \).

\[
\text{max. } \Delta_0(k) = \text{Max}_i \text{ Max}_k \Delta_0(k);
\]

\[
\text{min. } \Delta_0(k) = \text{Min}_i \text{ Min}_k \Delta_0(k).
\]

5. Taking distinguishing coefficient \( \zeta \) (traditional value equals to 0.5, and it can also be adopted by fuzzy method); The change of the distinguishing coefficient affects only the size of relative value, no more than to say the influence on the rank of the grey relational grade (Kun-Li Wen[6] et al.).

6. Calculating grey relational coefficient

\[
\gamma(x_0(k),x_i(k))\text{ by using Deng’s formula.}
\]

\[
\gamma(x_0(k),x_i(k)) = \frac{\Delta_0 + \zeta \Delta_{\text{max}}}{\Delta_0 + \zeta \Delta_{\text{max}}}
\]

7. Calculating grey relational grade(based on equal weights).

\[
\gamma(x_0,x_i) = \frac{1}{n} \sum_{k=1}^{n} r(x_0(k),x_i(k))
\]

8. Ranking the rank according to the magnitude of the grey relational grade.

### 3. Experimental Results

#### 3.1 Localization Grey Relational Analysis

Only one sequence \( x_0(k) \) is defined as the reference sequence, and the others are named as the inspected sequences. The findings are summarized in Table 1 (Ranking the first five influence factors in row and noting with V).

<table>
<thead>
<tr>
<th>Influence factor</th>
<th>Return of DJ</th>
<th>Return of CAC</th>
<th>Return of FTSE</th>
<th>Return of NK</th>
<th>Return of TX</th>
<th>Return of TE</th>
<th>Return of NDQ</th>
<th>Return of SP</th>
<th>Return of TF</th>
<th>Return of MAX*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return of DJ*</td>
<td>--</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Return of CAC*</td>
<td>V</td>
<td>--</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Return of FTSE*</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Return of NK**</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Return of TX*</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Return of TE*</td>
<td>V</td>
<td>--</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Return of NDQ***</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Return of SP***</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>Return of TF*</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
<td>V</td>
<td>V</td>
<td>V</td>
<td>--</td>
<td>--</td>
</tr>
</tbody>
</table>

*: The result (when \( \zeta \) equals to 0.5) is as same as the fuzzy model.

**: When the traditional way \( \zeta=0.5 \) is adopted, the result is not the same as the fuzzy method, that is, MAX,CAC,TE,TX,FTSE have significant return spillover effect on NK.

***: When the traditional way \( \zeta=0.5 \) is adopted, the result is not the same as the fuzzy method, that is, DJ,SP,FTSE,CAC,MAX have significant return spillover effect on NDQ.

#: Return of MAX is the maximum daily rate of return of index futures among the American and Eurasian markets.

### 3.2. Grey Model Construction

#### 3.2.1 The performance analysis of GM(1,1) prediction model (Referred in Table 2)

Table 2. Prediction performance of GM(1,1) model

<table>
<thead>
<tr>
<th>Predicted target</th>
<th>Prediction method</th>
<th>Optimal alpha, average error(%)</th>
<th>Residual check, average error(%)</th>
<th>Rolling check(5 points), average error(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return of DJ</td>
<td>24.137025</td>
<td>139.8005</td>
<td>91.858</td>
<td></td>
</tr>
<tr>
<td>Return of CAC</td>
<td>21.573325</td>
<td>53.1463</td>
<td>62.5179</td>
<td></td>
</tr>
<tr>
<td>Return of FTSE</td>
<td>22.85335</td>
<td>143.6674</td>
<td>1,320.71</td>
<td></td>
</tr>
<tr>
<td>Return of NK</td>
<td>34.9357</td>
<td>996.0035</td>
<td>308.0326</td>
<td></td>
</tr>
<tr>
<td>Return of TX</td>
<td>94.449475</td>
<td>1,693.26</td>
<td>419.8276</td>
<td></td>
</tr>
<tr>
<td>Return of TE</td>
<td>33.4687</td>
<td>155.884</td>
<td>185.2021</td>
<td></td>
</tr>
<tr>
<td>Return of NDQ</td>
<td>503.9656</td>
<td>996.0035</td>
<td>2,775.45</td>
<td></td>
</tr>
<tr>
<td>Return of SP</td>
<td>97.120375</td>
<td>1,038.79</td>
<td>244.7539</td>
<td></td>
</tr>
<tr>
<td>Return of TF</td>
<td>73.6585</td>
<td>322.6551</td>
<td>68.4565</td>
<td></td>
</tr>
</tbody>
</table>

2. GM(1,N) Model---The detailed findings are shown in Table 3.
Table 3. Main influence factor analysis of GM(1,N) Model (The priority is ranked 1,2,3…in row according to the impact scale on main/reference sequence.)

<table>
<thead>
<tr>
<th>Main influence factor</th>
<th>Return of DJ*</th>
<th>Return of CAC</th>
<th>Return of FTSE</th>
<th>Return of NK**</th>
<th>Return of TX</th>
<th>Return of TE</th>
<th>Return of NDQ</th>
<th>Return of SP</th>
<th>Return of TF</th>
<th>Return of MAX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Return of DJ</td>
<td>--</td>
<td>6</td>
<td>2</td>
<td>8</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>7</td>
<td>9</td>
</tr>
<tr>
<td>Return of CAC</td>
<td>1</td>
<td>--</td>
<td>2</td>
<td>9</td>
<td>6</td>
<td>5</td>
<td>8</td>
<td>4</td>
<td>7</td>
<td>3</td>
</tr>
<tr>
<td>Return of FTSE</td>
<td>1</td>
<td>5</td>
<td>--</td>
<td>6</td>
<td>4</td>
<td>8</td>
<td>9</td>
<td>2</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>Return of NK</td>
<td>1</td>
<td>6</td>
<td>3</td>
<td>--</td>
<td>7</td>
<td>4</td>
<td>9</td>
<td>5</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Return of TX</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>8</td>
<td>--</td>
<td>6</td>
<td>9</td>
<td>2</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Return of TE</td>
<td>1</td>
<td>7</td>
<td>3</td>
<td>9</td>
<td>4</td>
<td>--</td>
<td>6</td>
<td>2</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Return of NDQ</td>
<td>1</td>
<td>8</td>
<td>3</td>
<td>9</td>
<td>4</td>
<td>6</td>
<td>--</td>
<td>2</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>Return of SP</td>
<td>1</td>
<td>9</td>
<td>3</td>
<td>6</td>
<td>2</td>
<td>8</td>
<td>7</td>
<td>--</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Return of TF</td>
<td>1</td>
<td>8</td>
<td>4</td>
<td>9</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>2</td>
<td>--</td>
<td>6</td>
</tr>
</tbody>
</table>

4. Conclusion

The research shows that the forecast errors on the cross-continental / cross-national rate of daily returns of index futures by adopting GM(1,1) model is significant from 21.573325% to 2,775.45%, which implies that the prediction performance of GM(1,1) is not good (referred in Table 2).

Furthermore, in terms of GM(1,N) model, it reveals that DJ* index futures has the most significant influence/return spillover effect. On the contrary, NK** index futures has almost the least significant influence/return spillover effect. In addition, the return of NDQ has the greater impact on DJ, but fail to affect the other major rate of returns of index futures (shown in Table 3).

The above results can be contrasted to the Theodossiou and Lee’s[8] findings, that is the American stock market has mean spillover effect on the global stock market such as U.K., Canada and Germany. In a word, the American market is proved to be dominant in global stock and forward markets.

REFERENCES