

# Effectiveness Evaluation of Kill Chain Based on PCA, AHP and Entropy Weight Method

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**Abstract**—The Kill Chain is of great significance in forming systemic combat capabilities and improving combat effectiveness. Aiming at the evaluation of Kill Chain operational effectiveness, an evaluation indicator system is constructed. Principal components analysis is used to filtrate the original evaluation indicators, and the main indicators affecting the effectiveness of the kill chain are determined; afterwards, the evaluation model is established based on entropy weight method and Analytic Hierarchy Process. A quantitative assessment is conducted using kill chains formed by actual weapons, which verifies that the model is effective and feasible.

**Keywords**—kill chain; PCA; AHP; effectiveness evaluation

## I. INTRODUCTION

Since the United States Air Force (USAF) added the killing elements in C4ISR, the concept of Kill Chain had gradually emerged. The Kill Chain is the sequence of events that must be executed to achieve the expected objective, and the most common interpretation of the term is a six phase target cycle of Find, Fix, Track, Target, Engage, and Assess (F2T2EA). In order to conduct a successful military operation, the mutual cooperation and close connection between each link are the core concern of Kill Chain.

Effectiveness evaluation is an important research topic in the studies of Kill Chain. Jun LU pointed out that the closed-loop time is one of the most important indexes characterizing the efficiency of the kill chain [1]. He constructed three types of kill chain models based on the typical combat system and process, and then formalized the uncertainties such as completion time, completion status and transition status with stochastic methods to complete the derivation of closed-loop time. Bloye created a virtual scene based on the actual terrain, and deployed a series of active weapons at different locations [2]. He defined the combat and defense processes and used different methods to dispatch weapons to build a kill chain. Then, the simulation was conducted to calculate the closed-loop time. Kerrick and Shaw built a Colored Petri Net (CPN) from the architecture of the kill chain for Air-to-Ground targeting to assess the effectiveness of the kill chain [3]. Edward H. S. Lo and T. Andrew Au calculated the duration of each link in the kill chain based on the communication records kept in actual combat, and closed-loop time was obtained through further derivation [4].

In previous studies, the combat effectiveness evaluation of kill chain was mostly a system-level assessment but less detailed at the weapons level. The performance of each weapon that completes the designated task is undoubtedly one of the most

important factors affecting combat effectiveness of kill chain. In order to evaluate the effectiveness of the kill chain based on the performance parameters of weapons which build the kill chain, this paper constructs a kill chain effectiveness evaluation model based on Principal Component Analysis(PCA), Analytic Hierarchy Process (AHP) and Entropy Weight Method(EWM).

This paper is organized into 5 sections. Section 2 introduces the target and evaluation indicators of each phase of Kill Chain. The complete methodology framework and algorithm implementation steps will be described in section 3. And section 4 conducts an effectiveness evaluation of kill chains formed by actual weapons based on the algorithm. The paper ends with concluding remarks in section 5.

## II. KEY INDICATORS OF KILL CHAIN

Each link of the kill chain has different operational objectives. Even if the same weapon that is deployed in different phases of the kill chain, the performance indicators to be considered are different. Find. To complete the surveillance and reconnaissance of the enemy's object of possible military interest. The main task is to fuse and integrate sensors information, support real-time perception of the battlefield situation and enable commanders to grasp the battlefield changes opportunely. Therefore, this phase will focus on the weapon's reconnaissance capabilities.

Fix. To determine whether the new target identified in Find is a hostile target and whether it is worth attacking. Fix focuses on the performance of Radar IFF system.

Track. To continuously track the confirmed target, measure and report the target location. Track concentrates on the tracking capabilities of weapon.

Target. To formulate a military plan based on the intelligence of target and give instructions to weapons to commence attack. Target focuses on the weapon's capabilities includes target processing, command and control.

Engage. To engage the confirmed target. Engage concentrates on the combat capabilities of weapons.

Assess. To determine effects of Engage on targets. If the operational objectives are achieved, the closed-loop state of the kill chain can be achieved. Assess focuses on the reconnaissance and communication capabilities of weapons.

Based on the above analysis, key indicators for the effectiveness evaluation of kill chain are shown in TABLE I.

