

Collision risk estimation on parallel airways

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Abstract: In this paper, for purposes of airspace planning, by the theory of statistical-probabilistic and theory of traffic engineering a mathematical expression of the parallel airways based on the variable nominal distance and airspace planning parameters is given. By using the estimation formula proposed the average number of collisions between the parallel airways can be calculated and the impacts of airspace planning parameters on the collision risk can be shown. A method to characterize the collision risk when the aircraft are flying on parallel airways and the collision risk when the nominal distances on parallel airways are variable is given in the progress of airspace planning which may provide some theoretical basis for airspace planners to determine the lateral separation minimum, and may give some references basis for researchers of air traffic flow to study the capacity of airspace and air traffic controllers to keep airspace safe.

1. Introduction

In the field of air traffic management, the results of risk analysis will affect the actions performed of air traffic controllers because a main task of air traffic management is to contribute to risk reduction by keeping safe separation minima and administrations may use the methodology as a tool to estimate the collision risk in determining safe separation minima^[1,2]. Every change of the operating procedure implementation in the progress of planning airspace and air routes is preceded by risk analysis in order to make the flight safe^[3]. Airspace planners need to assess different scenarios for airspace development. Since aircraft safety level must be maintained when a new planning program is introduced in airspace, it is necessary to estimate the collision risk of this new planning program^[2], thus it is significant to integrate the collision risk estimation with airspace planning parameters in the revised airspace.

A collision risk model for long-range parallel air-routes over oceanic areas was given by Reich P. G. in 1966. Mr. Reich have studied the separation standards, collision risk and the quality of navigation which would be needed for safe reduction in standard^[4]. The Reich model has a great influence on collision risk model. The problems of safe separations are resolved well by Reich and the model is applied extensively^[5,6,7]. But the derivation of Reich model is a comparatively abstract one which does not make it easy to see what the key parameters are. Hsu researched the aircraft collision risk problem of intersecting air-routes for aircraft pairs flying on level air-routes at the same altitude with the invariable speed and both the Gaussian and the double double exponential model are used for the position error distributions to compute the aircraft overlap probability^[8]. A Post-Reich' event model was developed by Brooker for lateral collision risk in air traffic track systems and applied to lateral collision in the North Atlantic Region track system, which resolves the problems of the Reich model^[9]. It is a direct and concrete approach focusing on events, in contrast to Reich's

synthetic methodology. Some researches about the collision risk based on event model are given^[10,11]. In virtue of the model in reference[2], Han songchen and Qu yuling researched the collision risk model of terminal area airspace and around intersection of airways based on arrival time interval^[12,13].

The researches above of the collision risk on parallel airways are mainly apt to invariable nominal distances. At the same time in considering the situation of stochastic aircraft flow, the existing collision models are apt to use the distribution function of initial nominal distance separation on parallel airways. Though in reference [2] the time variable t is introduced in expressing the distance function, in the progress of derivation of collision probability the fixed time $t = T$ is used instead of the variable t . In contrast to the most exiting collision risk models which are mainly about the invariable nominal invariable, in this paper, the further research on collision risk based on the variable nominal invariable, time variable and initial time interval variable on parallel airways will be shown. Based on the references [2,12,13], this paper is intended for use by airspace planners as a basis for changing separation minima and presents a mathematical expression for the analysis of collision risk applicable to airspace planning parameters and lateral separation of aircraft on parallel airways.

2. Basic assumptions and definitions

- 1) It is assumed that the aircraft are flying along a straight line flight path at the same nominal level on parallel airways.
- 2) Denote one airway by K and another airway by L .
- 3) Denote the length of airway K by dis_K and the length of airway L by dis_L .
- 4) Denote the starting point and stopping point of airway K by A and B respectively. Denote the starting point and stopping point of airway L by C and F respectively.
- 5) Aircraft 1 is flying on the airway K and the type is i , and aircraft 2 is flying on the airway L and the type is j . $i = 1, 2, \dots, k$, $j = 1, 2, \dots, l$, k and l is the number of types of aircraft on the airway K and on the airway L respectively.
- 6) It is assumed that the aircraft 1 arrives at the starting point A earlier than the aircraft 2 arrives at the starting point C .
- 7) The time is $t = 0$ when aircraft 2 arrives at point C .
- 8) Denote the time interval between aircraft 1 arrives at point A and aircraft 2 arrives at point C by Δt and the time interval minimum by Δt_{\min} .
- 9) Denote nominal longitudinal direction by x , lateral direction by y , and vertical direction by z .
- 10) Denote the longitudinal distance between the starting point A of airway K and the starting point C of airway L by dis_x .
- 11) The units of this paper that haven't been given are nautical for distance, hour for time and knot for speed.

