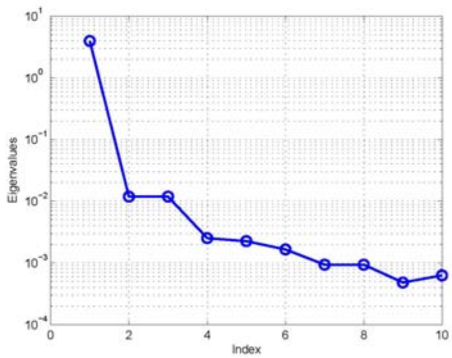
Resolution checks in stochastic space were conducted, and it was shown that third-order ($P = 3$) Legendre-chaos results in converged solution. For four-dimensional Legendre-chaos ($K = 4$), the total number of decomposition terms is 35. In figure 3, the convergence character of the block Jacobi iterative scheme is shown.

Since no analytical solution is available, the Monte Carlo simulation is employed to validate the Legendre-chaos solution. Here, the Monte Carlo computation is employed after the same KL decomposition. In this way the error from the Legendre-chaos decomposition is isolated, while the error introduced by the finite-term truncation of the KL decomposition, which is well understood, is excluded. The corresponding computational results associated with the $L^2(D)^2 \otimes L^2(\Gamma)$ norm has been shown in Figure 4.



Figuer 2. Eigenvalues of KL decompositon for $C(\mathbf{x}, \mathbf{z})$ with $b = 100$.

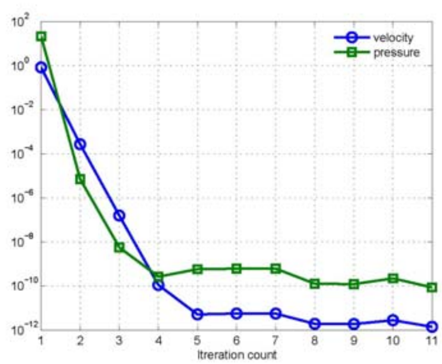


Figure 3. Convergence of error norm in term of iteration counts for the block Jacobi iterative scheme with $K = 4, P = 3$.

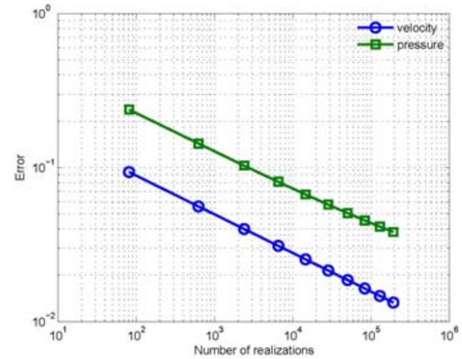


Figure 4. Comparison between the third-order Legendre-chaos solution and the Monte Carlo solution with various number of realizations.

VI. Conclusion

In this paper, we have presented a stochastic spectral method to the Stokes problems with random inputs. The essential finite dimensional noise assumption turns the original Stokes problem into a parametric saddle-point problem. It was solved by using the generalized polynomial chaos decomposition, together with a spectral Galerkin approximation in the spatial domain. We have employed a block Jacobi iteration technique to solve the system which governs the evolution of the chaos decomposition coefficients efficiently. The exponential correlation function was studied and applied in the computations. The Karhunen-Loeve decomposition is used to reduce the dimensionality of the stochastic space.

The exponential convergence rate is demonstrated for the model problem with $K = 1$. For more complicated Stokes problems, the Monte Carlo simulation is employed to validate the chaos solution. We have observed good agreement between the well-resolved chaos decomposition solution and the converged Monte Carlo simulation results.

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