Research on Measurement of Mass, Centroid and Eccentricity of Large-scall Axisymmetric Body

Gang LU
Department of technology of measurement and instrument, School of Mechanical Engineering, Nanjing University of Science & Technology, Nanjing, China

Aijun ZHANG
Department of technology of measurement and instrument, School of Mechanical Engineering, Nanjing University of Science & Technology, Nanjing, China
Email:zaj23@163.com

Changming WANG
Department of technology of measurement and instrument, School of Mechanical Engineering, Nanjing University of Science & Technology, Nanjing, China

Abstract—Pointed at low-precision of measurement of mass and centroid of large-scall axisymmetric body, measurement method is researched. After comparing, three-point method is adopted to design this system, and error is analyzed and compensated. According to experiments, measurement system has high-precision, and it can achieve requirement of engineering.

Keyword—Axisymmetric body, Mass, Centroid, Measurement, Three-point method

I. Instruction

In aerospace industry, we need to measure mass, centroid and eccentricity of large-scall axisymmetric body such as artificial satellite and spaceship in order to ensure that product accords with design requirements [1]. At present there are many kinds of method to measure centroid, such as suspension method, reaction force method, unbalanced moment method, multi-pivot weighing method, rotational balance method, inertia method and so on. Eccentricity is a parameter of axisymmetric body, and it is an application of measurement method for centroid. Eccentricity of axisymmetric body can be measured by multi-pivot weighing method, rotational balance method, and inertia method. Suspension method [2] can be used to measure centroid of regular and irregular body, but just small-scall object. Reaction force method can be used in measurement of large-scall object, such as vehicle. Unbalanced moment method is a method of static measurement, and it only can be used by regular small-scall object. Multi-pivot weighing method relies on static equilibrium principle, and it can be used to measure eccentricity distance of axisymmetric body with high-precision. Rotational balance method and inertia method are both dynamic measurement methods, and they are mainly used in eccentricity measurement of axisymmetric body. This system designs a suit of measurement system to measure mass, centroid and eccentricity by multi-pivot weighing method.

II. Measurement Theory of Mass, Centroid and Eccentricity

Measurement system adopts three-point method to measure mass, centroid and eccentricity of projectiles and rockets. Axisymmetric body is placed on a pedestal, which is supported by three sensors. According to moment equilibrium principle, mass, centroid and eccentricity can be measured at one time. Rectangular space coordinate system is built in Fig.1.

Coordinate values of three sensors’ supporting-pivot are \( S_1(x_1, y_1, 0) \), \( S_2(x_2, y_2, 0) \) and \( S_3(x_3, y_3, 0) \). X-axis of coordinate system passes through sensor’s supporting-pivot \( S_1 \), which means \( y_3 = 0 \), y-axis passes through center line of pedestal, and \( |x_1| = |x_2| = \frac{|x_3|}{2} \). Disposing sensors by this means is to make sure that pressure of each sensor is pretty much the same.

A. Measurement theory of centroid in the axial direction.

Centroid in the axial direction \( L \) is the distance between centroid of axisymmetric body and bottom of axisymmetric body. Projection of measurement device in plane \( OYZ \) is given in Fig. 2.

According to moment equilibrium principle, there is

\[
M \cdot g \times y_i = m_1 \cdot g \times y_1 + m_2 \cdot g \times y_2 + m_3 \cdot g \times y_3
\]

(1)

X-axis of coordinate system passes through sensor’s...
supporting-pivot, so there is \( y_3 = 0 \).

\[
y_e = \frac{m_1 \cdot g \cdot x_1 + m_2 \cdot g \cdot x_2}{M}
\]  
(2)

Centroid in the axial direction of standard object is known, and the distance between reference position and bottom of standard object can be measured by grating ruler, so coordinate value of x-axis can be calculated by Eq.2. So we can obtain calibration value of \( D \) by next equation.

\[
D = L_c + S_y - y_e
\]  
(3)

Where \( D \) is calibration value; \( L_c \) is centroid in the axial direction; \( S_y \) is the distance between reference position and bottom of standard object.

**B Measurement Theory of Eccentricity**

Eccentricity of axisymmetric body includes eccentric distance and eccentric angle. Projection of measurement device in plane \( o\Sigma \) is given in Fig. 3. We need to get weighing results in initial position and position after moving around axis of axisymmetric body for 90° to get eccentricity. If we can measure x-axis coordinate value of initial position \( x_c^{0°} \) and x-axis coordinate value of position after moving around for 90° \( x_c^{90°} \), we will get eccentric distance and eccentric angle. According to moment equilibrium principle:

\[
x_c = \frac{m_1 \cdot x_1 + m_2 \cdot x_2 + m_3 \cdot x_3}{M}
\]  
(4)

\( x_c^{0°} \) and \( x_c^{90°} \) can be measured according to equation (4), so eccentric distance and eccentric angle can be calculated by Eq.5 and Eq.6.

\[
e = \sqrt{(x_c^{0°})^2 + (x_c^{90°})^2}, \quad \theta = \arctan\left(\frac{x_c^{90°}}{x_c^{0°}}\right)
\]  
(5)

Where \( e \) is eccentric distance and \( \theta \) is eccentric angle.