3. The collision risk estimation formula

The number of aircraft involved in one collision is two. And the collision risk estimation on parallel airways will be shown. Lateral separation $S_{ij}^y(t, \Delta t) = S_y$ is a fixed value and vertical separation $S_{ij}^z(t, \Delta t) = 0$ for a given planning program by supposing that a randomly chosen pair of aircraft is at the same level. Clearly $V_i(t + \Delta t) - V_j t \cos \theta - \eta \cdot d_x$ is longitudinal nominal distance which is the function of t and varies with t , where, θ is 0° when the aircraft on airway K and the aircraft on airway L fly in the same direction, and θ is 180° when the aircraft on airway K and the aircraft on airway L fly in the opposite direction; η is 1 when aircraft 1 is flying from A to B and C is on the right side of A , and η is -1 when aircraft 1 is flying from A to B and C is on the left side of A ; V_i and V_j is the nominal speed of aircraft 1 and aircraft 2, respectively.

Letting $|\chi_{ij}(t)|$ be the distance in the direction of χ at $t \in [0, t_1]$, then for any fixed

$\Delta t \in [\Delta t_{\min}, dis_K/V_i]$ and any fixed $t \in [0, t_1]$, where $t_1 = \min\{dis_K/V_i - \Delta t, dis_L/V_j\}$, by the reference [13], we have

$$\chi_{ij}(t) = \varepsilon^x + S_{ij}^x(t, \Delta t). \quad (1)$$

Where, $\chi = x, y, z$, and $\varepsilon^x, \varepsilon^y, \varepsilon^z$ is the longitudinal, lateral and vertical error variable respectively.

In this paper, as in Doc9689-An/953^[2], an equivalent geometry that is a cylinder of λ_{xy} radius and height $2\lambda_z$ is used to characterize one aircraft.

Let $S_{ij}^x(t, \Delta t) = V_i(t + \Delta t) - \cos \theta V_j t - \eta \cdot d_x$, $S_{ij}^y(t, \Delta t) = S_y$, $S_{ij}^z(t, \Delta t) = 0$. By the reference [2], for any fixed $\Delta t \in [\Delta t_{\min}, dis_K/V_i]$ and any fixed $t \in [0, t_1]$ we have the collision probability of two aircraft as follow

$$P_{ij}(t, \Delta t; S_y) = P_{ij}^x(t, \Delta t; S_{ij}^x(t, \Delta t)) \times P_{ij}^y(t, \Delta t; S_y) \times P_{ij}^z(t, \Delta t; 0) \times \left(1 + \pi \lambda_x |\bar{z}| / (4\lambda_z V_{rel})\right). \quad (2)$$

where, $P_{ij}^x(t, \Delta t; S_{ij}^x(t, \Delta t)) = P(|\chi_{ij}(t)| \leq \lambda_x)$ and $\lambda_x, \lambda_y, \lambda_z$ is average aircraft height, average aircraft wingspan and average aircraft height respectively; V_{rel} is the relative speed of the two aircraft conditional on a horizontal overlap taking place; $|\bar{z}|$ is the relative speed of the two aircraft conditional on vertical direction.

Furthermore, we can have that the average number of aircraft pairs formed between one aircraft typed i on the airway K and the aircraft on the airway L in the time length t_1 is NP_i . Thus, by the reference [13], when aircraft is flying on the airway K for an hour, the average number of collisions coming from the aircraft on the airway L can be expressed as

$$CR = 2 \sum_{i=1}^k \sum_{j=1}^l (p_{Ki} p_{Lj} NP_i \frac{V_i}{dis_K} \int_{\Delta t_{\min}}^{dis_K/V_i} g(\Delta t) \times \frac{1}{t_1} \times \int_0^{t_1} P_{ij}(t, \Delta t; S_y) dt d\Delta t). \quad (3)$$

Where, $g(\Delta t)$ is the density function of Δt ; p_{Ki} is the proportion of the aircraft type i in the total aircraft on the airway K and p_{Lj} is the proportion of the aircraft type j in the total aircraft on the airway L .

