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Study on the Correlation Between Meso-Structure Fractal Characteristics and Macroscopic Parameters of Two Typical Rocks

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***Abstract*—The meso-structure characteristics of rock affect the macroscopic mechanical behaviors such as its fracture form. In this paper, through the uniaxial compression test and electron microscope scanning(SEM) test of two typical rocks of sandstone and limestone, the fractal characteristics of meso-structure and its influence on the macroscopic damage form are analyzed. The quantitative relation between the fractal dimension and the uniaxial compression strength(UCS) and the modulus of elasticity(E) is established. Results showed that the self-similarity of different lithology rock meso-structures is different. With the magnification of the meso-scanning image, the rock meso-structure fractal dimension is constantly changing, and the trend of sandstone and limestone is decreasing first and then increasing, the trend of the increase of limestone with multiple is slower than that in sandstone. There is a good inverse relationship between uniaxial compression strength and fractal dimension, while elastic modulus and fractal dimension have better power correlation.**

Keywords—rock, microstructure, fractal dimension, strength, macroscopic

I. Introduction

Fractal geometry is a new geometry subject to irregular geometry. The irregular and rough geometric figures in nature can be described so that people can go through disorder confusion and irregular form to explain the laws behind complex phenomenon[1]. Rock is a highly complex porous material, with large number of irregular voids randomly distributed. The shape, distribution and microstructure of these pores show self-similarity in statistical sense, and small scale phenomena often reproduce the characteristics of large scale, which conform to fractal features[2]. Fractal, as an effective means to study the self-similarity of systems, has been widely applied in rock mechanics in recent years and develops rapidly. Related scholars have found that fractal phenomena are very common in the field of rock mechanics. The fractal characteristics can be observed not only in the natural structure, physical property, distribution and geological structure of rock, but also of the parameters of rock joints or fracture surfaces, mechanical behavior and compressive strength of rock.

Amitava Ghosh and Jaak J K Daemend[3] used the joint photos of four rock faces of a deposit, and used the fractal dimension of joints to describe the characteristics of joints. Based on fractal theory, La Pointe[4] studied the network of rock structure surface and computer simulation of rock structural surface network that had good fractal characteristics. Academician Xie Heping[5-13] took the lead in studying the fractal theory in the field of rock mechanics, and estimated the fractal dimension of the joint roughness coefficient(JRC) of the rock. The empirical formula of the fractal dimension of the JRC and the joint roughness was obtained. The fractal dimension of tunnel surrounding rock is calculated by Zhang Peng[14] et al. It is found that the larger the fractal dimension, the worse the stability of the roadway surrounding rock is, the fractal dimension of the side wall of the roadway is slightly larger than the fractal dimension of the vault. Liu Jinghong[15] and others studied the crack propagation of rock material to the fracture process, and revealed the fractal characteristics of the crack propagation process in the rock. These studies provide a solid foundation for solving the problems of rock mechanics with fractal geometry method and quantitatively describing the complex physical properties of rocks.

II. Experimental and Fractal Methodology

A. Uniaxial Compression Test

The rock sample is taken from Xinwen mining area. In order to ensure the same humidity and moisture content with the site, the rock sample is transported to the laboratory after sealing treatment in the field. Then through three processes of drilling, cutting and grinding as standard specimen, the lithology of rock is determined by polarizing mirror and hydrochloric acid to determine rock lithology, and taking the lithology as the category, 10 typical sandstone and limestone with better homogeneity were selected for uniaxial compression experiments. Uniaxial tests were carried out by an electro-hydraulic servo-controlled testing system. Longitudinal and transverse strain gauge were arranged and balanced, considering a loading rate of 0.05 mm/s under the displacement control mode until the rock specimen fractured. In this way, UCS was obtained, as well as deformability modulus (E) and coefficient of Poisson (ν).

B. SEM Test

JSM-6510LV high vacuum scanning electron microscope was used in this experiment. The electron microscope can carry out the test of various materials such as rock, concrete and metal. The test system mainly includes electronic optical system, scanning system, signal detection and amplifying system, image display and recording system, power supply and vacuum system. The sample surface is bombarded with a fine focusing electron beam, and the surface or fracture morphology of the sample is observed and analyzed by the secondary electrons, the secondary electrons was produced by the interaction of the electron and the sample.