TABLE I. KEY INDICATORS OF KILL CHAINS

Phase	Indicators
Find	detection range(km), transmit power(kW), endurance(h), working frequency(MHz), pulse width( $\mu$ s), pulse repetition rate(kHz), antenna length(m), radome thickness(m), azimuth beam width( $^{\circ}$ ), elevation beam width( $^{\circ}$ ), antenna side lobe (dB)
Fix	transmit power(kW), received frequency(MHz), dynamic range(dB), sensitivity(dBv), output power(kW), MTBF(h), peak power(w), recognition range (km)
Track	target tracking capacity, angular accuracy( $^{\circ}$ ), velocity measurement range (kn) , clutter improvement factor(dB), operating range(km), endurance(h), MTBF(h)
Target	navigation capability, target processing capacity, azimuth( $^{\circ}$ ), data transfer rate(kbps), working frequency(MHz), operating range(km)
Engage	tactical range(km), service ceiling(m), maximum suspension weight(kg), maximum level flight speed(km/h), hitting probability(%), maximum attack range(km), maximum missile speed(km/h)
Assess	endurance(h), transmit frequency(MHz), received frequency(MHz), dynamic range(dB), sensitivity(dBv) , operating range(km) , MTBF(h)

### III. MODELING

Effectiveness evaluation method can be modeled since the key indicators of kill chain are explicit, and the most critical issue is to determine the weight of each indicator. There are many ways to determine the index weights, which can be roughly divided into subjective weighting method and objective weighting method. Subjective weighting method refers to people's subjective determination of the importance of various factors to be assessed, including analytic hierarchy process (AHP) and expert scoring method. The objective weighting method explores the relationship between the original data of indicators based on mathematical calculation to acquire the weight of each indicator, such as entropy weight method (EWM), mean square error method and principal component analysis (PCA).

Subjective weighting method relies extremely on experience so that information covered in the data is ignored. On the contrary, objective weighting method completely depends on data, which cannot reflect the preference for indicators of deciders. To overcome the shortcomings of subjective weighting method and objective weighting method, AHP combining with EWM was adopted. However, it can be seen from TABLE I that there are many evaluation indicators at each phase of kill chain. Therefore, when the judgment matrix is constructed in AHP, it is difficult to define the importance of the two indicators, resulting in the inconsistency of the judgment matrix. Consequently, PCA is used to filtrate the original evaluation indicators.

#### A. Data Standardization

Let the original data matrix be  $A=(a_{ij})_{m \times n}$ , where m is the number of objects to be evaluated and n is the number of indicators. Due to the differences in the units and magnitudes of the indicators, the original data need to be standardized as  $R=(r_{ij})_{m \times n}$ . There are two kinds of indicators: the bigger the better and the smaller the better. We take the following two different standardized methods:

$$r_{ij} = \frac{a_{ij}}{\max_j \{a_{ij}\} + \min_j \{a_{ij}\}} \quad (1)$$

$$r_{ij} = \frac{\max_j \{a_{ij}\} + \min_j \{a_{ij}\} - a_{ij}}{\max_j \{a_{ij}\} + \min_j \{a_{ij}\}} \quad (2)$$

#### B. Filtrate Indicators

PCA is a popular unsupervised statistical method to find key elements in a set of indicators. We filtrated the original evaluation indicators using PCA.

1) *Eigenvectors and eigenvalues of covariance matrix*: Let covariance matrix of R be  $C=(c_{ij})_{m \times n}$ :

$$c_{ij} = \frac{\sum_{k=1}^n (r_{ki} - \bar{r}_i)(r_{kj} - \bar{r}_j)}{n-1} \quad (3)$$

$\bar{r}_i$  and  $\bar{r}_j$  are the meanvalue of the  $i, j$  indicators, respectively. Eigenvalues of C are  $\lambda_1, \lambda_2, \dots, \lambda_n (\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_n)$  and the corresponding eigenvectors are  $e_1, e_2, \dots, e_n$ .

2) *Principal component matrix*: Proportion of total variation explained by the weights of  $\lambda_i$ :

$$p_i = \frac{\lambda_i}{\sum_{i=1}^n \lambda_i} \quad (4)$$

Select the first k eigenvectors such that  $\sum_{i=1}^k p_i \geq 0.9$ , then we got the principal component  $Z_i = R \cdot e_i (i=1, \dots, k)$ .

3) *Filtrate indicators*: Let  $Q = \sum_{j=1}^k p_i \cdot Z_i$ , then we can filter out indicators based on the coefficient of the original indicators in Q.

#### C. Effectiveness Evaluation

1) *AHP weights*: Hierarchy can be constructed based on the selected indicators. Then judgment matrix B can be constructed through a series of pairwise comparisons.  $\lambda_{\max}$  is the

maximum meanvalue of B and the corresponding eigenvectors is W. The component of W is  $\omega_j$ , which is the weight of the the corresponding indicator.