III. Error Analysis and Compensation

From measurement theory of mass, centroid and eccentricity we can see that mass error mainly comes from measurement error of sensors; centroid error mainly comes from measurement error of sensors, position error between each sensor and measurement error of grating ruler; eccentricity error mainly comes from measurement error of sensors, position error between each sensor and angle error brought from rotating axisymmetric body. In this paper, we only analyze eccentricity error. According to the measurement theory of eccentricity, eccentric distance and eccentric angle can be calculated by \( x_c \) of initial position and position after rotating for 90°. In our measurement system, we measure \( x_c \) of initial position, position after rotating for 90°, 180° and 270° to compensate error. We compensate error of mechanical structure by measurement of symmetrical position in order to minish error. From equation (4) error of \( x_c \) brought by measurement error of sensor is as follows:

\[
\sigma_{x_c} = \left[ \left( \frac{\partial x_c}{\partial m_1} \sigma_1 \right)^2 + \left( \frac{\partial x_c}{\partial m_2} \sigma_2 \right)^2 + \left( \frac{\partial x_c}{\partial m_3} \sigma_3 \right)^2 \right]^{1/2}
\]

\[
\frac{\partial x_c}{\partial m_1} \sigma_1 = m_2 (x_1 - x_2) + m_3 (x_1 - x_3)
\]

\[
\frac{\partial x_c}{\partial m_2} \sigma_2 = m_1 (x_1 - x_2) + m_3 (x_2 - x_3)
\]  
(6)

\[
\frac{\partial x_c}{\partial m_3} \sigma_3 = m_1 (x_1 - x_2) + m_2 (x_3 - x_2)
\]

Where \( x_1 = -50, x_2 = -50, x_3 = 100 \).

When we test standard object whose mass is 39269.7g and centroid is 320mm, error of \( x_c \) brought by measurement error of sensors is as follows:

\[
\sigma_{x_c} \approx \sigma_{x_1} \approx \sigma_{x_2} \approx \sigma_{x_3} \approx 0.000066\%
\]  
(7)

So eccentricity error is \( \sigma_e = 0.000047\% \). We can find that eccentricity error brought by measurement error is very small.

IV. System Testing

After sensors calibration, we use standard object to calibrate mass, centroid and reference position of grating ruler of eccentricity measurement device. Standard object’s length is 640mm, position of centroid is 320mm and mass is 39269.7g. Data and result of calibration is in Table 1.
Table 1  Data and result of calibration of mass, centroid and eccentricity

<table>
<thead>
<tr>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
<th>Total mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass [g]</td>
<td>12584.08</td>
<td>13935.24</td>
<td>12754.13</td>
</tr>
<tr>
<td>Grating ruler [mm]</td>
<td>268.88</td>
<td></td>
<td></td>
</tr>
<tr>
<td>result of calibration [mm]</td>
<td>D=583.719</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard object is tested by measurement system. Measurement result is in Table 2.

Table 2  Measurement result of mass, centroid and eccentricity

<table>
<thead>
<tr>
<th>Rotating angle</th>
<th>Sensor 1</th>
<th>Sensor 2</th>
<th>Sensor 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>12584.8</td>
<td>13935.2</td>
<td>12754</td>
</tr>
<tr>
<td>90°</td>
<td>12611.6</td>
<td>13909.4</td>
<td>12753.4</td>
</tr>
<tr>
<td>180°</td>
<td>12635.2</td>
<td>13887.1</td>
<td>12751.6</td>
</tr>
<tr>
<td>270°</td>
<td>12654.6</td>
<td>13865</td>
<td>12732.9</td>
</tr>
<tr>
<td>Grating ruler [mm]</td>
<td>268.75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measurement result</td>
<td>Total mass [g]</td>
<td>Centroid [mm]</td>
<td>eccentric distance [mm]</td>
</tr>
<tr>
<td>39274</td>
<td>320.127</td>
<td>0.00466</td>
<td>1.56</td>
</tr>
</tbody>
</table>

V. Conclusions

Measurement of axisymmetric body’s mass character has important significance in estimating and improving its work efficiency. Measurement system introduced in this paper can measure mass, centroid and eccentricity with high-precision. According to error analysis of measurement theory and testing of final product, the whole system has perfect functional and reliable performance.

References