A Behavioral Fuzzy Model For Analysis of Overreaction and Underreaction in the Brazilian Stock Market

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Abstract

In this paper empirical tests for the overreaction and underreaction hypothesis in the Brazilian stock market are presented. For these tests, due to the complexity of these phenomena, a new model based on the fuzzy set theory is proposed. It is shown that such model is strongly connected with two heuristics of behavioral finance: representativeness and anchoring. The proposed model is used to form portfolios based on financial indexes of open firms. The analysis is applied for stocks from petrol/petrochemical and textile firms, with financial indexes ranging from 1994 to 2005.

Keywords: Overreaction, Underreaction, Fuzzy Sets, Behavioral Finance, Stock Classification.

1. Introduction

The development of tools to help in the decision making of investors in the financial area has been object of intense research world over for some decades. Generally speaking, financial decisions aim at maximizing future profit and, in this context, the evolution of the basic premises has given birth to the appearance of clashing theories as, for instance, the theory of effective markets and the theory of behavioral finance, [11]. Each of these theories has been structured with basis on the fundamental contributions of a large number of authors. Nonetheless, the fact that each one of them contains in its origins works which have been awarded the Nobel Prize for Economics, such being the case of the portfolio theory proposed by H.M. Markowitz, [8], and the theory of behavioral finance proposed by D. Kahneman and V. L. Smith, [5], is undeniably of the utmost relevance. This is enough to give a clear idea of the importance of decision making in the economic theory.

The target of the present paper is the study of phenomena of overreaction and underreaction in what concerns the Brazilian stock market. Some of the papers which focus on similar problems in the Brazilian as well as in the international stock markets are: [3], for the American market, [6], for the Chinese market and [7], for the Brazilian market.

The bond still hardly investigated in the literature, as observed in [9], between the theory of fuzzy sets, originally proposed by L. A. Zadeh in 1965, [12], and the heuristics of the theory of behavioral finance is explored for the developments here presented. A methodology is developed for the rating of a set of stocks in the Brazilian market with basis on financial indexes of the corresponding firms. It is thus demonstrated that the proposed fuzzy behavioral model incorporates heuristic influences from the behavioral theory. Empirical tests for the overreaction and underreaction hypothesis are presented using data from 1994 to 2005.

The importance of phenomena of overreaction and underreaction lies in the fact that in decision making overreaction justifies the option for the use of the strategy named contrarian strategy while underreaction tends to justify the choice of the strategy of momentum.

2. Mathematical Basis

The fuzzy set theory possesses as one of its main characteristics the fact of allowing the treatment of linguistic variables, such as hot, very hot, high, low, advisable, not advisable, highly risky, etc.

The resulting property when considering linguistic variables to characterize objects is that, instead of belonging or not to a certain set, as stated by the classic set theory, these objects will have pertinence indexes associated with different sets. A detailed presentation of the main concepts of the fuzzy theory can be found in [12] and [13].

Definition 1: Let the set $X = \{x_1, x_2, ..., x_m\}$, $C_1, C_2, ..., C_n$ subsets of X and real numbers $0 \le \mu_i(x_j) \le 1,\ i = 1,2,...n$ e $j = 1,2,..., m$, such that, for every $j = 1,2,..., m$, one has $\sum_{i=1}^{n} \mu_i(x_j) = 1$. Under these conditions, $\mu_i(x_j)$ is denoted membership degree of the element x_j with respect to C_i.

The membership degree may be understood as a measure of the degree of affinity, similarity or compatibility among elements.

Among the techniques for the grouping or classification of elements in subsets of a given set, the Fuzzy Clustering Means – FCM algorithm has been proved to be an effective tool in those cases in which the features or attributes of the analyzed elements can be represented by a vector of real numbers. In such cases, the FCM algorithm allows identifying clusters of elements from a matrix of dimension $n \times p$, being n the number of elements and p the dimension of the vectors of features of these elements, [13]. As in the specific application of this paper the analyzed elements are grouped in 2 subsets only, the presentation is specified for this case. Thus, let $x_1, x_2, ..., x_m$ elements of X and consider the problem of grouping these elements in 2 subsets C_1 e C_2. The FCM algorithm determines the subsets C_1 and C_2 via the solution of the following problem.