4. The calculation and analysis of collision risk

4.1 Definitions of parameter values

In the following sections, navigation error and speed error are used to calculate collision risk. Generally speaking, the core distribution is Gaussian for some types of errors, some researchers assume that all navigation and tracking errors are Gaussian^[14,15,16]. Because the convolution of two Gaussian densities is another Gaussian density, which avoids the need for numerical convolution calculations, for the consideration of mathematical simplicity, in this paper Gaussian distribution model is selected for describing position errors. Then we have

$$\begin{aligned} \varepsilon^x &= \varepsilon_{1n}^x + \varepsilon_{1v}^x(t + \Delta t) + (-\varepsilon_{2n}^x - \varepsilon_{2v}^x t) \cos \theta \\ \varepsilon^y &= \varepsilon_{1n}^y + \varepsilon_{1v}^y(t + \Delta t) - (\varepsilon_{2n}^y + \varepsilon_{2v}^y t) \cos \theta \\ \varepsilon^z &= \varepsilon_1^z - \varepsilon_2^z \end{aligned}$$

Where, ε_{mn}^x and ε_{mn}^y ($m=1,2$) is the longitudinal and lateral navigational error variable of aircraft m respectively; ε_{mv}^x and ε_{mv}^y is the longitudinal and lateral speed error variable of aircraft m respectively; ε_m^z is the vertical error variable of aircraft m .

So $\varepsilon_{m\gamma}^r \sim N(0, \sigma_{m\gamma}^r)$ ($r=x, y, \gamma=n, v$), $\varepsilon_m^z \sim N(0, \sigma_m^z)$, $m=1,2$. In practical applications, the actual

distribution of deviations is likely to be acquired by the more information and data amassed in the concrete airspace. The situations of other distribution functions of deviations can be discussed by using the same method. It is assumed that errors are independent.

Computations are carried out for RNP (required navigation performance) value-4 in the following segments. The density function of navigation error distribution in the longitudinal direction and lateral direction can be specified on the basis of the RNP type of the airspace. So the standard deviation of navigation error may be determined by the RNP-n, i.e. the standard deviation of navigation error can be given by the value 4 of RNP multiplied by the factor 1/1.96. The standard deviation of speed error is 5.82 nautical mile per hour and $\sigma_m^z = 35/1852(m=1,2)$.

By assuming that the flow of along airways is Poisson distributed and the flow rate of airway L is λ_L , we can have the average number of aircraft on airway L in the time length

$$t_1 \text{ is } \sum_{k=1}^{\lambda_L} ((\lambda_L t_1)^k e^{-\lambda_L t_1} / (k-1)!).$$

Furthermore Δt is assumed to be uniform on $\Delta t \in [\Delta t_{\min}, dis_K/V_i]$. The density function of Δt can be given by $g(\Delta t) = 1/(dis_K/V_i - \Delta t_{\min})$, $\Delta t \in [\Delta t_{\min}, dis_K/V_i]$.

The collision risk estimation requires values of some basic parameters used in the calculation are presented in Table 1.

Table 1. Basic collision risk estimation parameter values

Parameter	λ_x /n mile	λ_y /n mile	λ_z /n mile	v_{rel} /kn	$ \bar{z} $ /kn	Δt_{\min} /h
Value	0.0417	0.0417	0.0114	35	1.5	0

4.2 The collision risk calculation of parallel airways

(1) For $\eta = -1$, $\lambda_c = 6$, $S_y = 12$, $d_x = 3$, $\theta = 0^\circ$, by Eq.3, we can have that the average number of collisions for different dis_K and dis_L , see Table 2.

