Analysis of Dynamic Mathematical Model of Bi-directional Contactless Inductive Power Transfer System Based on Generalized State-space Averaging Method

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Abstract—Based on generalized state-space averaging method (GSSA) and selective modal analysis method, a dynamic mathematical model was built to depict the dynamic characteristic of the bi-directional contactless inductive power transfer (CIPT) system. The model has already been tested by Matlab, and the simulation results illustrate that the model can give a good description of characteristics of bi-directional CIPT system.

Keywords—Bi-directional CIPT, Generalized state-space averaging method, Selective modal analysis method, System order reduction

I. INTRODUCTION

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The technology of CIPT has already been widely applied in a variety of power applications. Since it won’t generate any electric spark and abrasion, it is safety and meaningfully to introduce it to practical application [1]. But at certain times, loads which has already been charged don’t need to keep working continually, and the power grid maybe suffers the peak of power consuming. So, it is necessary to find a way to transfer the energy from loads to the grid to buff the power-consuming peak and to reduce the negative impact on it. At the same time, the natural wastage of electric energy of the batteries can be reduced to a lower level.

The bi-directional CIPT system can transmit the energy in both directions so when it comes the power-consuming peak, the energy-storage devices can provide some support to the grid to reduce the negative concussion [2]. Now it is one of the research focuses of power transmission. In order to obtain the greatest possible efficiency of power transmission capacity, researchers tried so many ways to make the primary and secondary subsystem coordinate closer with each other. But as the coils are loosely coupled and electromagic noise is inevitable, it’s difficult to achieve an ideal control system via setting up a traditional communication system.

This paper proposed a dynamic model to simulate the bi-directional system. To build the dynamic model, the mutual inductance electrical model should be set up, and then be analyzed with generalized state-space averaging method. The dynamic mathematical model of the system should be framed at the end.

II. THE ELECTRICAL MODEL OF THE SYSTEM

The schematic of the system is shown in Figure 1.

The equations of the system listed below can be drafted on the basis of the Kirchhoff’s voltage law and current law:

\[
\begin{align*}
\dot{V}_1 &= \dot{V}_2 + j\omega M_{12} + V_{c1} \\
\dot{i}_1 &= C_1 \frac{dV_{c1}}{dt} \\
\dot{i}_2 R_2 &= L_2 \frac{di_2}{dt} + j\omega M_{12} + V_{c2} \\
&\text{and} \\
\dot{i}_2 &= C_2 \frac{dV_{c2}}{dt}
\end{align*}
\]
The twelve state variables should be written as: zero-order and first-order Fourier coefficient, formulas with zero-order and first-order Fourier coefficient, formulas with

\[
\begin{aligned}
&i_1 = x_1 + jx_2, \\
&V_{c1} = x_3 + jx_4, \\
&i_2 = x_5 + jx_6, \\
&V_{c2} = x_7 + jx_8, \\
i_0 = x_9, \quad V_{c0} = x_{10}, \quad i_{20} = x_{11}, \quad V_{c20} = x_{12}
\end{aligned}
\]

And further more, the formula below can be drawn:

\[
\begin{aligned}
&i_{11} = i_{11}^*, \\
&V_{c11} = V_{c11}^*, \\
&i_{21} = i_{21}^*, \\
&V_{c21} = V_{c21}^*
\end{aligned}
\]

The Fourier form of formula (2) is

\[
s(t)_0 = 0, \quad s(t)_1 = -2j/\pi
\]

Formulas listed above can get the state space equation of zero-order and first-order couple with each other.

\[
\begin{aligned}
&i_R = L_1 \frac{di_1}{dt} + j\omega M_1 + V_{c1}, \\
i_1 = C_1 \frac{dV_{c1}}{dt}, \\
s(t)E_c = L_2 \frac{di_2}{dt} + j\omega M_1 + V_{c2}, \\
i_2 = C_2 \frac{dV_{c2}}{dt}
\end{aligned}
\]

in which and stands for the current in primary and secondary coil, \(V_{c1}\) and \(V_{c2}\) are the voltage of the compensation capacitors in primary and secondary subsystem respectively. \(S(t)\) is a switching function of the H-bridge resonant converter which can be shown as:

\[
s(t) = \begin{cases} 
1 & \text{if } T \leq t < (2m+1)T/2 \\
0 & \text{if } (2m+1)T/2 \leq t < (m+1)T 
\end{cases}
\]

where \(m\) is a positive integer.


