Increasing and Decreasing with Fuzzy Time Series

Ming-Tao Chou1 Hsuan-Shih Lee2

1Department of Aviation and Maritime Management, Chang Jung Christian University
2Department of Shipping and Transportation Management, National Taiwan Ocean University

Abstract

There is a significant problem associated with the fuzzy time series. That is a strict increasing and decreasing case. Under the discussion case, fuzzy time series model arise a continuous increasing/decreasing forecasting value. From the illustrative example, we can see that our definition not only only define the trend of the fuzzy numbers that represent the linguistic values of the linguistic variable more appropriately, but also can make the fuzzy time series reasonable.

Keywords: Fuzzy sets; Fuzzy forecasting.

1. Introduction

The main purpose of this article is to present a mathematical skill to construct a fuzzy time series model of a system. There has been a considerable of studies [1, 3-5] on fuzzy time series where fuzzy [6-9] implications are used to express forecasting rules. Many kinds of fuzzy time series model have been developed since Lee and Chou’s [5] paper.

In this article, we present a new definition for forecasting with fuzzy time series based on Lee and Chou’s method. The historical data of the Port of Kaohsiung used by Lee and Chou’s method [5] are adopted to justify that our method is as reasonable as Lee and Chou’s method but more complete than original method. In forecasting Port of Kaohsiung through, there is a significant problem associated with the fuzzy time series. That is the historical data shown strict increasing and decreasing situation. According to Lee and Chou’s method [5], we add a definition to conquer the attribution, based on the definition the fuzzy time series is more complete.

The remaining of the article is structured as follows. Section 2, presents the fuzzy time series definition. Section 3, we present a new definition for forecasting with fuzzy time series based on Lee and Chou’s method, our method is also evaluated in section 4. Finally, the conclusions are made in section 5.

2. Fuzzy sets and Fuzzy time series


Definition 2.1
A fuzzy number in real line \( \mathbb{R} \) is a fuzzy subset of \( \mathbb{R} \) that is normal and convex.

The fuzzy time series was first presented by Song and Chissom [3, 4]. There are a number of calculation methods to determine the fuzzy relation. Different methods will yield different results [3, 5, 10]. In their papers, they followed Mamdani’s method [3] to determine the relations. The main difference between the fuzzy time series and traditional time series is that the values of the former are fuzzy sets while the values of the latter are real historical numbers. The definitions of fuzzy time series are reviewed as follows.

Definition 2.2 [3, 4]
Let \( Y(t) (t = \ldots, 0, 1, 2, \ldots) \) be the universe of discourse on which fuzzy sets \( f_i(t) (i = 1, 2, \ldots) \) are defined and \( F(t) \) is the collection of \( f_i(t) (i = 1, 2, \ldots) \). Then \( F(t) \) is called a fuzzy time series on \( Y(t) (t = \ldots, 0, 1, 2, \ldots) \).

\( F(t) \) can be understood as a linguistic variable and \( f_i(t) (i = 1, 2, \ldots) \) as the possible linguistic values of \( F(t) \). Because at different times, the values of \( F(t) \) can be different, \( F(t) \) is a function of time \( t \). Also, since the universes of discourse can be different at different times, \( Y(t) \) is used for the universe at time \( t \).
Definition 2.3 [3, 4]
Let \( I \) and \( J \) be indices sets for \( F(t-1) \) and \( F(t) \) respectively. If for any \( f(t) \in F(t) \) where \( j \in J \), there exists \( f(t-1) \in F(t-1) \) where \( i \in I \) such that there exists a fuzzy relation \( R_i(t,t-1) \) and \( f(t) = f(t-1) \circ R_i(t,t-1) \) where ‘\( \circ \)’ is the max-min composition, then \( F(t) \) is said to be caused by \( F(t-1) \) only. Denote this as \( f_i(t) \rightarrow f_i(t) \) or equivalently \( F(t-1) \rightarrow F(t) \).