It is required that the sample must be dry and conductive. So in the preparation of the test, firstly, the sample is made to be suitable for the size of the electron microscope, secondly, the ethanol with water solubility and lower surface tension, and the ultrasonic cleaning was used to make the specimen cleaner; then the samples is fixed on the specimen piece seat, with the drying box to drying the sample; finnaly the dried samples were sprayed with gold in the vacuum coating system, which can avoid the charge accumulation under the electron beam irradiation. The coating thickness is generally 8~10mm.

C. Fractal Dimension of Rock Structure Surface

Fractal is a new method to study and describe the extremely irregular and disordered complicated structures, phenomena or behaviors. Its main characteristics are self-similarity and scale invariance of the system[16]. The parameter describing the fractal system quantitatively is called fractal dimension. Fractal dimension reflects the degree of complexity of the system. There are several methods to estimate fractal dimension, including Hausdorff dimension, box counting dimension, similarity dimension, information dimension, correlation dimension and capacity dimension. The box counting dimension is one of the most widely used dimensions. The idea of calculating the fractal dimension is to cover the geometric figures with the square lattice（$\delta\times\delta$）of different sizes. The total number of boxes needed to cover the geometric figures is N. There are the following relations between them:

$$N = a\delta^{-d} \tag{1}$$

Then, gradually changing the size of the side of the square grid, each time to count the number of squares covering the figure, then the box dimension of the figure can be expressed by the following formula:

$$\dim_B F = \lim_{\delta\to 0}\frac{\log N_\delta(F)}{-\log\delta} \tag{2}$$

Previous studies[17-19]and rock mechanics experiments[20] have shown that the meso-scopic space and particle size distribution of rocks are fractal in nature, with the statistical regularity of self-similarity and scale invariance, and the fractal dimension can be used to describe its complexity quantitatively. The SEM picture is the measurement and record of the meso-morphology of rock failure, reflecting the development and variation of its microscopic cracks and void particles. The picture appears the charicateristics of self-similarity, which is consistent with the fractal characteristics. The calculation of fractal dimension can well reflect the characteristics of rock failure and meso-scopic development, and the uniformity of rock fracture distribution.

For a picture of a rock meso-graph, it can only be divided into a finite size, that is, when δ reaches the scale of the pixel point, it is a two-dimensional set and no longer has the nature of fractal. When studying the fractal characteristics of an image, they must be restricted within a certain scale, and at least not less than the scale of a pixel. The calculation method is to split the image into a square with a border length of δ , and then count the number of squares that contain at least one crack point of the image, and take this number as *W*, change the size of the δ , and repeat the calculation process above. The fractal dimension of rock meso-morphology can be obtained from the logarithmic relationship between $N\left(\delta\right)$ and $1/\delta$ This process can be caculated in Matlab[21].

III. Results and Discussions

A. Results

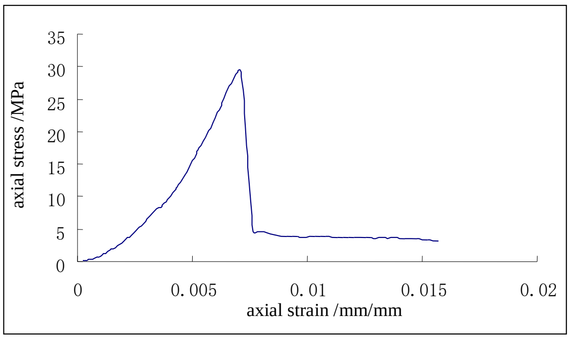
1) Results of uniaxial compression

Under uniaxial compression, rock produces longitudinal compression and lateral expansion. When the stress reaches a certain magnitude, the volume of the rock mass begins to expand and the initial crack occurs, and then the crack continues to develop and eventually leads to damage. In the experiment, displacement control is adopted in loading mode. In order to reduce the effect of the friction effect of the end surface on the strength of the specimen, the upper and lower ends of the specimen are loaded with the indenter with the same diameter as the specimen. Experimental results of uniaxial compression are shown in table 1. Taking one samples of each lithology as an example, the stress-strain curves of sandstone and limestone samples are shown in figure 1.