Given $x_1, x_2, ..., x_m$, described as vectors of dimension p, determine the vectors c_1 and c_2, also of dimension p, and $\mu_1(x_j) \geq 0$ and $\mu_2(x_j) \geq 0, \;\; j = 1,2,...,m$, such that $\mu_1(x_j) + \mu_2(x_j) = 1, \; j = 1,2,...,m$ and the function $\sum_{i=1}^{2}\sum_{j=1}^{m}[\mu_i(x_j)^2 \|x_j - c_i\|^2]$ is minimized.

The solution of such optimization problem is given by, [2]:

$$c_i = \frac{1}{\sum_{j=1}^{m}(\mu_i(x_j))^2}\sum_{j=1}^{m}(\mu_i(x_j))^2 x_j \quad i = 1,2 \quad (1)$$

$$\mu_i(x_j) = \frac{\frac{1}{\|x_j \quad c_i\|^2}}{\sum_{k=1}^{m}\|x_k - c_i\|^2} \quad i = 1,2 \quad j = 1,2,...,m \;\; (2)$$

Vectors c_i are called centers. If $\mu_1(x_i) > \mu_2(x_i)$ one says that the element x_i is associated to C_1 and if $\mu_2(x_j) > \mu_1(x_j)$ then x_j is associated to C_2.

As one can well observe, the calculation of c_i, through equation (1), depends on $\mu_i(x_j)$. These, on their turn, depend on c_i, according to the equation (2). The solution can be obtained iteratively, by the algorithm named FCM, whose steps are next described.

Step 1: Initiate with membership degrees, such that $\mu_1(x_j) + \mu_2(x_j) = 1, \; j = 1,2,...,m$ and $\mu_1(x_j) \geq 0$ and $\mu_2(x_j) \geq 0, \;\; j = 1,2,...,m$;

Step 2: Calculate the centers c_1 and c_2, by the equation (1);

Step 3: Recalculate the new membership degrees, via equation (2), by utilizing the centers obtained in step 2.

Repeat steps 2 and 3 until the objective function does not decrease, according to the assumed precision. In order to avoid local minima, it is usual to run the algorithm with different initial partitions. In the case of square-error based algorithms, if the final clustering is always the same, we get some confidence that the global minimum has been achieved, [13].

3. Behavioral Finance

According to the behavioral theory, individuals make decisions guided by heuristics, or practical rules, thinking in a way which deviates from the statistic rules.

Several heuristics influence decision making. In particular, the heuristics of representativeness and anchoring, [9], are directly related to the theory of fuzzy sets, in the way it is seen in this paper.

Briefly speaking, the heuristic of representativeness is associated with the similarity between the considered elements. In the context of decision making in economics, the individuals under the influence of the heuristic of representativeness tend to produce extreme predictions, or overreaction, in which former losers tend to be winners in the future and vice-versa, [13]. In the context of the fuzzy theory the similarity is directly related to the membership degree, which is a more suitable tool to describe such characteristic than statistic methods, [10].

The heuristic of anchoring establishes that people often base themselves on elements or conditions of reference in order to make decisions. The heuristic of anchoring, differently from the heuristic of reprentativeness, leads to excessive moderation in decision making, thus causing the underreaction phenomenon, in which former winners tend to be future winners, and former losers tend to be future losers, [13]. In terms of the theory of fuzzy sets, a decision based on this heuristic is focused on the element of stronger reference in the set, that is, the element of total membership, $\mu(x) = 1$, [9].

4. Behavioral Fuzzy Model

In this section, the methodology employed for the development of the behavioral fuzzy model is introduced. This model comprehends two steps: pattern recognition and stock rating. The data or features of the stocks utilized by the model are financial indexes of open firms, including some return indexes related to stock evaluation, profitability and debt. These indexes have been collected every trimester from the Economatica data base, [4], between the 4th trimester/1994 and the 3rd trimester/2005. The two steps of the proposed model are next described.

Step 1: in this step, named pattern recognition, the FCM algorithm classifies the stocks of a given group. This analysis has been based on data in the period between the 4th trimester/1994 and the 3rd trimester/2000. In each trimester t, the FCM algorithm was applied to the pattern matrix $n \times p$, in which each line corresponds to one firm of the group and each column corresponds to the financial indexes associated to the firms. Two clusters have been obtained and the average financial return that each cluster produces at the end of the trimester $t+1$ is calculated according to the equation (3).

$$r_{t+1} = \frac{1}{n}\sum_{i=1}^{n}\frac{P_{t+1}^{i} - P_{t}^{i}}{P_{t}^{i}} \qquad (3)$$

where P_t^i is the value of stock i at the end of trimester t and n is the number of stocks classified.

