

Enhanced Competitiveness of Multi-Agent System by Special Algorithm

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Abstract

Resource allocation in the econophysics of competing supermarket chains, modelled as a Multi-Agent System is addressed in the context of finding good method to increase the competitiveness of the company in networked economy. A special swapping algorithm which increased the local clustering effects of one Multi-Agent System is introduced and numerical work using Monte Carlo simulation demonstrate its effectiveness in enhancing the speed of market dominance as well as increasing the probability of dominance. Application of this model can be found in the problem of resource allocation of two supermarket chains competing for dominance in a hexagonal lattice, using the statistical mechanics of Ising spin.

Keywords: Multi-Agent Systems, Resource allocation, Strategy, Econophysics.

1. Introduction

The resource allocation problem is an interesting topic of econophysics. The problem considers the competition of two supermarket companies in a metropolis for dominance with the fastest speed [1,2], while maintaining a high probability of dominance. In our previous model of competing supermarket chain, the two companies compete for space in shopping mall to open their branches. A simple model to describe the dynamics makes use of the language of multi-agent system where the interaction of different supermarket branches is described by local spin model in statistical mechanics. For two competing supermarket chains, as there are two companies, a natural spin model to use is the Ising spin model [3]. To map the resource allocation problem to the Ising model, we assumed the metropolis is defined on a hexagonal lattice, though further investigation indicates that our conclusion for hexagonal lattice is quite general. In this lattice, each site represents a shopping mall, which can only accommodate one of the two supermarket chains. The company to which the supermarket belongs is

represented by the spin type of the site. The behaviour of supermarkets and Ising spins are similar. An Ising spin will be stabilised if a parallel spin type is in contiguity, and will be destabilized if an anti-parallel spin type is adjacent. Two adjacent supermarket branches of the same company will cooperate favourably, while two adjacent branches of different companies have to compete for customers and try to overrun each other. This leads to the introduction of the ferromagnetic Ising model which is discussed in section 2. The basic tool to evaluate the initial configurations is Monte Carlo simulation [4]. Our approach to the problem consists of three steps:

- (I) generate a random sample of initial configurations and evolve them with Monte Carlo simulation till one of the competing companies achieved market dominance, defined by a fixed market share of some value greater than 50%.
- (II) apply the swapping algorithm (which is discussed in section (4) to the same set of initial configuration for the targeted company.
- (III) evolve the processed initial configurations with Monte Carlo simulation again and observe how they are affected.

We compared the probability and speed for the same company to achieve dominance before and after step (II). It was found that the target company has a higher probability and faster speed to achieve dominance after the special algorithm is applied.

2. Definition of model

The resource allocation problem can be well modelled by the ferromagnetic Ising model. Let's consider an $N \times N$ hexagonal lattice with periodic boundary condition. We see the whole lattice as a metropolis, and every site on the lattice is interpreted as a shopping mall. We assume one and only one supermarket can be opened in each shopping mall and also there are only two supermarket companies in the model so as to simplify the problem. Each site i is associated with a spin x_i , which value can be either +1 or -1, also known as spin up or spin down,

representing the two competing companies. The interaction between the pairs of nearest neighbours is described by the energy E :

$$E = -\sum_{i>j} J_{ij} x_i x_j \quad (1)$$

where J_{ij} is one if i and j are nearest neighbour, and zero otherwise. If we interpret E as the cost of operating all the supermarkets of both companies, then a pair nearest neighbours will have their operation cost reduced because of cooperative benefit. On the other hand two competing nearest neighbours will require extra spending to attract customers and so they have a positive contribution in E . We note that energy in physics associated with the local spin configuration is interpreted as the cost of setting up such local configuration of branches of the two supermarket chains.

Apart from the interaction energy we also need to specify the initial configuration. Since we want to investigate the effect of various intelligent strategies for initial spin configuration, we assume an equal share of the market initially and seek for special spatial arrangement of resource of one company that will lead to dominance after evolution in a fair game, which we model by the dynamics of Monte Carlo method.

3. Monte Carlo Simulation and the Definition of Dominance

In the resource allocation model the Monte Carlo method is used to simulate the competition in the real world. In each Monte Carlo step, we first randomly pick a site i on the lattice. We consider the change in the interaction energy, ΔE , associated with the switching of the spin x_i . If $\Delta E \leq 0$ then the spin x_i will be switched. Otherwise spin x_i will be switched with the probability P_i

$$P_i = e^{-\Delta E/kT} \quad (2)$$

where k is the Boltzmann constant and is set to 1. T is the temperature of the lattice which account for the noises of the environment, such as, governmental restrictions, advertisement or any other perturbations with the character of thermal noise. We set $kT = 1$ for the model throughout the Monte Carlo simulation. This value is sufficiently low to guarantee that one of the two spins types will be able to achieve market dominance within a reasonable number of Monte Carlo steps [1,2,5]. If kT is set higher than a critical temperature, then the noise level is too high and no competitor can achieve market dominance. This is an undesirable situation as we cannot make any statement

on the strategic allocation of resource in the initial configuration, as any allocation lead to equal chance of market dominance.

After many steps of Monte Carlo simulation on a given initial configuration, one competitor will have market share more than 50%. Here we assume that once a competitor has 87.5% share of the market, it will eventually dominate (100% market share) the opponent, as it is very unlikely for the losing company to counter strike and so we stop the Monte Carlo simulation. Sometimes it is possible, though rarely happens, that neither company dominates: they have entered a deadlock and we declare a draw. We stop the Monte Carlo evolution in this case of a draw when the number of Monte Carlo steps reaches 400,000.