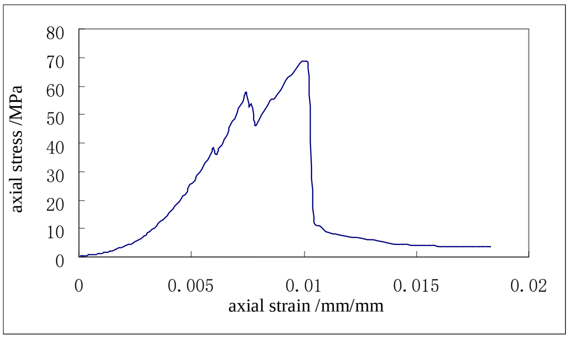
TABLE I. UNIAXIAL COMPRESSIVE STRENGTHS OF ROCKS

Rock type	**Uniaxial Compressive Strength, σ (MPa)**					**average**
Sandstone	1	2	3	4	5	52.10
	31.60	38.90	53.19	47.53	51.99	
	6	7	8	9	10	
	42.29	47.09	86.00	54.93	67.48	
Limestone	1	2	3	4	5	80.63
	71.01	68.81	95.56	81.54	77.72	
	6	7	8	9	10	
	101.83	97.89	69.82	70.30	71.85	

By comparing and analyzing the uniaxial compression test of each rock specimen, the stress-strain curve is divided into four parts: the close compression section of the rock meso-crack under compression, the elastic deformation stage, the plastic stage of the rock specimen failure and the post failure stage of the rock meso-fracture[22]. But from the above results, we can see that the mechanical behavior of each specimen is quite different, and the macroscopic failure characteristics of each lithology are also very different. The average strength of sandstone and limestone is 52.10MPa and 80.63MPa respectively, and sandstone joints are more, which reduces their compressive strength to some extent. The stress-strain curves of these two sedimentary rock specimens show strong brittleness.



（a）stress-strain curves ofsandstone specimen

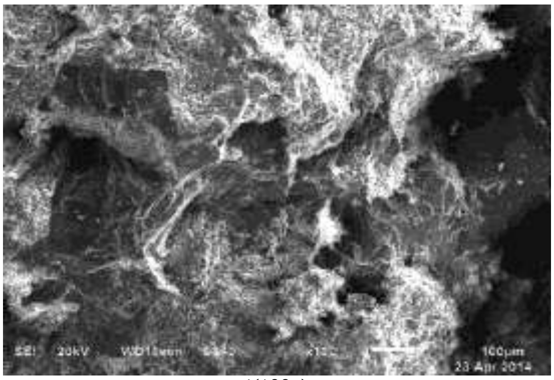


（b）stress-strain curves of sandstone specimen

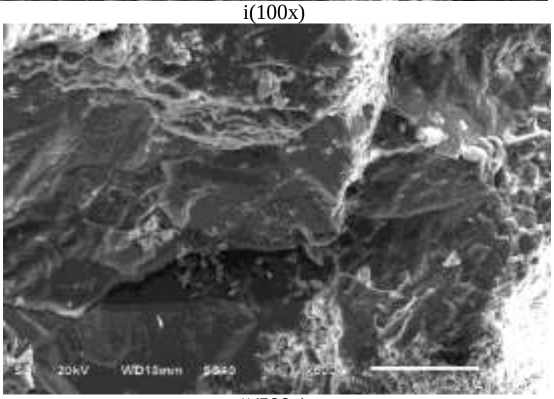
Fig. 1. Stress-strain curves of sandstone and limestone samples(one sample)

