

A Bi-normalized Frequency Estimation Algorithm

Zhaobi Chu, Yan Wang and Rui Zhang

School of Electrical Engineering and Automation ,Hefei University of Technology ,Hefei 230009,China

Abstract—This paper presents a bi-normalized frequency estimation algorithm which contains a two-dimensional state estimation equation and a one-dimensional frequency update rule coupled with it. The robustness of the algorithm is that the frequency estimation convergence is no longer subject to the actual value of the amplitude and frequency of the estimated signal. The asymptotic convergence of frequency estimation is demonstrated by Lyapunov stability theory, perturbation method, integral manifold and Mathieu equation, and the relationship between adaptive internal model algorithm and the proposed bi-normalization algorithm is analyzed. Simulation results verify the effectiveness of the proposed algorithm.

Keywords—frequency estimation; adaptive internal model; bi-normalization; robustness

I. INTRODUCTION

In harmonic detection and interference suppression of signals, frequency estimation has always been one of the core contents of the research. At present, there are a large number of methods for estimating the frequency of periodic signals, such as adaptive notch filtering algorithm^[1-4], the combination of internal model controller and adaptive algorithm^[5-7], phase-locked loop^[8], the second-order generalized integrator^[9], the combination of the second-order generalized integrator and the frequency-locked loop^[10], etc. These algorithms have been appropriately adjusted and improved for different models and application backgrounds, enriching solutions to the estimation of the periodic signal frequency. In addition, in order to eliminate the influence of amplitude, a normalized way is often used. At present, most of the proposed normalization methods^[15-16] are to eliminate the influence of amplitude on the convergence speed of the algorithm.

According to [1], a globally converged adaptive filtering algorithm for estimating the unknown frequency of a signal online was proposed. Then based on the above paper, a new algorithm which provided the range of parameters for convergent system was presented in [6]. Later theoretical Analysis of System Stability in the Range of Parameters was raised in [12]. The proposed adaptive internal model algorithm combines the internal model structure with an adaptive algorithm to eliminate the perturbation of periodic signals when performing frequency estimation.

In order to degrade the effect of frequency on the convergence of the algorithm and give the parameter range of system stability, we develop a bi-normalized frequency estimation algorithm. The state space equation of this scheme, the problem statement and the proposed algorithm is presented in section 2 of this paper. At the same time, the performance of the algorithm is analyzed, and the reference parameter range to ensure system convergence is obtained in Section 3. Section 4

shows simulations to verify the performance of the proposed algorithm.

II. PROBLEM STATEMENT

In Adaptive internal model algorithm^[7], input signal choose : $y(\tau) = A_0 \sin(\omega_0 \tau + \delta_0)$, A_0 , ω_0 , δ_0 is unknown and the state equations are:

$$\begin{cases} \frac{d\hat{x}_1}{d\tau}(\tau) = -\hat{\omega}(\tau)\hat{x}_2(\tau) \\ \frac{d\hat{x}_2}{d\tau}(\tau) = \hat{\omega}(\tau)\hat{x}_1(\tau) + K_f e \end{cases} \quad (1)$$

$$\begin{cases} \frac{d\hat{\omega}}{d\tau}(\tau) = K_e \frac{K_f \hat{x}_1(\tau)}{\hat{x}_1^2(\tau) + \hat{x}_2^2(\tau)} \\ e = y - \hat{x}_2 \end{cases} \quad (2)$$

$$\hat{A}(\tau) = \sqrt{\hat{x}_1^2(\tau) + \hat{x}_2^2(\tau)} \quad (3)$$

In which, $()' = d()/d\tau$ represents derivation of τ , $\hat{\omega}(\tau)$ and $\hat{A}(\tau)$ are the estimated value of frequency and amplitude respectively, $\hat{\omega}(\tau) > 0$. K_e and K_f are parameters that determine the accuracy of the frequency estimation and the rate of convergence respectively.

