Research on Route Optimization Problem of Road Transportation

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Abstract. The main research object of the paper is route optimization problem of road transportation, which focuses on motorcade transportation. It mainly includes: analysis of factors influence route optimization, establishing the model of motorcade transportation route optimization, researching the solution of route optimization algorithm, designing algorithm approaches, and etc.

Introduction

The route optimization problem is always a key research point of in computational science, operational research and geographic information science.

For road transportation, to satisfy the request of timing and economy, we need to choose the route with shortest distance, minimum diving time, minimal fuel consumption and minimal overall cost. And that research on choosing reasonable route is called route optimization problem of road transportation. Usually, it was shortened to be called route optimization problem or shortest route problem.

And for the problem of transportation time optimization and cost optimization, it usually achieves the algorithm realization by using basic method and thought of solving shortest route problem. And it needs the constraint conditions or weight process, and the support of low-level static or dynamic data. Many experts and scholars in the world study into this problem, but always narrow the problem as simple route problem and seldom consider the motorcade organization problem and road capacity. This paper mainly researches the route optimization during the process when motorcade organize road transportation. In this paper, we call it motorcade route optimization problem for short.

The definition of shortest path

For convenience, firstly, we give the definition of shortest route of traffic network.

A road network which consists of nodes and directed road sections is \( G = (V, A) \). \( V \) is the set of nodes in road network, \( V = \{v_1, v_2, \ldots, v_n\} \); \( A \) is the set of directed road sections in road network, \( A = \{a_1, a_2, \ldots, a_m\} \). For every directed road sections \( a \in A \), its starting point and terminal point is \((v_s, v_t)\), and the “impedance” is signed as \( w(a) = w_{ij} \). In traffic network, the “impedance” can be distance, time, cost and etc. Given the 2 nodes \( v_s, v_t \) in \( V \), and \( p \) is a route with \( v_s \) as the starting point and \( v_t \) as the terminal point. And we define the impedance of route \( p \) is the sum of road segments which make up of \( p \), and we signed it as \( w(p) = \sum_{a \in p} w(a) \). So we can describe the shortest route problem as below statement: in all of the routes with \( v_s \) as the starting point and \( v_t \) as the terminal point, we need to find the route \( p_0 \) which has minimum impedance, and make \( w(p_0) = \min_p w(p) \), then we call that \( p_0 \) is the shortest route from \( v_s \) to \( v_t \), the impedance of route \( p_0 \) is called the distance of shortest route from \( v_s \) to \( v_t \), and we sign it as \( d(v_s, v_t) \).
The classification of shortest path

In graph theory, generally speaking, shortest route problem can be divided to 3 types as below:

- To get the shortest route between 2 appointed nodes in road network;
- To get the shortest route between an appointed nodes and any other nodes in road network;
- To get the shortest route between any 2 nodes in road network.

For the first 2 types, the most common algorithm is mark-number method. The mark-number method is a general term of shortest route that we number the nodes, and gradually correct it by iteration. And the mark-number method can be classified into 2 kinds: mark-number setting and mark-number correction. The mark-number setting is the method that a node will be moved from temporary mark-number set to permanent mark-number set each time in iteration process of gradual correction. Usually, it is used to solve the shortest route problem that all road segment cost is nonnegative. The most typical mark-number setting method is Dijkstra algorithm; By contrast, the mark-number correction is the method that a node won’t be moved from temporary mark-number set to permanent mark-number set each time in iteration process, but just be corrected at that time. When all iteration are completed, all the mark-number of the nodes will be changed to permanent mark-number at the same time. The mark-number correction method can be used to solve the shortest route problem with some negative cost. The most typical mark-number correction method is Bellman-Ford algorithm.

For the third type problem, the common method is matrix iteration. By using this method, distance between any 2 nodes is specified by matrix elements, and shortest route will be worked out by gradually correcting the matrix elements in matrix calculation. The typical algorithms are Floyd algorithm and matrix multiple algorithm.

The model of motorcade route optimization

In practice, to make it easier to administrate and control all course of the transportation, or to enhance security in transportation, we always need formation as motorcade. In this case, we faced with new problem about the choice of shortest route. The solution for traditional shortest route is based on single vehicle. If we use this method on motorcade, it will cause the problem that the motorcade choose the road segment with bad road capacity. This will take more time to pass that road segment, reduce efficiency and increase transportation cost.

The factor influencing motorcade road passing time

The factor influencing motorcade road processing time may be viewed in below 3 categories. 

