Research on Pupil Center Location Based on Improved Hough Transform and Edge Gradient Algorithm

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***Abstract*—According to the poor efficiency of the traditional Hough transform algorithm to detect the pupil center, this paper proposed an edge gradient algorithm, which combines with improved Hough transform algorithm to detect the pupil center. At the beginning, a modified Hough transform algorithm was used to detect the pupil center, if more than one circle was found, the edge gradient algorithm was used to remove the excess circle. Experimental results show that our proposed method can not only improve the efficiency of pupil center location but also achieve better accuracy.**

Keywords-pupil detection; center position; Hough transform; the edge gradient.

I. INTRODUCTION

The pupil center location is an important aspect in the field of iris recognition technology, and it is also the first step to study on the rotation of the eyeball. We can diagnose certain diseases of the brain by researching the vibration characteristics of the pupil center. In general, iris and pupil are not strictly concentric, but their center is extremely close, therefore, we may find the pupil center through locating the iris center. At present, the research on pupil center is mostly based on circle detection technology. Hough transform is currently the most widely used method for circle detection. However, the traditional Hough transform is used in three-dimensional, which is very complex and time-consuming. Meanwhile, the detection rate is very low.

In this paper a new method based on the edge gradient algorithm combined with improved Hough transform has developed to detect the pupil center quickly and accurately. The first step of this process, to detect the input eye diagram by the improved Hough transform, if only one circle is found, that means the circle center is the iris center, but if the circle is more than one, go to the next step of this algorithm. The second step, we assume that the iris center in the input eye diagram is located at $(w/2, h/2)$, where w is width of the image, and h is the height of the image. Then the set of points of maximum modulus of the gradient obtained by calculating the gradient value of each point of the eye diagram is the iris edge. Getting the centers of the circles determined by any three points which are not collinear on the edge contour and calculating the Euclidean distances between those centers and the assumed iris center, and consider the circle center of the minimum Euclidean distance as the reference center.The third step, to calculate the Euclidean distances between the circle centers obtained in first step and the reference center calculated in second step. The circle center of the minimum Euclidean distance is the iris center.

II. IMPROVED HOUGH TRANSFORM

Hough transform can be applied to detect circle in image, mainly because it is applicable to any function in the form $g(v,c)=0$, where v is the coordinate vector, and c is the coefficient vector. The coordinates of point on the circle satisfy (1).

$$(x-c_1)^2+(y-c_2)^2=c_3^2 \qquad (1)$$

Where, (c_1, c_2) is the center coordinate of the circle, c_3 is the radius.

In equation (1), c_1, c_2 and c_3 ,are three parameters which can be mapped to a three dimensional rectangular coordinate system, so each accumulator $A(i, j, k)$ is equivalent to a cube structure graphic, as shown in Fig. 1.

Then the process of detection points on the circle is evolved into refining and gradually increasing the values of c_1 and c_2 in the $c_1c_2c_3$ plane, and then obtaining the value of c_3 which satisfies (1). The value stored in the accumulator $A(i, j, k)$ will become the number of many conical intersections in three dimensional parameter space.

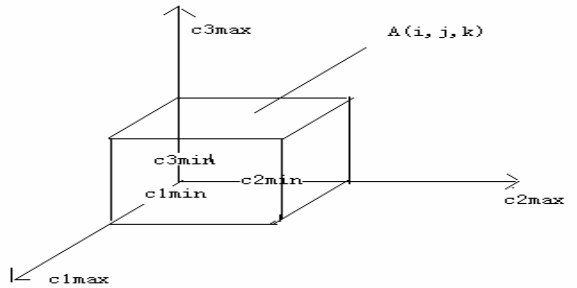

Figure 1. Architecture of accumulator mapping c1, c2 and c3 space

Although this process can detect circle theoretically in image, due to the parameter of the round function is many, and take into account the problem of computational complexity and detection efficiency in the three-dimensional space, the Hough transform algorithm is hard to truly apply to digital image processing algorithm for circle detection.

Considering the computational complexity of circle detection, the Improvement of Hough transform algorithm is also endless. This article proposes a new method that could drop Three-dimensional parameter space into two-dimensional parameter space.

First of all, the circle detection can be divided into two steps.

Step 1: Find the circle center in two-dimensional Hough transform space.

Step 2: Get the circle radius by statistic histogram. In fact, it is equivalent to a one-dimensional Hough transform space.