For convenience, choose a sinusoidal signal as input :

$$y(\tau) = 2 \sin(\omega_0 \tau + 0.4\pi) \quad (4)$$

The initial value ω_0 is 1Hz which jumps to 0.1Hz at $t=50s$.

Choose two sets of parameter values: $K_e=K_f=1$, $K_e=K_f=0.5$ and build simulink model to simulate according to the state space equation. Simulation results as flows.

From figure 1 and figure 2, we can notice that the robustness of the system will be obviously different when choose different parameters. The system is unstable and divergent when the frequency changes from 1rad/s to 0.5rad/s at $K_e=K_f=1$. However, when the other parameters are selected, the system is stable and convergent even if the frequency is smaller. We assume that the frequency probably has an influence on the parameters selection of the AIM algorithm

system.

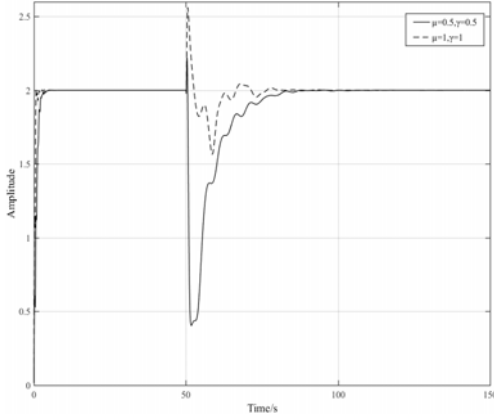


FIGURE I. AMPLITUDE FOLLOWING RESULTS OF DIFFERENT PARAMETERS BASED ON ADAPTIVE INTERNAL MODEL ALGORITHM

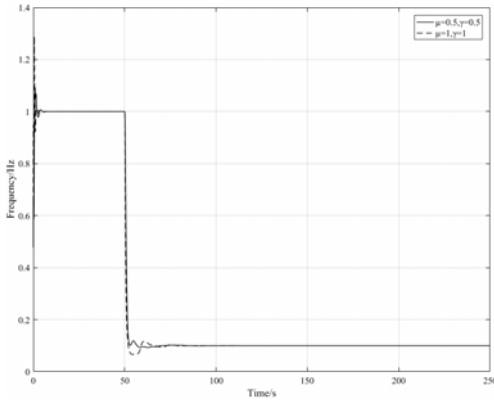


FIGURE II. FREQUENCY FOLLOWING RESULTS OF DIFFERENT PARAMETERS BASED ON ADAPTIVE INTERNAL MODEL ALGORITHM

In practical, we often deal with the tracking of frequency unknown signals and noise elimination. It can be seen that when the frequency is unknown, the accuracy of the model parameter selection is difficult, and multiple attempts are required. Therefore, we propose a new internal model algorithm solve the problem.

III. A BI-NORMALIZED FREQUENCY ESTIMATION ALGORITHM

According to the above simulation results, the relationship between the parameters K_e , K_f and frequency in the internal model algorithm proposed in [7] is existed. In order to eliminate the influence of frequency on parameter selection, the following algorithm is proposed:

$$\begin{cases} \hat{x}_1'(\tau) = -\hat{\omega}(\tau)\hat{x}_2(\tau) \\ \hat{x}_2'(\tau) = \hat{\omega}(\tau)(\hat{x}_1(\tau) + \mu(y(\tau) - \hat{x}_2(\tau))) \\ \hat{\omega}'(\tau) = \gamma\hat{\omega}^2(\tau) \frac{\hat{x}_1(\tau)(y(\tau) - \hat{x}_2(\tau))}{\hat{x}_1^2(\tau) + \hat{x}_2^2(\tau)} \end{cases} \quad (5)$$

The third of formula (5) is frequency update law. $\hat{\omega}$ is estimated frequency, μ and γ determine the accuracy of the frequency estimation and the rate of convergence respectively.

