

Sliding Mode Control of Permanent Magnet Synchronous Motor Based on Super-Twisting Algorithm

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Abstract—In this paper, the second order sliding mode control law was designed based on the super-twisting algorithm and the switching function $\text{sign}(s)$ was replaced by the saturation function $\text{sat}(s)$ for the torque and flux controller in the permanent magnet synchronous motor double closed loop control system. The stability of the system was proved by Lyapunov function. The traditional direct torque control method and the simulation results of this method were analyzed by using MATLAB / SIMULINK simulation tools. The results show that the torque ripple based on the direct torque control system of the super-twisting algorithm is reduced by 4 times compared with the traditional direct torque ripple. Speed response time is shorter. Speed fluctuation is smaller. System stability has improved significantly.

Keywords—super-twisting algorithm, sliding mode control, saturation function, lyapunov, stability

I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) have broad prospects in the fields of electric vehicles, robots and industrial control, due to their high power density, high efficiency, and high reliability [1-2]. However, the characteristics of the permanent magnet synchronous motor, such as nonlinearity and strong coupling, determine that advanced control strategies, which can solve the problems of non-linearity and poor adaptability of the control system, must be used such as adaptive control, neural network control, sliding mode control, etc. [3].

Sliding mode control (SMC) is known for its good robustness to system parameter changes and external disturbances [4-5]. There are chattering problems in the traditional sliding mode control. For these problems, many scholars at home and abroad have proposed many

suppression methods, such as approaching law, boundary layer method, high-order sliding mode, etc. Because the Super-Twisting Algorithm does not require the derivative of the sliding mode variable and has a unique, continuous sliding mode, it is widely used in various motor controls. [6-7]. In [8], a robust rotor magneto-rheological observer was designed using the super-twisting algorithm. In [9], the super-twisting controller (STC) was designed to control the flux tracking and velocity tracking of the motor. In [10], it applies the super-twisting sliding mode control algorithm to the full-bridge inverter, and in [11-12] they use the super-twisting sliding mode control algorithm for the motor speed control system. It is well known that the discontinuous switch function $\text{sign}(s)$ causes chattering. Boundary layer control and saturation functions are often used to eliminate chattering [13]. This article will use the saturation function $\text{sat}(s)$ instead of the switch function $\text{sign}(s)$. Using Lyapunov function proves the stability of the system. And combining simulation analysis proves the effectiveness of this method.

II. MATHEMATICAL MODEL OF PERMANENT MAGNET SYNCHRONOUS MOTOR (PMSM)

A. Mathematical model of PMSM in stationary coordinate system

The mathematical model of permanent magnet synchronous motor in the stationary coordinate system is [14]:

Stator voltage equation:

$$\begin{aligned} u_\alpha &= L \frac{d}{dt} i_\alpha + R i_\alpha - \omega_e \varphi_f \sin \theta_e \\ u_\beta &= L \frac{d}{dt} i_\beta + R i_\beta + \omega_e \varphi_f \cos \theta_e \end{aligned} \quad (1)$$

Stator flux equation:

$$\begin{aligned} \frac{d}{dt} \varphi_\alpha &= u_\alpha - R i_\alpha \\ \frac{d}{dt} \varphi_\beta &= u_\beta - R i_\beta \end{aligned} \quad (2)$$

Electromagnetic torque equation:

$$T_e = \frac{3}{2} p_n (\varphi_\alpha i_\beta - \varphi_\beta i_\alpha) \quad (3)$$

u_α, u_β --the components of the stator voltage vector in the stationary coordinate system; i_α, i_β --stator current vector components in the stationary coordinate system; $\varphi_\alpha, \varphi_\beta$ --magnetic flux component in the stationary coordinate system; p_n --number of pole pairs; L,R--stator inductance and stator resistance.

