From figure 1, we can see that numerical solution obtained by our method is good agrees with the exact solution and the accuracy of the method is very high.

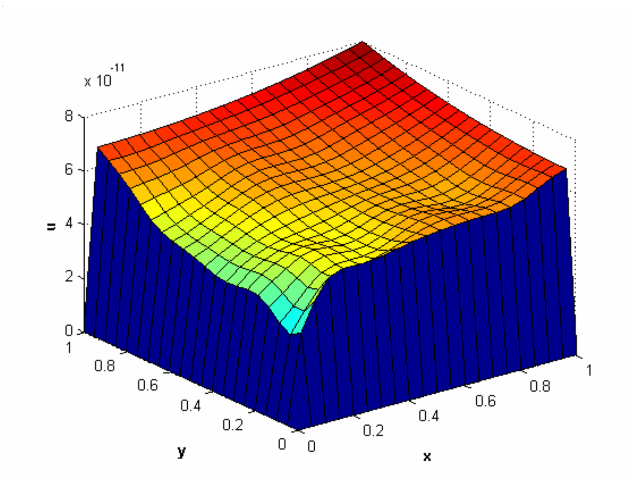

Figure 2. the absolute error of Poisson equation

B. Poisson equation

Consider the following Poisson equation

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-2u,(x,y)\in[0,1]\times[0,1] \quad (51)$$

with the Dirichlet boundary conditions

$$\begin{aligned} &u(0,y)=0,u(1,y)=\sin(1)\sin(y),\\ &u(x,0)=0,u(x,1)=\sin(x)\sin(1) \end{aligned} \quad (52)$$

has the exact solution is $u=\sin(x)\sin(y)$.

We solve the above problem by applying the technique described in Section III and have

$$\begin{aligned} \ddot{u}(x,y)&=C^T P_y^2\left[\Psi(x,y)-y\Psi(x,1)\right]\\ &\quad+y\left[\ddot{u}(x,1)-\ddot{u}(x,0)\right]+\ddot{u}(x,0),\\ u''(x,y)&=C^T P_x^2\left[\Psi(x,y)-x\Psi(1,y)\right]\\ &\quad+x\left[u''(1,y)-u''(0,y)\right]+u''(0,y), \end{aligned} \quad (53)$$

$$\begin{aligned} u(x,y)&=C^T P_x^2 P_y^2\left[\Psi(x,y)-x\Psi(1,y)-y\Psi(x,1)\right]\\ &\quad-xyC^T P_x^2 P_y^2\Psi(1,1)+g_3(x,y)-yg_3(x,1)+g_4(x,y), \end{aligned} \quad (54)$$

where $g_3(x,y)=x\left[u(1,y)-u(1,0)-u(0,y)+u(0,0)\right]+xy\left[u'(0,0)-u'(1,0)\right]+u(0,y)-u(0,0)$ and

$g_4(x,y)=y\left[u(x,1)-u(x,0)\right]+u(x,0)$.

Substituting (53) and (54) into (51), we obtain

$$C^T\Lambda=g(x,y) \quad (55)$$

where

$$\begin{aligned}\Lambda&=P_y^2\left[\Psi(x,y)-y\Psi(x,1)\right]+P_x^2\left[\Psi(x,y)-x\Psi(1,y)\right]\\ &\quad+2P_x^2P_y^2\left[\Psi(x,y)-x\Psi(1,y)-y\Psi(x,1)-xy\Psi(1,1)\right]\end{aligned}$$

and

$$\begin{aligned} g(x,y)&=-x\left[u''(1,y)-u''(0,y)\right]-u''(0,y)\\ &\quad-y\left[\ddot{u}(x,1)-\ddot{u}(x,0)\right]-\ddot{u}(x,0)\\ &\quad-2\left[g_3(x,y)-yg_3(x,1)+g_4(x)\right] \end{aligned}.$$

From figure 2, we can see that numerical solution obtained by our method is full agrees with the exact solution.

CONCLUSION

In this paper, we give a brief proof about the general procedure of two-dimensional operational matrices of integration, and then develop a solution of PDEs by using the two-dimensional operational matrices of integration. The main benefits of the proposed method are its computation-effective (only need a small number of collocation points guarantees the necessary accuracy) and universality (the same approach is applicable for a wide class of linear PDEs).

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