Next, analysis the stability of the proposed bi-normalization algorithm. Let $t = \omega_0\tau_0 + \delta_0$, $dt = \omega_0 d\tau_0$, then τ domain transform to t domain and normalize the amplitude. The state equation of t domain as follows:

$$\begin{cases} \dot{x}_1 = -\theta x_2 \\ \dot{x}_2 = \theta(x_1 + \mu(\sin t - x_2)) \\ \dot{\theta} = \gamma\theta^2 x_1(\sin t - x_2)(x_1^2 + x_2^2)^{-1} \end{cases} \quad (6)$$

In reference[4], let $\varepsilon = \sqrt{\gamma}$, errors define as:

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} = \begin{bmatrix} x_1 - \cos t \\ x_2 - \sin t \\ \varepsilon^{-1}(\theta - 1) \end{bmatrix}$$

Then error equation can be rewritten as a disturbance system:

$$\dot{e} = A_1(t)e + \varepsilon g_1(t, e) + \varepsilon g_2(t, e) \quad (7)$$

In which,

$$A_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & -\mu & 0 \\ 0 & 0 & 0 \end{bmatrix}, g_1(t, e) = \begin{bmatrix} -\sin te_3 \\ \cos te_3 \\ -\cos te_2 \end{bmatrix},$$

$$g_2(t) = \begin{bmatrix} -e_2 e_3 \\ -e_3(\mu e_2 - e_1) \\ -(1 + \varepsilon e_3)^2(e_1 + \cos t)e_2 + \cos t \end{bmatrix}$$

In order to describe the stability of system (7), its linear periodic nominal system is considered:

$$\dot{e} = A_1 e + \varepsilon g_1(t, e) \quad (8)$$

The eigenvalue of matrix A_1 are:

$$\lambda_{1,2} = -0.5\mu \pm 0.5\sqrt{\mu^2 - 4}, \lambda_3 = 0$$

And the perturbation $g_1(t, e)$ is continuous bounded and has a continuous first derivative. According to theorem VII 7.1 in [15], exists a constant $\varepsilon_0 > 0$, for fixed ε which range in $0 \leq \varepsilon \leq \varepsilon_0$, there is always an integral

manifold: $S_\varepsilon = \{(e, \mathbf{x}) : e_1 = \tilde{e}_1(t, \varepsilon), e_2 = \tilde{e}_2(t, \varepsilon)\}$

And satisfy:

$$\tilde{e}_1(t, 0) = 0, \quad \tilde{e}_1(t + 2\pi, \varepsilon) = \tilde{e}_1(t, \varepsilon)$$

$$\tilde{e}_2(t, 0) = 0, \quad \tilde{e}_2(t + 2\pi, \varepsilon) = \tilde{e}_2(t, \varepsilon)$$

In addition, the third equation of formula (8) derived again can get:

$$\ddot{e}_3 + \varepsilon^2 \cos^2 t e_3 = \varepsilon(\mu \cos t + \sin t) e_2 - \varepsilon \cos t e_1$$

On the integral manifold S_ε , let $e_1 = e_2 = 0$:

$$\ddot{e}_3 + (0.5\varepsilon^2 + 0.5\varepsilon^2 \cos 2t) e_3 = 0 \quad (9)$$

According to the reference [1], the near necessary condition for the stability of the system can be written as $\varepsilon^2 = \gamma < 2$.

In addition, according to the theorem 4.14 of the literature [14], it can be seen that if the exponential stable equilibrium point of the system (8) is $e = 0$, there is a sufficiently small ε , make $e = 0$ is also the exponential stable equilibrium point of the system (6).