III. MATHEMATICAL MODEL OF PMSM IN ROTATING COORDINATE SYSTEM

The mathematical model of permanent magnet synchronous motor in the rotating coordinate system is [14]:

$$\begin{aligned} \varphi_r &= \int (u_d - R i_d) dt \\ \frac{d i_q}{dt} &= \frac{1}{L} (u_q - R i_q - \omega_e \varphi_d) \\ \frac{d T_e}{dt} &= \frac{3}{2} p_n \varphi_f \frac{d i_q}{dt} \end{aligned} \quad (4)$$

Get them:

$$\begin{aligned} \frac{d \varphi_r}{dt} &= u_d - R i_d \\ \frac{d T_e}{dt} &= \frac{3}{2L} p_n \varphi_f (u_q - R i_q - \omega_e \varphi_d) \end{aligned} \quad (5)$$

Make $K = \frac{3}{2L} p_n \varphi_f$, then,

$$\frac{d T_e}{dt} = K (u_q - R i_q - \omega_e \varphi_d) \quad (6)$$

φ_r --stator flux; u_d, u_q --stator voltage vector components in a rotating coordinate system; i_d, i_q --stator current vector components in a rotating coordinate system, T_e --electromagnetic torque, φ_f --permanent magnet flux, ω_e --

electrical speed of the motor, φ_f --the component of the stator flux on the d-axis of the rotating coordinate system.

IV. SLIDING MODE VARIABLE STRUCTURE CONTROL BASED ON SUPER-TWISTING

A. Super-Twisting based controller design

For a controlled system [15]:

$$\frac{dx}{dt} = u(t) + \delta(t) \quad (7)$$

Including: $x \in R^n$ is state variable; $\delta(t)$ is time-varying disturbance; $u(t)$ is "Super-twisting" control rate.

Hypothesis:

$$u(t) = u_1(t) + u_2(t) \quad (8)$$

make:

$$\begin{aligned} u_1(t) &= -k_1 |x|^{1/2} \text{sign}(x) \\ \dot{u}_2 &= -k_2 \text{sign}(x) \end{aligned} \quad (9)$$

The sliding mode switching function for designing the super-twisting torque and flux controller is:

$$s_\varphi = \varphi_r^* - \varphi_r \quad (10)$$

$$s_T = T_e^* - T_e \quad (11)$$

Where: φ_r^* is given stator flux linkage; φ_r is actual feedback stator flux; T_e^* is given torque; T_e is actual feedback torque.

The design torque and flux controller is:

$$\begin{aligned} u_d &= K_1 |s_\varphi|^r \text{sign}(s_\varphi) + \int_0^t K_2 \text{sign}(s_\varphi) dt \\ u_q &= K_3 |s_T|^r \text{sign}(s_T) + \int_0^t K_4 \text{sign}(s_T) dt + \omega_e \varphi_d \end{aligned} \quad (12)$$

Where K_1, K_2, K_3 , and $K_4 > 0$ are gains.

The switch function $\text{sign}(s)$ used in the traditional super-twisting algorithm is a discontinuous function that can cause chattering in the system. Introducing the saturation function $\text{sat}(s)$ instead of the switch function $\text{sign}(s)$ here not only guarantees switching characteristics of the switching function, but also reduces chattering and system discontinuities. The torque and flux controller can be designed to:

$$\begin{aligned} u_d &= K_1 |s_\varphi|^r \text{sat}(s_\varphi) + \int_0^t K_2 \text{sat}(s_\varphi) dt \\ u_q &= K_3 |s_T|^r \text{sat}(s_T) + \int_0^t K_4 \text{sat}(s_T) dt + \omega_e \varphi_d \end{aligned} \quad (13)$$

In the formula, the value of r in the super-twisting sliding mode controller is $0 < r < 1$. To select a suitable r value, the control error is approached to 0, reducing system chatter. Take $r=1/2$ here.

The saturation function $\text{sat}(s)$ is expressed as:

$$sat(s) = \begin{cases} 1, s > \sigma \\ \frac{s}{\sigma}, |s| \leq \sigma \\ -1, s < -\sigma \end{cases} \quad (14)$$

Where, σ is the thickness of the boundary layer. When $|s| > \Delta$, the power term can ensure that the system state approaches the sliding mode at a large speed; When $|s| \leq \Delta$,