Road traffic capacity

Road traffic capacity refers to maximum number of vehicles passing a section of a road per unit time. The measuring unit is quantity of vehicles/hour. Road traffic capacity is a kind of performance, a physical quantity measuring road traffic dispersion. When traffic is overload (beyond the road traffic capacity), it will be congested even be traffic jam. Road traffic capacity is an important road index, the basic data which is necessary to road plan and design, and it’s one of specific parameters of traffic control.

According to the nature and using requirement of road traffic capacity, it is divided into basic traffic capacity, possible traffic capacity and design traffic capacity.

Basic traffic capacity (also called theoretical traffic capacity) refers to maximum quantity of standard vehicles passing one-lane per hour in ideal state of road, traffic control and circumstance, whatever the service level. The formula is as below:

\[ N = \frac{3600}{h_i} = \frac{1000V}{L + \frac{V}{3.6}t + 0.00394 \frac{V^2}{\phi}} \]  

(1)

\( h_i \) : Minimum safety headway of road running vehicles (s);
Driving speed of road running vehicles (km/h);  
Average length of vehicles (m);  
Adhesion coefficient between vehicles and road;  
Reaction time of driver (s).

Possible traffic capacity refers to maximum quantity of standard vehicles passing one-lane per hour in actual state of road, traffic control and circumstance, whatever the service level. The formula is as below:

Design traffic capacity refers to traffic capacity of one-lane in its service level in actual state of road, traffic control and circumstance. For the same lane, every service level has different traffic capacity.

In some cases, road traffic capacity also depends on traffic density of artificial regulation. Traffic density is the vehicles density degree on one lane, that is to say, it is the quantity of vehicles in one lane for unit length at one instant of time.

**Road passing time of single vehicle**

Road passing time for single vehicle depends on 2 variables: average speed \( \bar{V} \) and road length \( L \). The road passing time of single vehicle for Road section \( i \) is:

\[
t_i = \frac{L_i}{\bar{V}_i}
\]

(2)

**Motorcade passing time for a section**

Motorcade passing time for a section is the time that all the vehicles in motorcade (from first one to last one) can pass the road section. According to the definition,

\[
t'_i = Qh_i = \frac{Q}{C_i}
\]

(3)

\( t'_i \): Motorcade passing time for section \( i \) (unit: hour);  
\( Q \): The vehicles quantity of motorcade;  
\( h_i \): Average headway on section \( i \);  
\( C_i \): Road traffic capacity of section \( i \).

And \( C_i = n_iN_i \), \( n_i \) is the lanes available of section \( i \), \( N_i \) is the traffic capacity of single lane on section \( i \). The motorcade is always by a single lane road, so \( n_i = 1 \), \( C_i = N_i \).

**Setting motorcade route optimization model**

According to above influencing factor, we can know that motorcade section passing time consists of 2 pats: road passing time of single vehicle and motorcade passing time for a section. So we can set the passing time function of motorcade for a section:

\[
T_i = \frac{Q}{C_i} + t_i
\]

(4)

\( T_i \): Passing time on section \( i \); and other parameter is the same as above.

However, the passing time when motorcade all passes route \( p \) is not the just addition of the passing time for all section. When the motorcade passes the previous section, at the same time, it’s also passing the latter section. So the final passing time of motor route should include the passing time for one section only. And this section is the section which has minimum traffic capacity in the entire route. So, the passing time for route \( p \) is:

\[
T(p) = \max\left(\frac{Q}{C_i}\right) + \sum_{i=1}^{m} t_i
\]

(5)

\( m \): Shows that the route consists of \( m \) sections.
Usually, the optimization purpose for motorcade shortest route is to find route $P^*$ in given road network to make the passing time for all vehicles in motorcade be shortest. So, we can set the model of shortest route for motorcade as below:

$$T(P^*) = \min T(p) = \min \left[ \max \left( \frac{Q}{C_i} \right) + \sum_{i=1}^{m} t_i \right]$$  \hspace{2cm} (6)$$

**Algorithm analysis of motorcade shortest route**

According to formula (5) and (6), the motorcade passing route time is divided into 2 parts: one is the time when motorcade pass the road segment which has minimum actual traffic capacity, and the other one is the single vehicle passing time for each road segments. So, to work out the motorcade shortest route, we can start with these 2 parts.

**Shortest Route for Single Vehicle**

For shortest route of single vehicle, there are some ripe methods, about dozens of methods, just like Dijkstra, Floyd, Dantzing, Double-sweep algorithm and etc. And Dijkstra algorithm is most widely used.