The cluster with larger average financial return is called good and the one with smaller average financial return is called bad. In this step, the classification of the groups as good or bad has been possible only at the end of the trimester $t+1$, that is, the classification is a posteriori.

Step 2: the aim of this step is to classify, at the end of trimester t, the cluster of stocks with performance supposedly good or bad at the end of trimester $t+1$. Thus, differently from step 1, the aim is to set a classification a priori.

For this second step the clusters formed by the centers of the 1st trimesters, 2nd trimesters, 3rd trimesters and 4th trimesters from 1994 to 2000 are considered separately. Then, the FCM algorithm is applied again in order to identify a winner and a loser center for each set of 1st trimesters, 2nd trimesters, 3rd trimesters and 4th trimesters.

For the classification of a stock at the beginning of a particular trimester, it is enough to calculate the membership degrees related to the winner and the loser centers corresponding to that trimester. For each trimester, the group of promising stocks will be called winner portfolio and the group of non-promising stocks will be called loser portfolio. For the numerical results, the classification of stocks was made in the period between the 1st trimester/2001 and the 3rd trimester/2005.

5. Tests of hypothesis

The phenomena of overreaction and underreaction have been largely investigated through empirical research, [3], [7]. The procedure for the empirical tests of such hypothesis along with the tests of statistic significance presented in this paper are similar to those presented in [3].

Firstly, by using the winner and the loser centers, stocks are classified as described in section 3. Thus, winner and loser portfolios for each trimester are formed. Next, the corresponding residual return is calculated for each week of the trimester, according to the equations (4)-(6), for the winner portfolio.

$$RR_{t+1,j}^{W} = r_{t+1,j}^{W} - r_{t+1,j}^{Ibov} \qquad (4)$$

$$r_{t+1,j}^{W} = \frac{1}{n}\sum_{i=1}^{n}\frac{P_{t+1,j}^{i} - P_{t}^{i}}{P_{t}^{i}} \qquad (5)$$

$$r_{t+1,j}^{Ibov} = \frac{Ibov_{t+1,j} - Ibov_{t}}{Ibov_{t}} \qquad (6)$$

where $RR_{t+1,j}^{W}$ is the residual return for the portfolio in the week j of the trimester $t+1$, $r_{t+1,j}^{W}$ is the return for the portfolio in the week j of the trimester $t+1$, $r_{t+1,j}^{Ibov}$ is the return associated with the Bovespa (the stock market of Sao Paulo/Brazil) in the week j of the trimester $t+1$, $P_{t+1,j}^{i}$ is the value of the stock i, of the portfolio at the end of the week j of the trimester $t+1$, P_t^i is the value of the stock i of the portfolio at the end of trimester t, $Ibov_{t+1,j}$ is the Bovespa index at the end of week j of the trimester $t+1$, $Ibov_t$ is the Bovespa index at the end of trimester t and n is the number of stocks in the portfolio. Similar calculations are performed for the loser portfolio.

From the residual returns corresponding to the weeks of each trimester, the average residual returns RRM_t^W and RRM_t^L are calculated, from the 1st trimester/2001 up to the 3rd trimester/2005. The hypothesis of overreaction says that $RRM_t^W \quad RRM_t^L < 0$, [7]. In order to evaluate whether the difference between average residual returns in each trimester is meaningful, a test of statistic-t is performed. The null hypothesis to be tested is $H_0 : RRM_t^L - RRM_t^W = 0$, against the alternative hypothesis of overreaction $H_A : RRM_t^W \quad RRM_t^L < 0$. Similarly, the alternative hypothesis for underreaction is $H_A : RRM_t^W - RRM_t^L > 0$.

6. Results

In this section the behavioral fuzzy model proposed in section 3 is utilized to form portfolios and, next, tests for the hypothesis of overreaction and underreaction are performed by using the methodology of section 4. The table 2 shows the results of this procedure for trimesters from 2001 to 2005 in the case of stocks of the petrol/petrochemical sector. In this case evidences of statistically meaningful overreaction are present.