2) Results of SEM test

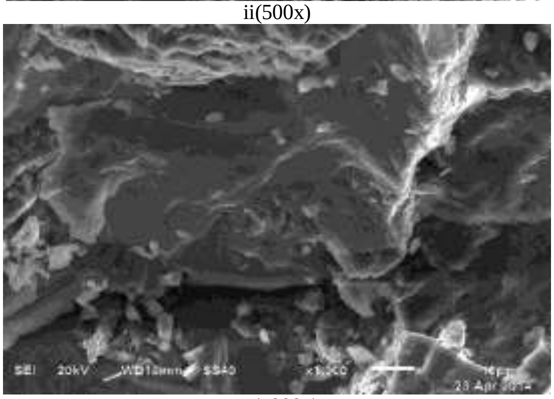
10 samples of sandstone and limestone were sampled, and the SEM of rock was subjected to secondery electronic imaging. Each sample gets 4 picture of different maginification, 100x, 500x, 1000x and 2000x respectively. Taking one samples from each lithology as an example, the SEM image is shown in figure 2.



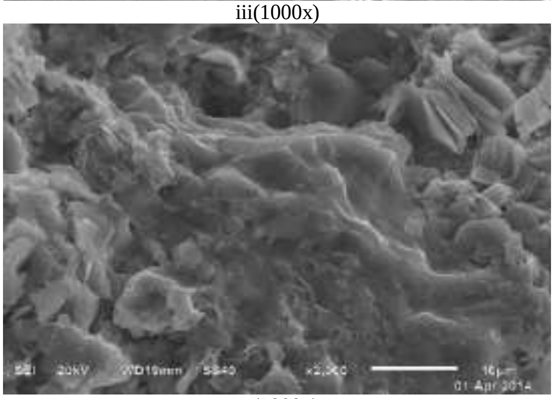
i(100x)



ii(500x)

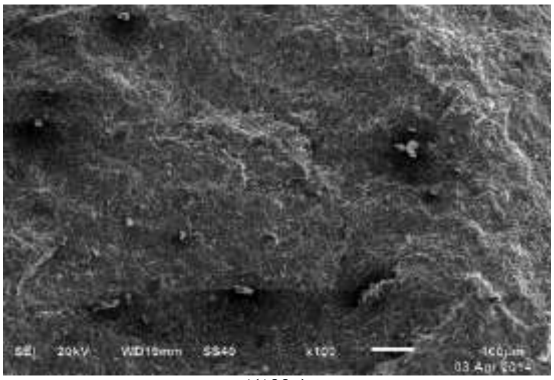


iii(1000x)

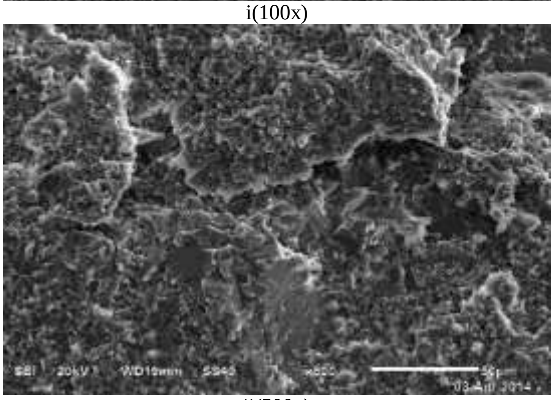


iv(2000x)

（a）SEM image of sandstone specimen (S-1#)



i(100x)



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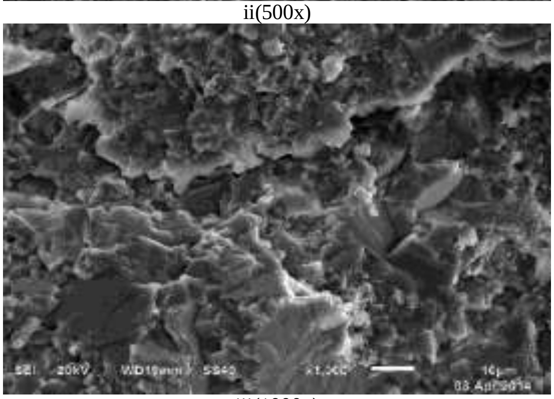
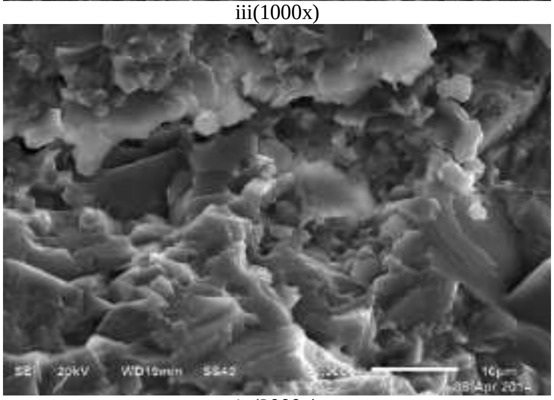


iii(1000x)



iv(2000x)