In general, it is assumed that the circle tangent *m* intersect with the circle at the point (x, y), then make the perpendicular line *n* of the straight line *m* through this point, and the straight line *n* will pass through the circle center. To find the circle center, above conditional proposition was considered to be continuously tenable. The normal component which crosses the circle center can be obtained by using the gray-scale edge detection operator (for example, sobel operator). As shown in Fig. 2, the three-dimensional parameters (x, y, θ) space can be mapped into the two-dimensional parameter (a, b) space, where (a, b) is the circle center coordinates, and (x, y, θ) respectively are the point coordinate and the angle between normal vector and the x-axis. After (x, y, θ) is mapped to the two-dimensional parameter space, It will produce many straight lines (a pair of parameters correspond to a straight line), the intersection of these straight lines can determine the center coordinates. Of course, the center coordinates obtained by this method may be many.

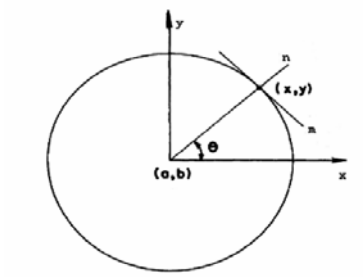

Figure 2. The relationship between (x, y, θ) parameter space and (a, b) parameter space

From Fig. 2:

$$TAN(\theta) = (y-b)/(x-a) \tag{2}$$

$$b = TAN(\theta)a + (y - xTAN(\theta)) \tag{3}$$

The circle radius can be calculated by (4).

$$\delta = (x - a_0)^2 + (y - b_0)^2 \tag{4}$$

Where, (a_0, b_0) is the coordinate of the circle center obtained from step (1). Through traversing all point (x_k, y_k), we can get a statistical histogram of the related radius value, then the maximum peak (The number of pixels which supports the circle center are most) is the size of the radius. As shown in Fig. 3.

This algorithm was run on the .NET platform. The eye diagram data was from the iris image database CASIA, All required parameters of algorithm have been strictly set in experiment.

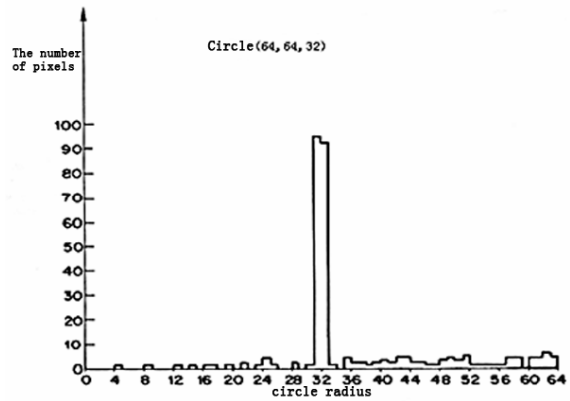

Figure 3. The statistical histogram used to calculate the circle radius.

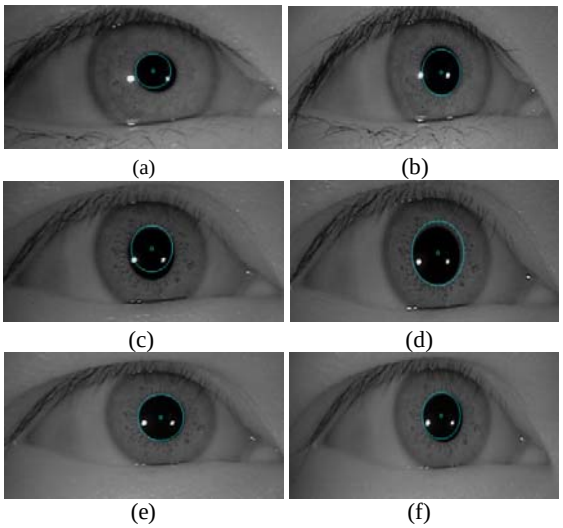

Figure 4. (a)-(f) shows the results of pupil detection using improved Hough Transform

As the picture in Eye Gallery was captured under very good lighting conditions, the experimental result was satisfactory. But in many eye pictures, more than one circle was found in experiment, as shown in Fig. 5.

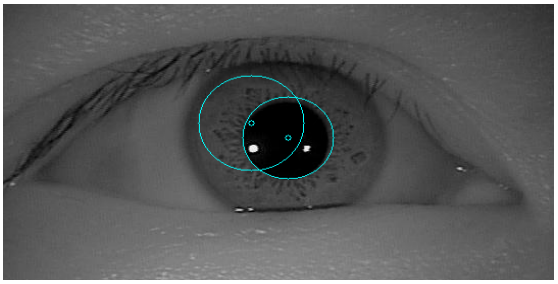

Figure 5. Multiple circle detected through improved Hough Transform

III. EDGE-GRADIENT ALGORITHM

In digital image processing, the gradient operation (differential algorithm) is extremely widely applied. We can assume that the grayscale function of an original image is expressed by $f(x, y)$, the gradient vector of any point in the image can be expressed by (5).

$$Grad[f(x,y)] = [\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}]^T \tag{5}$$

Where $Grad[f(x,y)]$ is the gradient of the pixel point (x, y) in image, $\frac{\partial f}{\partial x}$ is the variance rate of gray level in the horizontal direction (x axis) and $\frac{\partial f}{\partial y}$ is the variance rate of gray level in the horizontal direction (y axis).