B. System stability proof

For s_φ , s_T time derivative can be obtained:

$$\begin{aligned} \frac{d}{dt} s_\varphi &= -K_1 |s_\varphi|^{1/2} sat(s_\varphi) - \int_0^t K_2 sat(s_\varphi) dt + Ri_d \\ \frac{d}{dt} s_T &= -KK_3 |s_T|^{1/2} sat(s_T) - \int_0^t KK_4 sat(s_T) dt + KRi_q \end{aligned} \quad (15)$$

Make $K_5 = KK_3$, $K_6 = KK_4$. Select Lyapunov function as [16]:

$$V = \xi_1^T P_1 \xi_1 + \xi_2^T P_2 \xi_2 \quad (16)$$

Make

$$\begin{aligned} \xi_1 &= \begin{bmatrix} |s_\varphi|^{1/2} sat(s_\varphi) \\ \int_0^t K_2 sat(s_\varphi) dt \end{bmatrix} \quad \xi_2 = \begin{bmatrix} |s_T|^{1/2} sat(s_T) \\ \int_0^t K_6 sat(s_T) dt \end{bmatrix} \\ P_1 &= \frac{1}{2} \begin{bmatrix} 4K_2 + K_1^2 & K_1 \\ K_1 & 2 \end{bmatrix} \quad P_2 = \frac{1}{2} \begin{bmatrix} 4K_6 + K_5^2 & K_5 \\ K_5 & 2 \end{bmatrix} \end{aligned}$$

(17)

If $\Delta_1 > 0$, $\Delta_2 > 0$, $|Ri_d| \leq \Delta_1 |s_\varphi|^{1/2}$, $|KRi_q| \leq \Delta_2 |s_T|^{1/2}$,

$$\begin{aligned} K_1 &> 2\Delta_1 \\ K_2 &> K_1 \frac{\Delta_1^2}{8(K_1 - 2\Delta_1)} \\ K_5 &> 2\Delta_2 \\ K_6 &> K_5 \frac{\Delta_2^2}{8(K_5 - 2\Delta_2)} \end{aligned} \quad (18)$$

Then $\dot{V} < 0$. When equation (18) is satisfied, the sliding mode motion can be achieved and the system approaches the phase plane $s_\varphi = \dot{s}_\varphi = 0$, $s_T = \dot{s}_T = 0$.

V. SIMULATION ANALYSIS

With MATLAB / SIMULINK simulation tools, the traditional PMSM-DTC system and the method presented in this paper are simulated and compared. The permanent magnet synchronous motor parameters in the simulation are shown in Table 1.

The sliding mode control block diagram based on the super-twisting algorithm described in this paper is shown in Figure 1. This control method combines the sliding mode control, direct torque control, and space vector modulation to effectively control the permanent magnet synchronous motor.

TABLE I. PERMANENT MAGNET SYNCHRONOUS MOTOR RATED PARAMETERS

Parameter	Value
Stator resistance/ Ω	2.875
Stator inductance/mH	8.5
Permanent magnet flux/Wb	0.175
Number of pole pairs	4
Moment of inertia /(kg \cdot m ²)	0.001
Damping coefficient /(N \cdot m s)	0.008

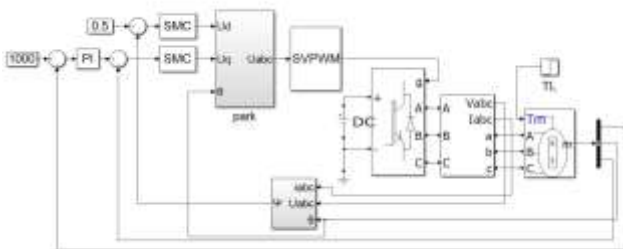
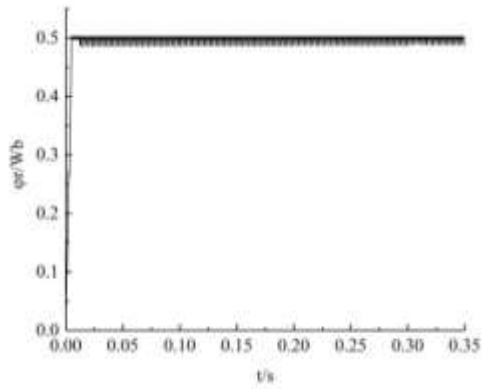