（b）SEM image of limestone specimen (H-1#)

Fig. 2. Fig.1 SEM image of the specimens (Each type of rock one cases)

In the rocks selected in this paper, both sandstone and limestone belong to sedimentary rocks, and the geological genesis of the two rocks are different. Sedimentary rocks, formed by weathering and denudation of the mother rock on the surface, are formed by transporting, depositing, and harden diagenesis. It is a secondary rock that has been accumulated and cemented once again after the rock is broken. In the process of diagenesis, the diagenesis and early evolution of sedimentary rocks determine that microstructures within the rocks vary with their causes and internal structures, such as some clastic structures, some with crystalline structures, etc. After the formation of rocks and the long-term geological evolution, they can form sedimentary rocks. These characteristics have certain effects on their mechanical properties.

It can be seen from the above SEM images that the mesoscopic structure of sandstones is more complex, and some of the sandstone mesoscopic appears mound monopoly, which is corresponding to the characteristic of lack of pressure bearing capacity of the fissure and the reduction of the adhesion between each particle. The cementation patterns of particles vary according to composition and size of cutting grain. Concretely, it can be seen from image of 100x that large lumps combine sandstone, with clay attached and no clay attached to the surface, the directional arrangement is not obvious, the overall surface structure is looser, the secondary one is more, the mineral and mineral have poor cementation surface, and the large particle crystal is distributed around the cavity; From the images of 500-1000x, the failure section of sandstone is unevenness, The whole surface structure is still made up of large particles and blocks. The bedding is obvious, the block structure is close, the clay particles are attached to the surface of the block, and the debris particles play a skeleton role. From 2000x of the images, clay particles are clustered together by strip like bodies, and the surface of the block is attached to the lamellar mineral.

The development of cracks and voids in limestone is relatively uniform. When the magnification is 100x, the sheet body is clearly directed, the arrangement is extended along the curve, and the large granular bodies are embedded in the clay minerals, the tight pores occupy small areas; when the magnification increased to 500-1000x, the sheet structure of the rock is more obvious, and the small particles are scattered on large prothallial. That is, it plays the role of skeleton particles and plays the role of cementation. The shape is irregular and the size, shape and distribution of the micropores are more obvious. When the magnification is 2000x, it is observed that tiny clay minerals are confused and clown together, attached to the surface, while small particles are filled in large particles, and the combination of particles is more compact.

B. Fractal Characteristics of Microscopic Structure

According to the method of box counting dimension, fractal dimension calculation program has been compiled by Matlab. The fractal dimension of 100x, 500x, 1000x and 2000x of meso morphology of different rock is calculated. The fractal dimension of each specimen can be obtained by obtaining the average value. The results are shown in table 2, where sandstone events are taken as an example, and the fitting figures of image fractal dimensions under 100x to 2000x are shown in figure 3.

TABLE II. FRACTAL DIMENSION FOR DIFFERENT MAGNIFICATION

Rock type	**D_c of different magnification**				
	500	***1000***	***2000***	***5000***	***average***
Sandstone	2.09	2.08	2.11	2.13	2.10
Limestone	2.08	2.07	2.08	2.10	2.08