Definition 2.4 [3, 4]
If for any \( f(t) \in F(t) \) where \( j \in J \), there exists \( f(t-1) \in F(t-1) \) where \( i \in I \) and a fuzzy relation \( R_i(t,t-1) \) such that \( f(t) = f(t-1) \circ R_i(t,t-1) \), let \( R(t,t-1) = \bigcup_{i} R_i(t,t-1) \) where ‘\( \bigcup \)’ is the union operator. Then \( R(t,t-1) \) is called the fuzzy relation between \( F(t) \) and \( F(t-1) \) and define this as the following fuzzy relational equation:

\[
F(t) = F(t-1) \circ R(t,t-1).
\]

Definition 2.5 [3, 4]
Suppose \( R(t,t-1) = \bigcup_{i} R_i(t,t-1) \) and \( R(t,t-1) = \bigcup_{i} R_i(t,t-1) \) are two fuzzy relations between \( F(t) \) and \( F(t-1) \). If for any \( f(t) \in F(t) \) where \( j \in J \), there exists \( f(t-1) \in F(t-1) \) where \( i \in I \) and fuzzy relations \( R_i(t,t-1) \) and \( R_i(t,t-1) \) such that \( f(t) = f(t-1) \circ R(t,t-1) \) and \( f(t) = f(t-1) \circ R(t,t-1) \), then define \( R(t,t-1) = R_i(t,t-1) \).

Definition 2.6 [3, 4]
Suppose \( F(t) \) is caused by \( F(t-1) \) only or by \( F(t-1) \) or \( F(t-2) \) or... or \( F(t-m) \) \((m > 0)\). This relation can be expressed as the following fuzzy relational equation:

\[
F(t) = F(t-1) \circ R(t,t-m)
\]

or

\[
F(t) = (F(t-1) \cup F(t-2) \cup ... \cup F(t-m)) \circ R(t,t-m)
\]

Then equation 2-1 or 2-2 is called the first-order model of \( F(t) \).

Definition 2.7 [3, 4]
Suppose \( F(t) \) is caused by \( F(t-1), F(t-2), ..., \) and \( F(t-m) \) \((m > 0)\) simultaneously. This relation can be expressed as the following fuzzy relational equation:

\[
F(t) = (F(t-1) \times F(t-2) \times ... \times F(t-m)) \circ R(t,t-m)
\]

Then equation 2-3 is called the mth order model of \( F(t) \).

To simplify the assumption, Chen assumed \( F(t) \) is singleton. That is, \( F(t) \) is a fuzzy set not a set of fuzzy sets. The simplified fuzzy time series is defined as follows.

Definition 2.8 [5]
\( F(t) \) is a fuzzy time series if \( F(t) \) is a fuzzy set. The transition is denoted as \( F(t-1) \rightarrow F(t) \).

Definition 2.9 [5]
The universe of discourse \( U = \{ D_L, D_U \} \) is defined such that \( D_L = D_{\min} - \frac{s}{\sqrt{n}} t_u(n) \) and \( D_U = D_{\max} + \frac{s}{\sqrt{n}} t_u(n) \) when \( n \leq 30 \) or \( D_L = D_{\min} - \frac{s}{\sqrt{n}} z_u \) and \( D_U = D_{\max} + \frac{s}{\sqrt{n}} z_u \) when \( n > 30 \), where \( t_u(n) \) is the \( 100(1 - a) \) percentile of the \( t \) distribution with \( n \) degrees of freedom and \( z_u \) is the \( 100(1 - a) \) percentile for the standard normal distribution, that is, if \( Z \) is \( N(0,1) \) distribution then \( P(Z \geq z_u) = a \).

Definition 2.10 [5]
Assume there are \( m \) linguistic values under consideration. Let \( A_i \) be the fuzzy number that represents the \( i \)th linguistic value of the linguistic variable where \( 1 \leq i \leq m \). The support of \( A_i \) is defined to be

\[
\left\{ D_L + (i-1) \frac{D_U - D_L}{m}, D_L + i \frac{D_U - D_L}{m}, D_L + (i-1) \frac{D_U - D_L}{m}, D_L + i \frac{D_U - D_L}{m}, 1 \leq i \leq m - 1 \right\}
\]

\[
\left\{ D_L + (m-1) \frac{D_U - D_L}{m}, D_L + m \frac{D_U - D_L}{m}, D_L + (m-1) \frac{D_U - D_L}{m}, D_L + m \frac{D_U - D_L}{m}, i = m \right\}
\]

3. A new definition for forecasting with fuzzy time series

In the following, we propose a new definition to forecast by fuzzy time series. Generally, our method is a modification of the method proposed by Lee and Chou [5]. We add a new definition is more complete than Lee and Chou method due to the fact that we define stability concept of the fuzzy time series that represent the increasing and the decreasing case more appropriately.