Fig.1 Sliding mode control block diagram based on super-twisting algorithm

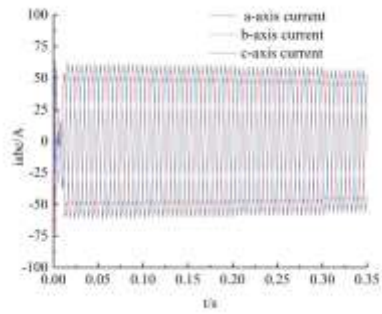
The system simulation parameters are set as follows: the motor given speed is 1000r/min, and the given stator flux is 0.5Wb. When $t = 0s$, the load torque $T_L = 0N \cdot m$ is

given; when $t = 0.1s$, the load torque $T_L = 2N \cdot m$ is given; when $t = 0.2s$, the load torque $T_L = 5N \cdot m$ is given; when $t = 0.3s$, the load torque $T_L = 10N \cdot m$ is given.

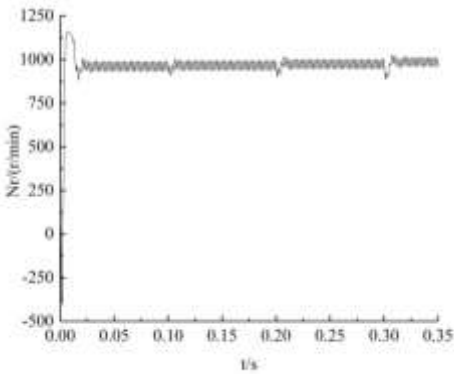
Figure 2 is the traditional direct torque control simulation response diagram. From (a)-(d) are the flux response, three-phase current, speed and torque simulation response diagrams.



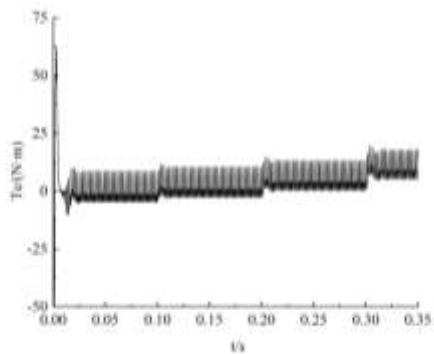
(a) Flux simulation response



(b) Three-phase current simulation response



(c) Speed simulation response

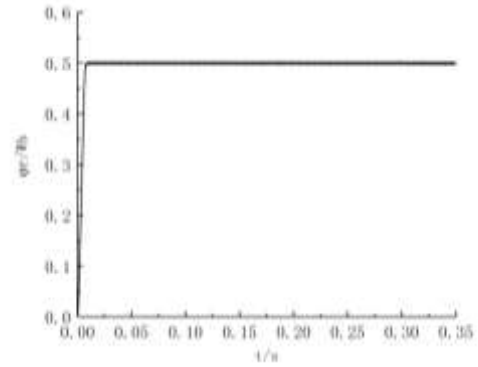


(d) Torque simulation response

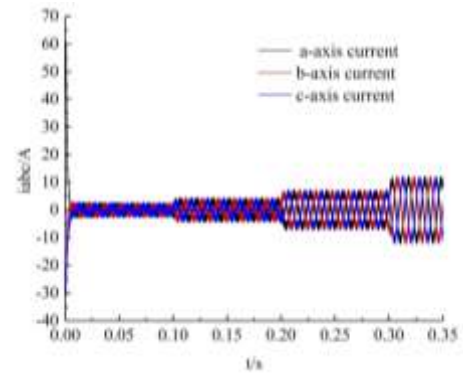
Fig.2 Traditional direct torque control simulation response diagram

Figure 3 is the simulation response diagram of the direct torque control based on the super-twisting algorithm. From

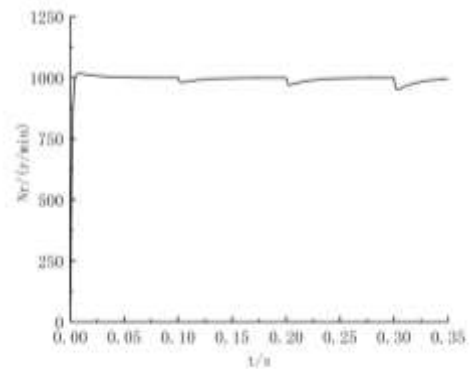
(a)-(d) are the simulation responses of the flux linkage, three-phase current, speed and torque respectively.