Dijkstra mark-number method is one of most effective algorithms to solve the shortest route problem. Its basic idea is to gradually get the shortest route according to the mark-number of every node. There are 2 kind of mark-number: one is temporary mark-number, showed in $T$; the other one is permanent mark-number, showed in $P$. $T$ mark shows the shortest upper bound from start node to that node. It may vary depending on different route to that node. $P$ mark shows the minimum right of way from start node to that node, and it will not vary. The mark-number process can be divided in 2 steps:

**Step 1.** Modify $T$ mark-number. Supposing $v_i$ is newly generated mark-number node $P$, we will examine all arc segments $v_iv_j$ which start from $v_i$. If $v_j$ is mark-number node of $P$, we won’t mark node $v_j$; If $v_j$ is mark-number node of $T$, we will modify it as below:

$$T(v_j) = \min \left[ T(v_j), P(v_i) + w_{ij} \right]$$

$T(v_j)$ in square bracket is original $T$ mark-number of $v_j$.

**Step 2.** Produce $P$ new mark-number. Its rule is to modify the minimum value to mark-number $P$ in existing mark-number $T$.

Repeat above steps, until the end mark-number $T$ is modified to mark-number $P$.

**Motorcade passing time**

According to known condition of $Q$ (quantity of vehicles in motorcade) and $C_i$ (actual traffic capacity of road segment), we can get $Q/C_i$ (motorcade passing time for each road segment in road network).

<table>
<thead>
<tr>
<th>Road $R$</th>
<th>Actual Traffic Capacity</th>
<th>Motorcade Passing Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_1$</td>
<td>$C_{R_1}$</td>
<td>$t_{g1}$</td>
</tr>
<tr>
<td>$R_2$</td>
<td>$C_{R_2}$</td>
<td>$t_{g2}$</td>
</tr>
<tr>
<td>$R_3$</td>
<td>$C_{R_3}$</td>
<td>$t_{g3}$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$R_n$</td>
<td>$C_{R_n}$</td>
<td>$t_{gn}$</td>
</tr>
</tbody>
</table>

**Basic idea of finding shortest route**
In road network, we will find shortest route \( p_1 \) for single vehicle and the maximum motorcade passing time on this route by using Dijkstra algorithm, and then delete all the road segments that the passing time is greater than or equal to that road segment (including that road segment itself), so we can get a simplified road network. In that simplified road network, we will find shortest route \( p_2 \) for single vehicle. If \( p_1 > p_2 \), make \( p_2 \) is the shortest route; otherwise, \( p_1 \) is still the shortest route. Repeat the above steps, until there isn’t any route from the beginning to the end in road network. The final route will be the shortest route for motorcade.

The result shows that there is a shortest route \( p_n \) for single vehicle which is shorter than route \( p_1 \) in road network. And it is incompatible with original solution that \( p_1 \) is the shortest route for single vehicle in road network. So there is no shortest route with

\[
\max \left( \frac{Q}{C_i} \right)_{p_1} > \max \left( \frac{Q}{C_i} \right)_{p_i}
\]

in road network.

The algorithm steps of the shortest route for motorcade

\textbf{Step0:} Initialize \( T(p_0) = M \), \( M \) is the maximum;

\textbf{Step1:} Find the shortest route \( p \) for single vehicle by using Dijkstra algorithm,

\[
t'_p = \max \left( \frac{Q}{C_i} \right), \quad T(p) = \max \left( \frac{Q}{C_i} \right) + \sum_{i=1}^{m} t_i
\]

\textbf{Step2:} If \( T(p_0) > T(p) \), then \( T(p_0) = T(p), \ p_0 = p \);

\textbf{Step3:} Delete all the road segments which satisfied \( t'_{R_i} \geq t'_p \), and get simplified road network;

\textbf{Step4:} If there is some route from the beginning to the end, return to step \( 1 \);

\textbf{Step5:} \( p_0 \) is the shortest route for motorcade in road network, and \( T(p_0) \) is the time when passing the shortest route.

\section*{Conclusion}

This paper proposes a algorithm solution on Route Optimization Problem of Road Transportation. If we take it into practical application, we still need some low-level basic data (including static or dynamic data) and the technique of data acquisition and handling. Further more, if we search the route with minimum fuel consumption and combined fuel economy, we still need to add related constraint conditions and weight database. And this is the problem the authors will focus on next.

\section*{Reference}