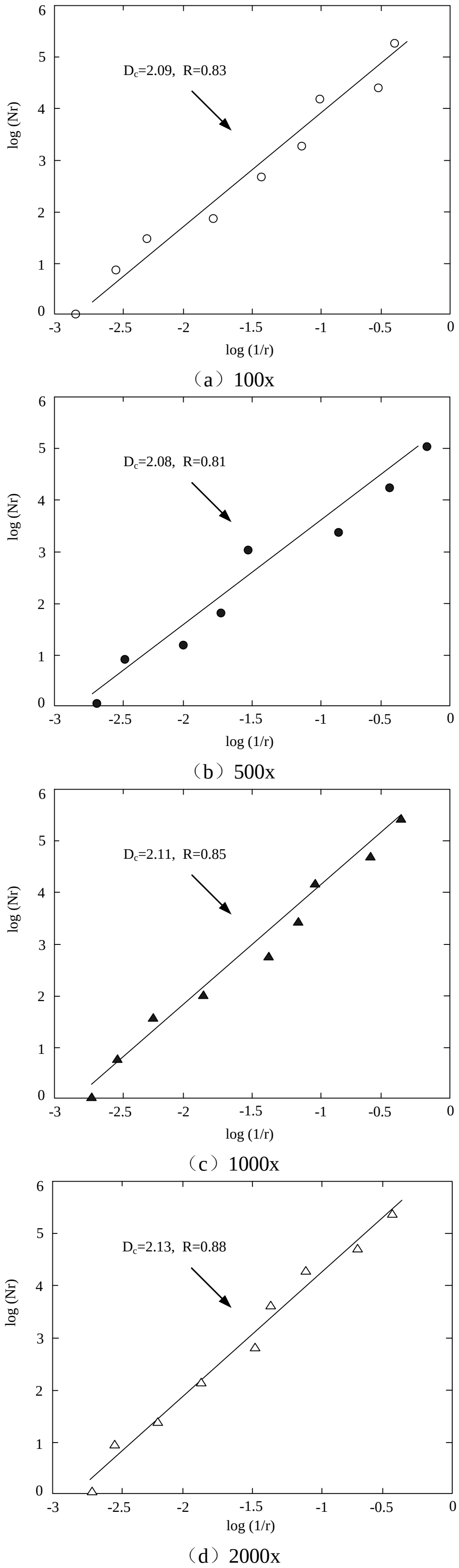


Fig. 3. Fractal dimension of sandstone variations with magnification

The SEM test and statistical analysis showed that fractal characteristics of mesoscopic crack exist only in a specific interval, especially large multiple or very small multiples do not conform to the fractal characteristics. The meso morphology of 100x, 500x, 1000x and 2000x is selected to calculate the fractal dimension and analyze the fractal characteristics. Rock specimen selected in this test is also a fractal in statistical significance. The fractal dimension results are statistically studied, and the variation trend of fractal dimension and correlation coefficient under different magnification is obtained.

The results showed that the fractal dimension of sandstone is higher than that of limestone. This rule is closely related to the genesis, lithology, compressive strength and fracture development of rocks. Different lithology results in different density of fracture development and fractal dimension. The more dense and hard the lithology, the lower the fractal dimension; conversely, the higher the lithology is, the lower the strength, the higher the fractal dimension.

In figure 4, with the increase of magnification, fractal dimension of rock specimen is constantly changing. Typical sandstone and limestone have similar change characteristics. All of them show the trend of "first decreasing and then increasing", and the size of the fractal dimension is closer. The trend of the growth of the limestone with multiple is slower than that of the sandstone, which is closely related to the mesoscopic self similarity. Figure 5 shows the variation of fractal dimension correlation coefficient of the rock. The general trend is consistent with the variation of fractal dimension, it means that the self similarity of rock meso structure is varies from its scanning scale, that is, different rock magnification show different fractal dimensions. Therefore, the accuracy of the fractal dimension description method not only depends on the rock lithology, but also to the magnification.