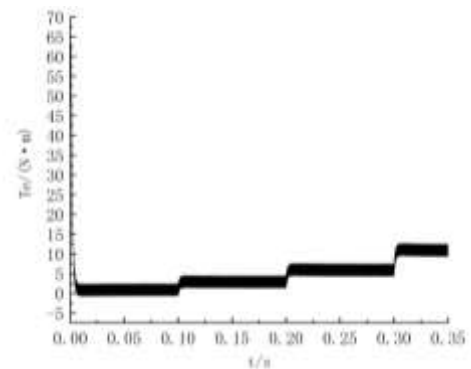
(a) Flux simulation response



(b) Three-phase current simulation response



(c) Speed simulation response



(d) Torque simulation response

Fig.3 Direct Torque Control Simulation Response Diagram Based on Super-twisting Algorithm

Figure 4 shows the comparison of the motor speed response of the two methods. Through comparison of the simulation results, under the zero load speed in the traditional direct torque control, the overshoot is severe and the response time is long. When the load torque changes, the speed fluctuation is large and the system stability is poor. In the direct torque control based on the super-twisting algorithm, the speed response time is short, the speed fluctuation is small, and the system stability is obviously improved.

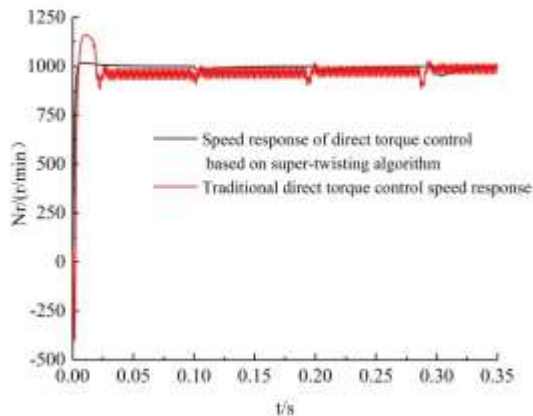


Fig.4 Comparison of two methods of motor speed response

Figure 5 shows a comparison of the torque response of the two methods. By comparing the simulation results, the traditional direct torque control torque ripple is $13.5\text{N}\cdot\text{m}$. Based on the super-twisting algorithm, the direct torque control torque ripple is $3.25\text{N}\cdot\text{m}$. Traditional direct torque control is based on four times the direct torque control ripple of the super-twisting algorithm. It can be seen that the system stability based on the super-twisting algorithm direct torque control is better.

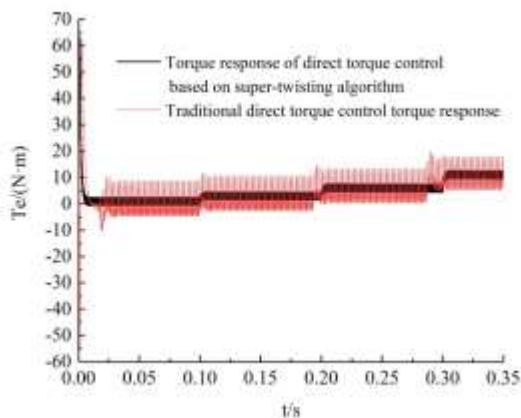


Fig.5 Comparison of two methods of motor torque response

VI. CONCLUSIONS

It can be seen from the above simulation results that during the process of the motor rising to the reference speed in the conventional direct torque control system, the overshoot of speed is severe. And when the load torque changes, the rotational speed response is relatively slow and the torque ripple is severe. In the super-twisting algorithm

direct torque control system in the process of motor up to the reference speed, the speed has some overshoot, but it can quickly return to a given speed. When the load torque changes, the speed response time is shorter and the speed fluctuation is smaller. Torque ripple is reduced by a factor of 4 over traditional direct torque control system torque ripple. System stability has improved significantly.

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