Trimester/ year	$RRM_t^V - RRM_t^P$	Test-t	Over/under -reaction
1st trim/2001	19,017	6,297*	Under
2nd trim/2001	10,626	4,331*	Under
3rd trim/2001	11,784	3,202*	Under
4th trim/2001	-0,463	-0,201	Over
1st trim/2002	-9,974	-2,788*	Over
2nd trim/2002	-7,439	-2,215*	Over
3rd trim/2002	-16,948	-5,014*	Over
4th trim/2002	-8,867	-3,275*	Over
1st trim/2003	-8,729	-2,202*	Over
2nd trim/2003	-13,514	-1,804**	Over
3rd trim/2003	6,382	1,26	Under
4th trim/2003	-5,474	-1,537	Over
1st trim/2004	-2,726	-1,282	Over
2nd trim/2004	-1,749	-0,523	Over
3rd trim/2004	-16,598	-3,076*	Over
4th trim/2004	16,306	2,654*	Under
1st trim/2005	-1,099	-0,676	Over
2nd trim/2005	-2,024	-1,351	Over
3rd trim/2005	-3,847	-1,792**	Over

Table 2 - Average residual returns and Test-t for the petrol/petrochemical sector statistically meaningful at the level of (*)5% and (**)10%.

The same procedure has been applied for stocks of the textile sector, for which evidences of underreaction were observed.

7. Conclusion

In this paper, a model for stocks rating is proposed, with the purpose of the realization of empirical tests of the overreaction and underreaction hypothesis in the Brazilian stock market.

The proposed model is based on the theory of fuzzy sets, which on its turn is closely related to the theory of behavioral finance, thus suggesting the name behavioral fuzzy model. To form the portfolios, the proposed model utilizes financial indexes of open firms. Two sets of stocks are used in this study: stocks of the petrol/petrochemical sector and of the textile sector. The petrol/petrochemical sector presents statistically meaningful evidences of overreaction while the portfolio formed by stocks of the textile sector presents evidences of underreaction. This fact, added to the existing bond between fuzzy sets and the theory of behavioral finance points to influences of the heuristics of representativeness and anchoring in the behavior of the Brazilian stock market. For future work, the development of neural network based models may be a promising research line.

8. REFERENCES

[1] Amir, E.; Ganzach, Y.. "Overreaction and Underreaction in Analysts' Forecasts". *Journal of Economic Behavior & Organization*, v. 37, p. 333-347, 1998.

[2] Bezdek, J. C.. "Pattern Recognition with Fuzzy Objective Function Algorithm". *Plenum Press, New York and London*, 1981

[3] De Bondt, W. F. M.; Thaler, R. H.. "Does the Sock Market Overreact?" *Journal of Finance*, v. 40, n.3, p. 793-807, 1985.

[4] Economática Ltda. "Software of Support to Investors", www.economatica.com.br, acessed in 2004

[5] Kahnemann, D.. "Maps of Bounded Rationality: A Perspective on Intuitive Judgment and Choice". *Prize Lecture*, December 2002.

[6] Kang, J., Liu, M., Ni, Sophie X.. "Contrarian and Momentum Strategies in the China Stock Market: 1993-2000". *Pacific-Basin Finance Journal*, v.10, p. 243-265, 2002.

[7] Kimura, H.. "Aspectos Comportamentais Associados às Reações do Mercado de Capitais – Behavioral Aspects Associated with the Reactions of the Stock Market". *RAE-eletrônica*, v. 2, n. 1, jan./jun. 2003.

[8] Markowitz, H.M.. "Portfolio Selection". *New York: John Wiley & Sons*, 1959.

[9] Peters, E.. "Chaos and Order in the Capital Markets", *2ª edição. New York: Wiley*, 1996.

[10] Peters, E.. "Simple and Complex Market Inefficiencies: Integrating Efficient Markets, Behavioral Finance, and Complexity". *The Journal of Behavioral Finance*, v. 4, Nº 4, 2003.

[11] Shiller, R.J.. "From efficient market theory to behavioral finance. Cowles foundation for research in economics", *Yale University. Cowles foundation discussion*, paper No. 1385. http://cowles.econ.yale.edu/, oct/2002

[12] Zadeh, L.A.. "Fuzzy sets". *Information and Control*, vol. 8, p.338-353, 1965.

[13] Zimmermann, H.J.. "Fuzzy Set Theory and its Aplication". *Kluwer Academic*, Boston, 1996.