The actual state variables of the system can be depicted as:

\[
x(t) = x_0e^{-j\omega t} + x_1e^{j\omega t}
\]

To describe \(i_1, V_{c1}, i_2\) and \(V_{c2}\) with the model formed by zero-order and first-order Fourier coefficient, formulas with 12 variables should be written as:

\[
\begin{aligned}
&i_{11} = i_{11}^*, \\
&V_{c11} = V_{c11}^*, \\
&i_{21} = i_{21}^*, \\
&V_{c21} = V_{c21}^*
\end{aligned}
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To describe \(i_1, V_{c1}, i_2\) and \(V_{c2}\) with the model formed by zero-order and first-order Fourier coefficient, formulas with 12 variables should be written as:

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\end{aligned}
\]

The actual state variable of the system can be depicted as:

\[
x(t) = x_0e^{-j\omega t} + x_1e^{j\omega t}
\]


\[
\begin{bmatrix}
\frac{-R}{L_1} & \omega & -1 & 0 & 0 & \omega M & 0 & 0 \\
-\omega & 0 & \frac{-1}{L_4} & \omega M & 0 & 0 & 0 & 0 \\
1 & \frac{-1}{C_1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{C_1} & -\omega & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{\omega M}{L_4} & 0 & 0 & 0 & \omega & 0 & -\frac{1}{L_2} \\
0 & 0 & 0 & 0 & \frac{1}{C_2} & 0 & 0 & \omega \\
0 & 0 & 0 & 0 & 0 & \frac{1}{C_2} & -\omega & 0 \\
\end{bmatrix}
\]

\(A_i = \begin{pmatrix}
B = \begin{bmatrix}
0 & -2 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -2 & \frac{\omega M}{L_4} & 0 & 0 \\
\end{bmatrix}
\end{pmatrix} \quad \text{(13)}
\]

\(C = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

\( \text{Formula (8) to (14) can formate a GSSA model of the bi-directional CIPT system.}
\]

The formula of the state variables can be written on the basis of formula (3) as:

\[
\begin{align*}
i_1 &= 2x_1 \cos (\omega t) - 2x_1 \sin (\omega t) + x_9 \\
V_{c1} &= 2x_3 \cos (\omega t) - 2x_4 \sin (\omega t) + x_{10} \\
i_2 &= 2x_5 \cos (\omega t) - 2x_6 \sin (\omega t) + x_{11} \\
V_{c2} &= 2x_7 \cos (\omega t) - 2x_8 \sin (\omega t) + x_{12}
\end{align*}
\]

\( \text{IV. SIMULATION OF THE DYNAMIC MATHEMATICS MODEL}
\]

The simulation results of the model based on GSSA method and the model based on direct electric circuit topology are shown in figures below.

The parameters of system comparative simulation are:

\[
\begin{align*}
L_1 &= 130 \mu H \\
L_2 &= 130 \mu H \\
M &= 130 \mu H \\
C_1 &= 0.2034 \mu F \\
C_2 &= 0.2034 \mu F \\
f &= 40 \text{kHz} \\
R_1 &= 27 \Omega \\
R_2 &= 27 \Omega \\
E_g &= 100 V \\
E_e &= 100 V
\end{align*}
\]

\(i_1 \text{ based on GSSA method and circuit topology}
\]

\(i_2 \text{ based on GSSA method and circuit topology}
\]
in analyzing the technology of bi-directional contactless inductive power transfer. The representative system was fully analyzed by the method, and the simulation results were compared to results simulated by circuit topology. Apparently, the deviation is acceptable. The models proposed in this paper can be used in system designing and stability assessing for a bi-directional CIPT system.

REFERENCES