4. Special Algorithm to Enhance the Competitiveness of one Company

We have discussed the process involve in simulation and specified the restrictions of the initial configurations. If we generate the initial configurations randomly, we would expect that the probability for one spin type to achieve dominance is the same as the other spin type. Our motivation is to seek an intelligent way of arranging resource to enhance the winning probability of one targeted spin types. Let's denote the targeted spin type to be spin up.

Next, we consider the effect of local clustering of the spin up on winning probability. We choose this feature to investigate because it is natural to consider cooperation between same spin type will strengthen the alliance against hostile takeover. To begin, we first define the number of triangle of spin up, T^U , to be the number of groups of three sites that are all nearest neighbour (forming "a triangle of friends") and have spin up type. Similarly, we denote the number of spin down, T^D , for the number of groups of three sites that are all nearest neighbour. Also let's denote the difference in number of triangle to be $G = T^U - T^D$. Now, we define our swapping algorithm. In each step of the swapping algorithm we first choose randomly two sites with one spin up and one spin down. (These two sites need not be nearest neighbours). If we switch the spin type of this pair of sites we will end up in a new configuration with a new difference in number of triangle: G' . Define $\Delta G = G' - G$. A criterion to activate the spin swapping is that G' should be larger than G so that there are more local clustering for spin up. Thus, we swap spin if $\Delta G \geq 0$, otherwise we record this as an unsuccessful swap. When the number of unsuccessful swap is too high, say, above a preset number F , then we stop the swapping algorithm.

5. Result and Discussion

In our simulation we set $N = 20$ and $F = 400$. At the onset of simulation we generated 100 random configurations, and for every configuration we evolve them with Monte Carlo simulation for 1000 times. Excluding all those cases that entered a deadlock we compute the probability p for spin up to achieve dominance for the configuration. We have also measured the average speed V to dominance for the case when spin up wins, $\langle V \rangle_U = 1/t_U$, where t is the Monte Carlo steps elapsed before dominance is achieved and $\langle \rangle_U$ refers to the average taken over those cases among 1000 simulations where spin up has achieved dominance. Each site i is associated with a connectivity, C_i , which is defined as the number of its neighbouring site that has the same spin type as i 's. The distribution of connectivity of spin up is then the probability distribution function of the element of the set $\{C_i | x_i = I\}$. The distribution of connectivity of spin down is defined similarly.

After generating and measuring p and $\langle V \rangle_U$, of interest on the random initial configurations, we repeat our numerical experiment on initial configurations after application of swapping algorithm. We measure p and $\langle V \rangle_U$ again on the 100 enhanced configurations and compare the two sets of results in Table 1. We see that our special algorithm which maximizes G can increase the difference of the competitiveness of the two spins. This is intuitive as a larger difference G in number of triangle leads to higher level of local cooperation, which globally leads to higher chance of dominance. Fig. 1 displays the distribution of connectivity for both spin types before and after the application of our swapping algorithm. We can observe that spin up after the application of our swapping algorithm, which becomes the winning spin with higher probability, has a broader distribution compare with its counterpart. This broad distribution can serve as a signature of the winner.

We have observed in our simulation that forming triangle is an essential point for a spin type to defeat its counterpart in the pursuit of market dominance. To apply our result in the real world a supermarket company should open its branch strategically in order to form more triangles and try to destroy the triangles of its competitor. We see also from this simple model the virtue of strategic allocation of resource. Once we recognize the value of local clustering effect, here measured by G , we can design algorithm to generate good initial configuration: good in the sense of high probability of market dominance and fast dominance in a fair competitive world, as modelled here by Monte Carlo evolution. K.Y. Szeto acknowledges the

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Table 1: The probability and speed to dominance of spin up before and after the application of the special algorithm

Before the application of the special algorithm		After the application of the special algorithm	
p	$\langle V \rangle_U$	p	$\langle V \rangle_U$
0.50	0.036	0.55	0.041
± 0.07	± 0.008	± 0.07	± 0.010
G		G	
0 ± 8		74 ± 5	

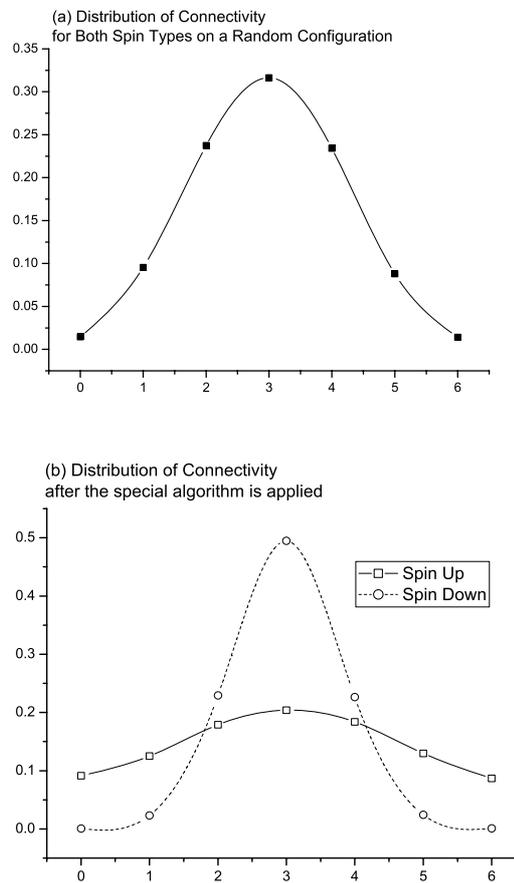


Fig. 1: (a) Distribution of Connectivity of either spin type on a random configuration. (b) Distribution of Connectivity of the two spin types after the application of the special algorithm.

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