Table 2. Results for different dis_K and dis_L

dis_K /n mile	dis_L /n mile	v_i /kn	v_j /kn	CR
300	280	480	480	9.5924×10^{-9}
200	150	480	480	3.6319×10^{-9}
100	150	480	480	1.8330×10^{-9}
100	100	480	480	1.8330×10^{-9}
300	280	600	600	4.5743×10^{-9}
200	150	600	600	1.9959×10^{-9}
100	150	600	600	1.6469×10^{-9}
100	100	600	600	1.6469×10^{-9}
300	298	480	480	9.5924×10^{-9}
200	202	480	480	3.6319×10^{-9}
100	102	480	480	1.8330×10^{-9}

The results given in the Table 2 show the influences of the length of airway and speeds of aircraft on the collision risk CR . By the results of calculation we found that the longer dis_K , the greater collision risk conditional on other parameter values fixed and the smaller speed, the greater collision risk when other parameter values are fixed. By the model proposed in this paper we can analyze the main reasons as follows: The first main reason is that when other parameter values are fixed the longer dis_K , the longer flight time on airway K which causes the greater extrapolation errors; The second main reason is that the smaller average speed, the longer flight time conditional on other parameter values fixed. By the analysis we can have the conclusion that the collision risk is also increasing with the increased the proportion of aircraft type which is of the smaller speed when other parameter values are fixed.

(2) The collision risk which there are two types of aircraft on parallel airways is considered. Consider the collision risk where we have three cases: ① Let the speed of aircraft be 480 and 480, respectively; ② Let the speed of aircraft be 600 and 600, respectively; ③ Let the speed of aircraft be 480 and 600, respectively, and let the proportion be 95% and 5%, respectively. Then for $\eta = 1$, $\lambda_c = 6$, $d_x = 3$, $dis_K = 300$, $dis_L = 280$, $\theta = 0^\circ$, by Eq.3, we can have that the average number of collisions for different S_y , see Fig. 1.

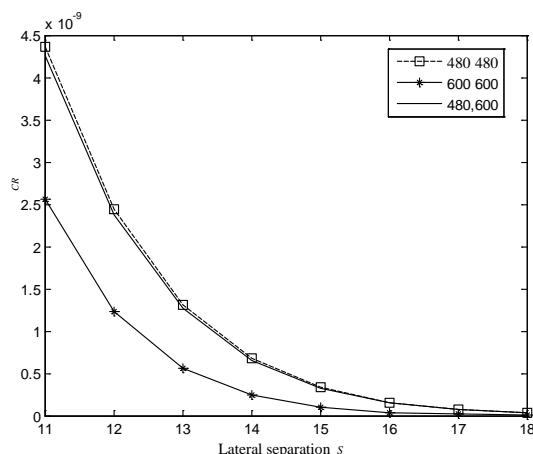


Fig.1. The relation between the number of collisions and lateral separation S_y

The results of Fig. 1 show that collision risk CR varies with the lateral separation S_y . The collision risk will decrease with the increased lateral separation S_y . It is a very obvious result the wider separation, the less collision risk.

(3) Let the speed of aircraft be 480 and 480, respectively. For $dis_K = 180$, $dis_L = 180$, $S_y = 12$, by Eq.3, we can have that the average number of collisions CR when η , d_x , λ_c and θ take different values, see Table 3.

Table 3. CR for different η , d_x , λ_c and θ

η /n mile	d_x /n mile	λ_c /number of flight/h	θ /(°)	CR
-1	3	6	180	1.0693×10^{-9}
1	20	6	180	1.8084×10^{-9}
-1	10	10	0	4.8963×10^{-9}

Table 3 shows the results of the estimation associated with variations of η , d_x , λ_c and θ . The impact of parameter values on the collision risk can be seen in Table 2 and Table 3. From results given in Table 2 and Table 3, it is possible to calculate, for some given parameter values in the progress of airspace planning, the average number of collisions on parallel airways.

5. Conclusions

1) The collision risk when the nominal distances between aircraft are variable on parallel airways can be characterized by the method proposed in this paper. This paper describes the methodology to determine lateral separation minima for use on parallel airways. These theories could provide some references for planners to set the lateral separation and the reporting interval.

2) A method to research the collision risk based on some airspace planning parameters on parallel airways is given. Airspace planners need to estimate the collision risk when some related parameter values change in different scenarios in the progress of the airspace planning. The methodology presented in this paper provides a framework by which airspace characteristics, the number of flight per hour, speeds of aircraft, the proportion of the aircraft type and RNP which will give some

theoretical basis for airspace planners to determine related parameter values and then determine lateral safe separation minima for parallel airways operations.

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