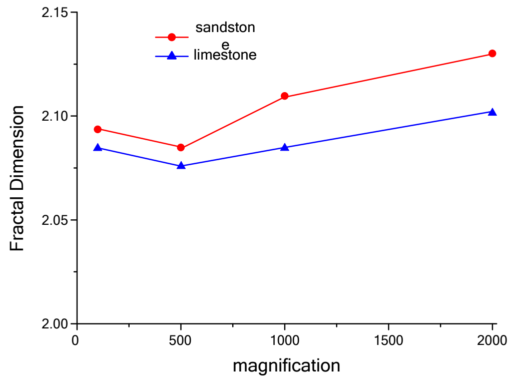


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Fig. 4. Fractal dimension variations with magnification

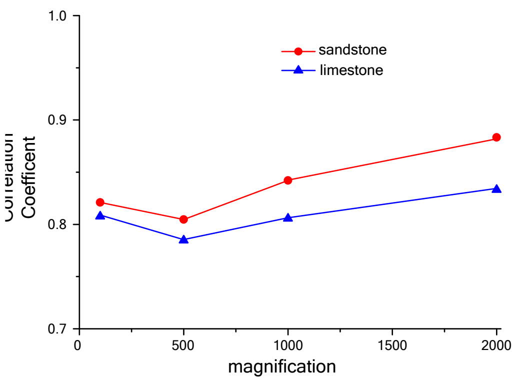

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Fig. 5. Correlation coefficent variations witch magnificatio

From the microscopic fractal dimension and the SEM image, it can be seen that the fractal dimension of the rock failure morphology is closely related to its self similarity. The complexity and roughness of its micro morphology will gradually appear with the magnification of multiple and be reflected in fractal dimension. The meso morphology of sandstone is more complex, so fractal dimension is higher. The limestone is characterized by simple texture and low fractal dimension, so there is a certain correlation between the rock fractal dimension and its meso morphology.

C. Correlation between Fractal Characteristics and Mechanical Parameters of Meso structures

Rock is a heterogeneous and anisotropic brittle material, which contains various fissures, voids and microstructural planes. In essence, it is composed of particles of various morphologies and components that are different from each other. Specifically, mineral composition, cementation mode and crystal spatial structure determine the strength characteristics. In order to find the quantitative relation between the fractal characteristics and the mechanical parameters of the rock meso structure, fractal dimension of sandstone and limestone and the compressive strength and modulus of elasticity are fitted. It was find that the correlation coefficient between fractal dimension and strength is low when the two type rock mixed together. It is necessary to fit the parameter according to the lithology, and get the fractal dimension-UCS, fractal dimension-E curve of various types of rocks.

For each type of rock, the scatter points corresponding to each specimen are drawn in a rectangular coordinate system, with the fractal dimension as the abscissa, the intensity or the elastic modulus as the ordinate. It can be seen that as a whole, there is a reverse relationship. When all the lithologies are fitted together, the correlation coefficient falls below 0.5. Thus, in general, the relationship between the two parameters can be described with an anti correlation. The UCS, E and fractal dimension of all kinds of rocks are fitted, and diagram was obtained.

The form of a fractal dimension - UCS fitting line is: y=ax+b

In the formula: y is the rock uniaxial compression strength, a and b are fitting coefficients, and x is fractal dimension.

TABLE III. THE FITTING PARAMETER TABLE BETWEEN UCS AND FRACTAL DIMENSION

fitting parameter	sandstone	limestone
a	-548.84	-387.94
b	1205.8	887.54
correlation coefficent	0.9321	0.8749

The power equation of fractal dimension - E fitting is in the form of $y=ax^b$

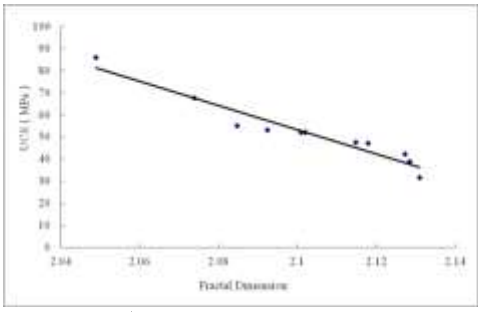
In the formula: y is the elastic modulus of rock, a and b are the fitting coefficients, and X is the fractal dimension.

The relationship among UCS, E and fractal dimension is obtained by fitting the above formula. The fitting parameters are shown in table 3-4, and the fitting figures are shown in Figure 6-7.