So there exists $0 < \gamma^* < 2$, when $0 < \gamma < \gamma^* < 2$,

from algorithm (5) and formula (3) there are:

$$\hat{x}_1(\tau) \rightarrow K_0 \cos(\omega_0 \tau + \delta_0), \quad \hat{\omega}(\tau) \rightarrow \omega_0$$

$$\hat{x}_2(\tau) \rightarrow y(\tau) = K_0 \sin(\omega_0 \tau + \delta_0)$$

$$\hat{A}(\tau) = \sqrt{\hat{x}_1^2(\tau) + \hat{x}_2^2(\tau)} \rightarrow K_0$$

Compare algorithm (2) and (5) gets

$$K_f = \mu \hat{\omega}(\tau), K_e = \gamma \omega(\tau) / \mu$$

So the necessary condition for the convergence of algorithm (2) is $K_e K_f < 2\omega_0^2$. Observe figure 1 and figure 2, $K_e = K_f = 1$, $\omega_0 = 1 \text{ Hz} = 2\pi \text{ rad/s}$, $K_e K_f = 1 < 2\omega_0^2 = 8\pi^2$, the system meets the above necessary condition while $\omega_0 = 0.1 \text{ Hz} = 0.2\pi \text{ rad/s}$, $K_e K_f = 1 > 2\omega_0^2 = 0.08\pi^2 = 0.79$, it does not satisfy the condition. So the curves corresponding to $K_e = K_f = 1$ in Figure 1 and Figure 2 do not converge when $\omega_0 = 0.1 \text{ Hz}$.

IV. SIMULATION

In order to verify the effective of the proposed algorithm, build simulation model according to (5) and (3) in MATLAB/Simulink, then make simulation experiment on the

signal of formula(4). In order to ensure the comparability with the adaptive internal model algorithm (2), the two sets of parameters are selected, and the simulation results are shown in Figure 3 and Figure 4.

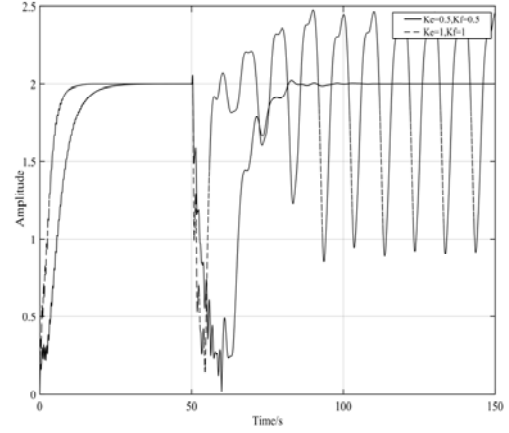


FIGURE III. AMPLITUDE FOLLOWING RESULTS OF DIFFERENT PARAMETERS BASED ON BI-NORMALIZED ALGORITHM

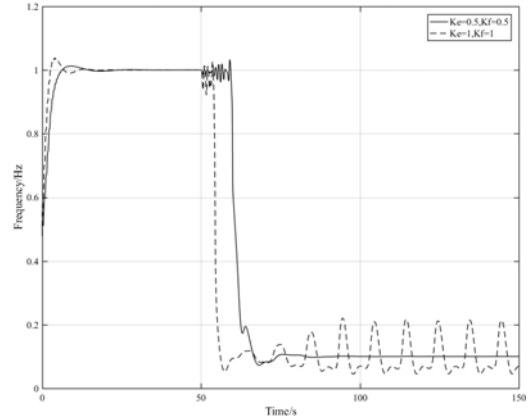


FIGURE IV. FREQUENCY FOLLOWING RESULTS OF DIFFERENT PARAMETERS BASED ON BI-NORMALIZED ALGORITHM

Compare Figure 1 and Figure 3, Figure 2 and Figure 4 respectively, we can find that the proposed algorithm still holds asymptotic convergence in small frequency conditions. Its convergence speed is faster and the overshoot is smaller which shows better robustness.

V. CONCLUSION

This paper proposed a bi-normalized frequency estimation algorithm based on the adaptive internal model algorithm which make the frequency estimation convergence is no longer related to the amplitude and frequency of the estimated signal. The convergence of the algorithm is illustrated. The simulation results show that the proposed algorithm still can follow the amplitude and frequency of the signal quickly even if the estimated frequency value is small, improve the robustness of the algorithm and extend the applicable range of the algorithm.

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