Research of Workflow nets Application Based on the Petri Nets

Qi Dabin
University of Science and Technology Liaoning,
Anshan, China, 13610981503
qdbinner@163.com

Yao Jun
Chemical plant of Ansteel stock company,
Anshan China

Li Qiuju
Anshan Normal University Liaoning
China

Abstract—This paper introduced basic knowledge of Petri nets and workflow, and invariant analysis technique of Petri nets models structure, and integrate the real application circumstances of the workflow of receive document in document management, constructs an integrated workflow model based on Petri net. Further more, the author utilize invariant analysis technique to provide the verification of the model.

Key words-Petri Nets; Workflow nets; Performance analysis; Receive document

In recent years, Petri-net theories developed concurrency, synchrony, enology and net topology as primary content. Petri-net application has refer to every field of computer science, especially, workflow application field. Petri-net refered to the better maths base and visual modeling, and it has become implement of workflow study on analyzing and modeling.

I. PETRI-NET DEFINITION AND FUNCTION RULE.

Petri-net definition: Petri-net is defined as four-dimension groups PN(P, T, F, M0), P is named place sets, P=(P1,P2,…),T is transition sets, T=(T1,T2,…), F is flow relation, M0 is initial marking. In Petri-net, "O" denotes a place (middle state); rectangle denotes a transition; the arrowhead from P to T( or from T to P) denotes a arc in flow relation.

Dynamic action is denoted by token, namely symbol "·" in Petri-net. T is activated by condition which token amounts from T to P are greater than arc amounts from P to T, namely "·" has occurring authority in "·" : for all “·∈P, ·∈·”, M(p) ≥ W(p,t), “·” enables “·” to occur, “·” is “·” input sets. W(p, t) is arc amounts from P to T. If T is activated , token distribution will occur to alter in Petri-net, token amounts with transition will reduce in input place, but token amounts will increase in its output place ( "·") . “·” make “·” become new marking · ( successor marking): for all “·∈P,

\[
M'(p) = \begin{cases} 
M(p) - W(p,t) & \text{if } p \in t \\
M(p) + W(t,p) & \text{if } p \notin t \\
M(p) - W(p,t) + W(t,p) & \text{if } p \notin t \cap t' \\
M(p) & \text{if } p \in t \cap t'
\end{cases}
\]

II. WORKFLOW-NET BASED ON PETRI-NET


A. 1 Workflow net definition

Definition 1:

One Petri-net is Workflow-net, W=(P, T, F), iff:

1. existing two places in W: place  is input place, i = φ; place o is output place, o = φ.

2. Increasing a transition i between Place o and i, (namely , then w is a strong connectivity Petri-net.

Definition 2:

One Petri-net is reliable, iff:

1. for any a reachable state from i to M, existing a executing sequence M which reaching state o , i is initial state and o is terminative state. Namely :

\[
\forall M(i \rightarrow M) \rightarrow M \rightarrow o
\]

2. for any a reachable state from i to M, if existing a token in o at least, the state is terminative state, now , the Workflow-net only has a token in o, other places should be empty. Namely :

\[
\forall M(i \rightarrow M \land M \geq o) \rightarrow (M = o)
\]

3. no existing deadlock in w, then Petri-net is flexible. Namely :

\[
\forall t \in T, \exists M, M, then (i \rightarrow M \rightarrow M)
\]

B. Basic structure on Workflow net

![Figure 1 Serial structure](image1)

![Figure 2 Parallel structure](image2)
III. ANALYSIS METHOD THEORY BASED ON INVARIANT

The method uses algebra calculation to transact system, and supposes basic net of Petri-net is simplex, namely \( \forall p \in P, p \cap p^* = \emptyset \). \( T_{\text{invariant}} \) explains that marking will resume initial state after transition sets occur, \( T_{\text{invariant}} \) reflects transition sets making state return, utilizing it to study periodicity performance, etc. \( P_{\text{invariant}} \) reflects one weighted conservation on all token amounts in part places, utilizing it to study mutex, dead lock and error detection, etc.

In Petri-net analysis technology, it has great merit for Petri-net to apply invariant analysis method to confirm Petri-net performance based on simple linearity algebra equation.

A. Petri-net state equation.

In Petri-net, \( m_k \) denotes Petri-net marking after running \( k(\geq 0) \) time, then the marking after running “\( k+1 \)” times is:

\[
m_{k+1} = m_k + Cu_k, k \geq 0
\]  

(1)

\( C \) is incidence matrix, \( u_k \) is activated count vector, it is one “\( n \times 1 \)” vector, the \( i \) element denotes activated times by transition \( t_i \) in running “\( k+1 \)” , formula (1) is named Petri-net state equation. All tokens in place is not negative, so running legally will ensure: \( m_k + Cu_k \geq 0 \), for all \( k \geq 0 \)

B. \( P_{\text{invariant}} \) and \( T_{\text{invariant}} \)

\( P_{\text{invariant}} \)

One \( P_{\text{invariant}} \) is one “\( n \times 1 \)” vector \( x(x \geq 0) \), and meets condition:

\[
x^T C = 0
\]  

(2)

left multiply “\( x^T \) n” both sides in formula (1) :

\[
x^T m_{k+1} = x^T m_k + x^T C u_k
\]  

(3)

Beginning on deducing ‘gradually from “\( k=0 \)” :

\[
x^T m_0 = x^T m_1 = x^T m_2 = x^T m=\text{constant}
\]

(4)

Formula (4) denotes the sum of initial tokens which all weighted places by \( p_{\text{invariant}} \) are constant, namely the sum of weighted tokens in all places from “\( m0 \)” to reachable “\( m \)” are constant, the process is called that the places are covered with \( p_{\text{invariant}} \).