In this article, we apply difference test to understand whether information are in stable state. If the information location is in unstable state, do the second difference to divide and continue assaying, recursion until the information are in stable state again. Let \( d(t) - d(t-1) \) is difference value, \( d(t) \) is \( i^{th} \) historical data, \( d(t-1) \) is \( (t-1)^{th} \) historical data respectively.

Definition 3.1
For test \( H_0: \text{non fuzzy trend} \) against \( H_1: \text{fuzzy trend} \), the test given by

\[
P(f(t) > f(t-1)) \geq 1 - \lambda,
\]

\[
P(f(t) > f(t-1)) \geq 1 - \lambda,
\]

\[
P(f(t) > f(t-1)) = P(f(t-1) > f(t)) = 0.5,
\]

\[
0 < \lambda < 0.5.
\]

Where Critical region:
Our forecasting algorithm based on fuzzy time series works as follows:

**Step 1:**
Let \( d(t) \) be the history data under consideration and \( F(t) \) be the fuzzy time series. Difference test to understand whether information are in stable state according to definition 3.1., recursion until the information are in stable state again. Where critical region \( C^* = \{ C \mid C_1^{-1} + C_2^{-1} > C_1 = C_2^{*} \times (1 - \lambda) \} \).

**Step 2:**
Determine the universe of discourse \( U = [D_L, D_U] \) according to definition 2.9.

**Step 3:**
Following the definition of 2.10, define \( A_i \) by letting its membership function
\[
\begin{align*}
for \ x \in [D_L + (i-1) \frac{D_L - D_U}{m}, D_L + i \frac{D_L - D_U}{m}] \text{ where } i \leq m-1 \\
&= 0 \text{ otherwise.}
\end{align*}
\]

**Step 4:**
Then \( F(t) = A_i \) if \( d(t) \in \text{supp}(A_i) \), where \( \text{supp}(\cdot) \) denote the support.

**Step 5:**
Derive the transition rule from period \( t-1 \) to \( t \) and denote it as \( F(t-1) \rightarrow F(t) \). Aggregate all transition rules. Let the set of rules be
\[
R = \{ r_i : P_j \rightarrow Q_k \},
\]

**Step 6:**
The value of \( d(t) \) can be predicted by fuzzy time series \( F(t) \) as follows:
Let \( T(t) = \{ P_j \mid d(t) \in \text{supp}(P_j) \}, \) where \( r_i \in R \) be the set of rules fired by \( d(t) \), where \( \text{supp}(P_j) \) is the support of \( P_j \). Let \( \text{supp}(P_j) \) be the median of \( \text{supp}(P_j) \). The predicted value for \( d(t) \) is
\[
\sum_{r_j \in T(t-1)} \frac{\text{supp}(Q_j)}{|T(t-1)|}.
\]

### 4. Forecasting history data with our method

We verify our method by forecasting the container through by harbors in Taiwan area from 1973 to 2003 as shown in Table 1.

**Step 1:**
Let \( d(t) \) be the history data under consideration and \( F(t) \) be the fuzzy time series. Difference test to understand whether information are in stable state according to definition 3.1., recursion until the information are in stable state again. 

**Difference one**
\[
C^* = \{ C \mid C_1^{*1} \times (1 - 0.2) \} = 372.
\]

**Difference two**
\[
C^* = \{ C \mid C_1^{*2} + C_2^{*2} \} = 214 < \{ C \mid C_1^{*2} \times (1 - 0.2) \} = 348.
\]

**Step 2:**
Determine the universe of discourse \( U = [D_L, D_U] \) according to Table 1.

**Step 3:**
Assume the following linguistic values are under consideration: extremely few, very few, few, some, many, very many, and extremely many. Following Definition 2.10, the supports of fuzzy numbers that represent the linguistic values are given by
\[
\begin{align*}
\text{supp}(A_1) &= [179,430,208,765) \\
\text{supp}(A_2) &= [28,675,236,780) \\
\text{supp}(A_3) &= [236,780,444,885) \\
\text{supp}(A_4) &= [444,885,652,990) \\
\text{supp}(A_5) &= [652,990,861,095) \\
\text{supp}(A_6) &= [861,095,1,069,200) \\
\text{supp}(A_7) &= [1,069,200,1,277,302).
\end{align*}
\]

**Step 4:**
The fuzzy time series \( F(t) \) is given by
\[
F(t) = A_i \text{ when } d(t) \in \text{supp}(A_i).
\]

### 5. Forecasting Process

**Step 5:**
The transition rules are derived from Table 1. For example, \( F(1974) \rightarrow F(1975) \) is \( A_1 \rightarrow A_2 \). All transition rules obtained from Table 1 are shown in Table 2.