Getting the gradient module value by Taking a modular operation on Gradient vector at the point (x, y), then $Grad[f(x,y)]$ is defined as (6).

$$GradM(x,y) = |Grad[f(x,y)]| = \left(\left(\frac{\partial f(x,y)}{\partial x} \right)^2 + \left(\frac{\partial f(x,y)}{\partial y} \right)^2 \right)^{\frac{1}{2}} \tag{6}$$

Where $Grad[f(x,y)]$ is the modulus of gradient.

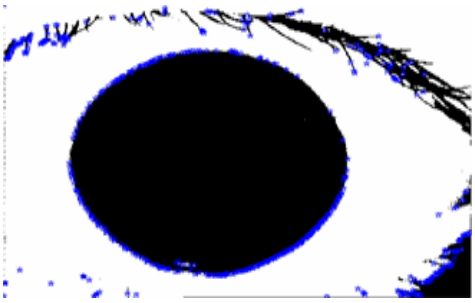

Figure 6. The maximum gradient point set in binary image.

In binary image, grey level only has two values: 0 and 1. Therefore, the modulus of the gradient at each point only possibly takes three values, 0,1 and $\sqrt{2}$. It is only at the edge of the iris and some noise point, the modulus of the gradient is the largest. The square of modulus value is 2. Then we may be able to mark the candidate point on iris edge contour by calculating the square of the gradient magnitude value of each pixel point in the image, as shown in Fig. 6.

In geometry, it is shown that three points in a plane, not in a straight line, determine a unique circle which passes through those three points. Therefore we can get a circle determined by any three points not in a straight line on the Iris edge contour. After getting overall Iris edge contour by computing the gradient modulus values, the circle center determined by any three points not in a straight line on the Iris edge contour was obtained. Because the Iris is located in or near the center position in the image, in order to remove noise interference, we may first Assume that the center of the iris is located at $(w/2, h/2)$, where w is the image width, h is the image height. By comparing the Euclidean distances between the assumed center and the centers obtained through the edge gradient detection, we can get the coordinate of the center that are closest to the assumed circle center, as shown in Fig. 7.

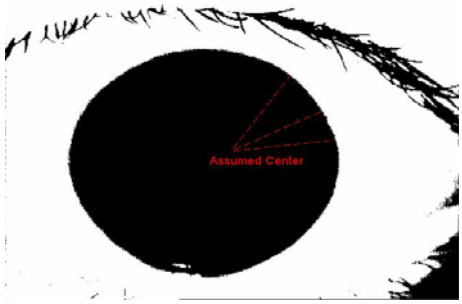

Figure 7. Three points that marked the iris edge in binary image.

Next, we need to find the real center determined by three points that are not in a straight line. It is assumed that the real center coordinate is (x_0, y_0), the three-point coordinates marked through above process at the edge contour respectively is (x_1, y_1), (x_2, y_2), (x_3, y_3). They have the same Euclidean distance from the real center of the circle, according to the definition of circle,

$$(x_0 - x_1)^2 + (y_0 - y_1)^2 = (x_0 - x_2)^2 + (y_0 - y_2)^2 \tag{7}$$

$$(x_0 - x_2)^2 + (y_0 - y_2)^2 = (x_0 - x_3)^2 + (y_0 - y_3)^2 \tag{8}$$

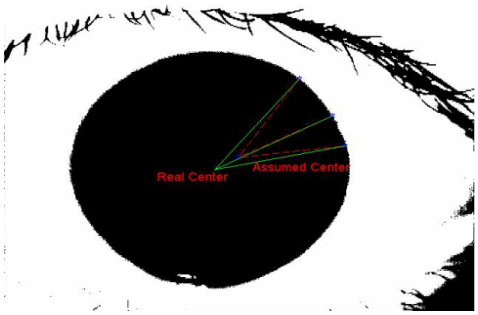

Figure 8. The real center obtained by edge-gradient method

We can get a Binary Linear Equation Group with respect to (x_0, y_0) through (7) and (8), then the real center coordinates is finally obtained, as shown in Fig. 8.

IV. OUR PROPOSED METHOD

In this paper, we use a new method which combines the edge gradient algorithm with improved Hough transform. The flowchart is shown in Fig. 9.