Based on incidence matrix, utilizing \( p_{\text{invariant}} \) to analyze limit and conservation performance in Petri-net structure:

(1) supposes it has only one “\( n \times 1 \)” vector \( x(x>0) \), makes \( x^T C \leq 0 \), then the Petri-net structure is limit.

(2) supposes it has only one “\( n \times 1 \)” vector \( x(x>0) \), makes \( x^T C = 0 \), then the Petri-net structure is conservation.

\( T_{\text{invariant}} \)

One \( T_{\text{invariant}} \) is one “\( m \times 1 \)” vector \( y(y \geq 0) \), and meets condition:

\[
Cy = 0
\]  

(5)

Suppose one transition sequence is activated (the sequence count vector is “\( u \)” ), token return from initial marking to initial marking, then based on formula (2):

\[
m_k = m_0 + Cu
\]  

(6)

Concluded: \( Cu = 0 \) Then “\( u \)” is one \( T_{\text{invariant}} \), namely “\( y=u \)”, it denotes that element \( (\text{element } k \geq 0) \) in \( T_{\text{invariant}} \) is times activated by transition in transition sets returning from \( m_0 \) to \( m_0 \). It is important applying validity of transition sets in \( T_{\text{invariant}} \) to analyze activity of Petri-net structure.

Estimating Petri-net structure reliability to utilize limit and activity.

Petri-net invariant is not exclusive, calling invariant which is not other invariant linearity combination basic invariant from linearity algebra, supposing that the order of matrix is \( r=\text{rank}(C) \), then the matrix has “\( n-r \)\( P_{\text{invariant}} \) and “\( m-r \)\( T_{\text{invariant}} \). The two invariants are attained by solving linearity equation \( x^T C=0 \) and \( Cy=0 \).

IV. ANALYSIS OF APPLYING PROBLEM

Document management adapted to come true in OA(Office Automation) system, document is transferred from a dept to another dept, which based on workflow technology. Sending document and receiving document are center of document management, utilizing workflow-net technology to realize document transferred is efficient.

A. Workflow-net reliability analysis.

Figure 5 is dispatch document manage workflow model, utilizing it to analyze workflow-net model construction, analysis and the same technology.
The flow process as follows: user drafts document, submitting, secretary transacts document with flow of serial structure and parallel structure based on sending document request. After checking , transacting document has two choice: sign or draft. In all the flow process, administrator can check and print flow process tracking table at any moment.

Figure 6 is workflow-net model of Figure 5. It includes six transition: draft, revise, leader1 and leader2 check, auditing, sign. Workflow process commences on token p1, and p8 in the end, the role denotes transition in each process

Increasing a transition t1(return document) , if transition t5 is not past , the document will be returned t2 via t7, now, any one token exists in p2 or p7, t2 will occur , the improvement model (Figure 3)accords with not only Petri-net theory but sending document workflow in practice.

Further more, the improvement model is free choice Petri-net model. For any two transition of the improvement model t1 and t2, only existing ˈ t1∩ t2 ≠ ∅, then ˈ t1⊂ t2, for example, ˈ t1⊂ t4⊂ t3, the same as , the improvement model is proved as right structure via the analysis.

B. Workflow-net invariant analysis method

As follows, Resorting of invariant analysis method, for workflow model based on Petri-net , utilizing invariant analysis method, analyzing to accept document in document management, construct accept document management workflow model as Figure 8.

The model is reliable from theory, in accord with workflow-net definition, however, in practice, the model has logic error: executing t2, appearing and-join of Parallel structure. t2 has two input places, ˈ t2 = (p2,p7), only existing tokens in p2 and p7, t2 can occur, the conclusion is not right. In practice, any one token exists in p2 or p7, namely drafting document is finished or auditing didn’t pass, workflow should enter directly into revising state. The model is revised as Figure 7
analysis method and Workflow-net theory to analyze the performance of model.

1) Workflow-net activity

Workflow-net activity can be proved by constructing states equation, based on invariant analysis method. By formula \( M = M_0 + CU \), supposing initial marking \( M_0 = (1,0,0,0,0,0,0,0) \), terminating marking \( M = (0,0,0,0,0,0,1) \), constructs the incidence matrix:

\[
C = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
U = (1,1,1,1,1,1) \text{, pass transition sequence } T_1 \ldots T_6, \text{ all the workflow process is from initial marking to terminating marking, so the Workflow-net is active.}
\]

2) Workflow-net boundary

Boundary problem may transform states equation to solve problem. For model (Figure 9), \( Y \geq 0, CY \leq 0 \), so the Workflow-net is not boundary, the model should be improved. Improving model is as follows Figure 10.

![Photo](unnamed.png)

Figure 10  accept document Workflow-net improvement model

In improvement model(Figure 10), Increasing a transition \( t_7 \), accept document process didn’t execute one time ,but execute circularly.

constructs the incidence matrix over again:

\[
C = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
CY \leq 0 \Rightarrow \begin{cases}
-y_1 + y_2 \leq 0 \\
y_1 - y_2 \leq 0 \\
y_2 - y_3 \leq 0 \\
y_3 - y_4 \leq 0 \\
y_4 - y_5 \leq 0 \\
y_5 - y_6 \leq 0 \\
y_6 - y_7 \leq 0
\end{cases}
\]

\( \gamma \geq 0 \), so the improvement model is boundary.

V. CONCLUSION

This article introduced basic knowledge of Petri-net and Workflow-net, utilizing Workflow-net definitions and invariant analysis method to analyze Petri-net performance. In the complicated OA system, applying Workflow-net to construct model, applying reachable tree and incidence matrix method to analyze system feasibility, reliability, security ,etc. in practice, the methods have an better effect on Petri-net performance analysis. The methods make system project and performance optimal, more application of the technology demands to study and explore.

REFERENCES