**Step 6:**
We are going to forecast based on Table 2. The forecasting results from 1974 to 2003 are shown in Table 3. The forecasting process of year 2004 are illustrated in the following. The same process can be applied to other years.
Year 2004 To predict the enrollment of 2004, let $t = 2004$. Since $d'(t - 1) = d'(2003) = 486,119 \in \text{supp}(A_4)$, $T(t - 1) = T(2003) = \{r_1: A_1 \rightarrow A_2, r_2: A_1 \rightarrow A_3, r_3: A_1 \rightarrow A_4\}$. Thus the forecasted value for difference one $d(t) = d(1972)$ is

$$\sum_{r_j(2003)} \text{supp}(Q_j) = \text{supp}(A_1) + \text{supp}(A_2) + \text{supp}(A_3) \approx 340,832.$$ 


Table 1. Fuzzy historical TEUs

<table>
<thead>
<tr>
<th>Year</th>
<th>TEUs</th>
<th>d(t)</th>
<th>d'(t)</th>
<th>d''(t)</th>
<th>Fuzzy TEUs</th>
</tr>
</thead>
<tbody>
<tr>
<td>1973</td>
<td>370,372</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1974</td>
<td>391,354</td>
<td>20,982</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td>473,353</td>
<td>81,999</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1976</td>
<td>661,866</td>
<td>188,513</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1977</td>
<td>755,258</td>
<td>93,392</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1978</td>
<td>1,062,561</td>
<td>307,303</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979</td>
<td>1,374,192</td>
<td>311,831</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>1,644,376</td>
<td>269,984</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1981</td>
<td>1,787,758</td>
<td>143,382</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1982</td>
<td>1,902,264</td>
<td>114,506</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1983</td>
<td>2,429,310</td>
<td>527,046</td>
<td></td>
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<tr>
<td>1984</td>
<td>3,026,846</td>
<td>597,536</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>3,075,151</td>
<td>48,305</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>4,104,953</td>
<td>1,029,802</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>4,772,339</td>
<td>667,386</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1988</td>
<td>4,941,022</td>
<td>168,683</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td>5,263,091</td>
<td>322,069</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1990</td>
<td>5,463,566</td>
<td>200,475</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1991</td>
<td>6,129,667</td>
<td>666,101</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>1992</td>
<td>6,178,712</td>
<td>49,205</td>
<td></td>
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<tr>
<td>1993</td>
<td>6,824,973</td>
<td>646,101</td>
<td></td>
<td></td>
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<tr>
<td>1994</td>
<td>7,307,304</td>
<td>482,531</td>
<td></td>
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<tr>
<td>1995</td>
<td>7,665,178</td>
<td>357,874</td>
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</tr>
<tr>
<td>1996</td>
<td>7,866,995</td>
<td>201,817</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1997</td>
<td>8,520,199</td>
<td>655,204</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>8,858,211</td>
<td>338,012</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1999</td>
<td>9,757,648</td>
<td>899,437</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2000</td>
<td>10,510,762</td>
<td>753,114</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2001</td>
<td>10,427,714</td>
<td>-83,048</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2002</td>
<td>11,608,634</td>
<td>1,180,920</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2003</td>
<td>12,094,753</td>
<td>486,119</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Fuzzy transitions derived from Table 1.

<table>
<thead>
<tr>
<th>r_1: A \rightarrow A</th>
<th>r_2: A \rightarrow A</th>
<th>r_3: A \rightarrow A</th>
</tr>
</thead>
<tbody>
<tr>
<td>r_4: A \rightarrow A</td>
<td>r_5: A \rightarrow A</td>
<td>r_6: A \rightarrow A</td>
</tr>
<tr>
<td>r_7: A \rightarrow A</td>
<td>r_8: A \rightarrow A</td>
<td>r_9: A \rightarrow A</td>
</tr>
</tbody>
</table>

5. Conclusions

In this article, we add a useful definition to improve original method [4] to forecast the strict increasing and decreasing case based on fuzzy time series. From the illustrative example, we can see that our definition not only define the trend of the fuzzy numbers that represent the linguistic values of the linguistic variable more appropriately, but also can make the fuzzy time series reasonable. In summary, our method is as accurate as Lee and Chou method [4] but more reasonable in forecasting the strict increasing and decreasing case.

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7. References