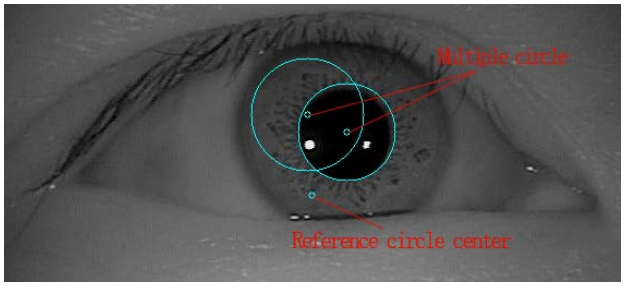

Figure 9. The figure marked many circle centers

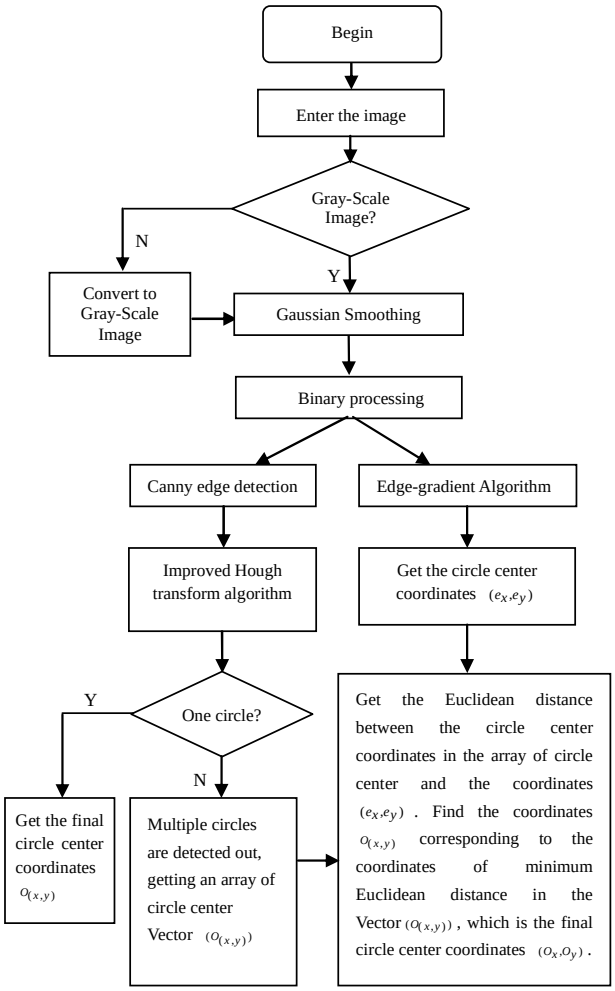

Figure 10. The steps of our proposed method for puplil detection

To remove the Noncompliance circles, we need firstly find the reference center position of the iris through the edge of the gradient method, as shown in Fig. 10.

Next, through comparing the Euclidean distances between the detected center and reference center of the circle, removing the unwanted circle. As shown in Fig. 11, where the Euclidean distance $d_1 > d_2$.

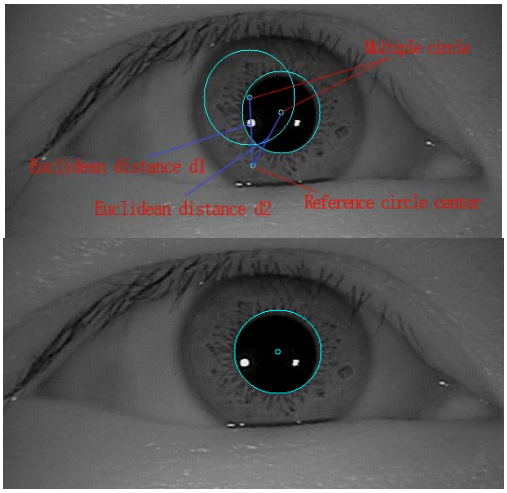

Figure 11. The final circle removed the excess circles

For the other eye diagram in Eye Gallery experimental results are shown in the following table.

Total number of pictures	**Number of pictures detected circle**		**Detection rate of circle**	**Number of pictures removed extra circle**	**Removal rate of extra circle**
	One circle	***Multiple circle***			
350	282	218	100%	218	100%

In the table, Detection rate of circle is calculated as number of pictures detected circle divided by total number of pictures. Removal rate of extra circle is calculated as number of pictures removed extra circle divided by number of pictures detected out multi-circle.

V. CONCLUSION

The experimental result proves the effectiveness of our proposed method. First of all, an improved Hough transform algorithm that could drop Three-dimensional parameter space into two-dimensional parameter space is applied. The complexity of the algorithm is greatly reduced, and the efficiency is greatly improved. Secondly, our proposed method does not require high quality eye-diagram, only ordinary eye image. Such as White-Spot is appeared in eyes under light conditions, the result of the experiment will not be affected. Thirdly, the Pupil Center positioning method in this article is mainly based on the geometric characteristics of circle which any three points not in a straight line determine a unique circle, it is also available for other

special condition, for example, the eyes are not fully open or the pupil dose not fully display.

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