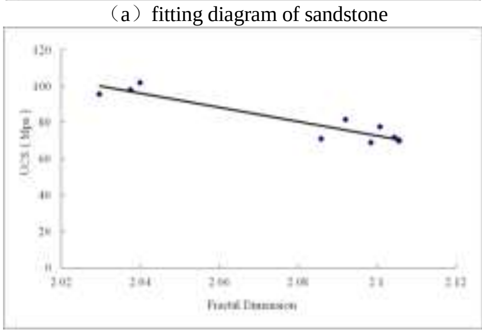
It can be seen from the fitting results that the fractal dimension of rock meso structure has correlation with its mechanical parameters. Obviously, there is a good inverse relationship between UCS and fractal dimension, the correlation coefficient is above 0.8, while the E and fractal dimension have a better power correlation, the correlation coefficient is above 0.7.

TABLE IV. THE FITTING PARAMETER TABLE BETWEEN E AND FRACTAL DIMENSION

fitting parameter	sandstone	limestone
a	1.78×10^7	5.10×10^9
b	-10.22	-17.78
correlation coefficent	0.8400	0.7418

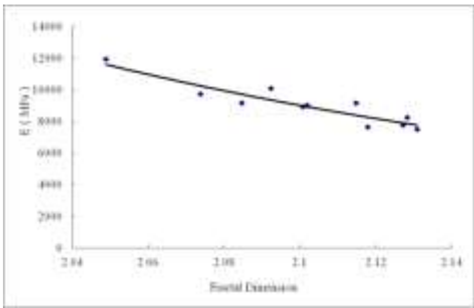


(a) fitting diagram of sandstone

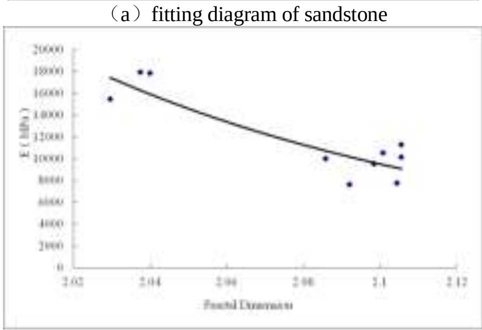


(b) fitting diagram of limestone

Fig. 6. The fitting relationship graph between UCS and fractal dimension (linear)



(a) fitting diagram of sandstone



(b) fitting diagram of limestone

Fig. 7. The fitting relationship graph between E and fractal dimension (power)

This indicates that the rock micro morphology is self similarity and can be studied by fractal theory. Fractal can well reflect the characteristics of rock fracture development and the uniformity of fracture distribution. It is reasonable to characterize or measure rock mechanical properties by using the fractal dimension of rock meso morphology. At the same time, it also shows that rock mechanical properties are closely related to the minuteness structure, and the minuteness structure also affects the fractal characteristics of rock failure. Therefore, fractal dimension can be regarded as one of the comprehensive indexes to reflect rock mechanical properties, and rock mechanical properties can be measured by fractal dimension. Fractal dimension is different from the general quantitative indexes such as the granularity distribution of rock and the number of fractured strips. It not only reflects the development and particle size of the rock micro morphology, but also reflects the distribution and roughness of rock particles, so it is a more scientific quantitative index.

IV. Conclusions

- The typical sandstone and limestone specimens obtained from the deep mining area were tested by uniaxial compression experiments and SEM tests. The Matlab calculation program was compiled by box counting dimension method, the meso morphology of 100x, 500x, 1000x and 2000x of rock failure was obtained, and the fractal dimension of sandstone and limestone rocks was calculated.
- The average fractal dimension of sandstone is larger than that of limestone. The microscopic fractal dimension of the SEM image under different magnification showed that the fractal dimension of the rock failure morphology is closely related to its self similarity, the complexity and roughness of its micro morphology will gradually appear with the magnification and reflected in the fractal dimension.
- The relationship between the rock meso fractal dimension and the parameters of uniaxial compression strength and modulus of elasticity was established. The results showed that there is a good inverse relationship with the uniaxial compressive strength. The correlation coefficient with the compressive strength is over 0.8 and has a good power correlation with the elastic modulus, the correlation coefficient is above 0.7.

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