2) *EWM weights*: The entropy of the i-th indicator can be written as

$$e_j = -\frac{1}{\ln m} \sum_{i=1}^m p_{ij} \ln p_{ij} \quad (5)$$

where

$$p_{ij} = \frac{r_{ij}}{\sum_{i=1}^m r_{ij}} \quad (6)$$

Then we can get the EWM weights  $\alpha_j$ :

$$\alpha_j = \frac{1 - e_j}{n - \sum_{j=1}^n e_j} \quad (7)$$

3) *Combined weight*: Combined AHP weights and EWM weights then we got the combined weights  $\varepsilon_j$ :

$$\varepsilon_j = \frac{\alpha_j \omega_j}{\sum_{j=1}^n \alpha_j \omega_j} \quad (8)$$

4) *Effectiveness evaluation*: After we known the hierarchy and the weights, we can calculate the effectiveness priorities of the kill chains to be evaluated.

#### IV. CASE ANALYSIS

Four kill chains that are shown in TABLE II are formed by actual weapons. We will evaluate the effectiveness of the kill chains in this section.

TABLE II. KILL CHAINS

Serial Number	Find	Fix	Track	Target	Engage	Assess
1	E-3	E-3	E-3	E-3	F-15E	E-3
2	S-100B	S-100B	S-100B	S-100B	F-15E	S-100B
3	E-3	E-3	E-3	E-3	su30MK2	E-3
4	S-100B	S-100B	S-100B	S-100B	su30MK2	S-100B

We filtrated indicators based on PCA at first. Take the Find as example, the raw data and the standardized data that are shown in TABLE III derived from professional books [5,6].

TABLE III. RAW DATA AND STANDARDIZED DATA

Indicators	Raw Data				Standardized Data			
	E-2C	E-3	A50	S-100B	E-2C	E-3	A50	S-100B
detection range(km)	268	320	230	300	0.4872	0.5818	0.4181	0.5454
transmit power(kW)	3.9	8.5	20	3	0.1695	0.3695	0.8695	0.1304
endurance(h)	360	510	450	330	0.4285	0.6071	0.5357	0.3928
working frequency(MHz),	450	3300	3700	3300	0.1084	0.7951	0.8915	0.7951
pulse width( $\mu$ s)	13	1	1	6	0.0714	0.9285	0.9285	0.5714
pulse repetition rate(kHz)	0.3	100	25	250	0.0011	0.3995	0.0998	0.9988
antenna length(m)	7.3	9.1	9.4	8	0.4371	0.5449	0.5628	0.4790
radome thickness(m),	0.8	1.8	1.8	0.6	0.3333	0.75	0.75	0.25
azimuth beam width( $^{\circ}$ )	6.6	1	0.7	0.7	0.0958	0.863	0.9041	0.9041
elevation beam width( $^{\circ}$ )	20	5	4.2	9	0.1735	0.7933	0.8264	0.6280
antenna side lobe( $^{\circ}$ )	-34	-50	-24	-35	0.5405	0.3243	0.6756	0.5270

Analyze raw data using PCA, the total variance explanation is shown in TABLE IV. And the cumulative of variance of the

first three components are equal to 100%, which means the first three components are principal components.

TABLE IV. TOTAL VARIANCE EXPLAINED

Component	Initial Eigenvalues			Extraction sums of Squared Load		
	Total	% of Variance	Cumulative%	Total	% of Variance	Cumulative%
1	6.322	57.477	56.970	6.322	57.477	56.970
2	2.915	26.501	83.978	2.915	26.501	83.978
3	1.762	16.022	100.000	1.762	16.022	100.000
4	9.833E-17	8.234E-15	100.000			
5	5.041E-17	4.221E-15	100.000			

Component	Initial Eigenvalues			Extraction sums of Squared Load		
	Total	% of Variance	Cumulative%	Total	% of Variance	Cumulative%
6	3.523E-17	2.950E-15	100.000			
7	2.458E-17	2.058E-15	100.000			
8	-1.508E-17	-1.263E-15	100.000			
9	-2.128E-17	-1.782E-15	100.000			
10	-5.531E-17	-4.632E-15	100.000			
11	-1.713E-16	-1.435E-14	100.000			

The component matrix is shown in TABLE V. Therefore, we can get the PCA composite score Q:

$$Q = 0.2441r_5 + 0.2393r_3 + 0.2322r_{10} + 0.2160r_7 - 0.2083r_{11} + 0.2074r_9 + 0.2039r_4 + 0.1997r_8 + 0.1743r_1 + 0.0820r_6 + 0.0608r_2 \quad (9)$$

TABLE V. COMPONENT SCORE COEFFICIENT MATRIX

Indicators	Principal Component		
	1	2	3
detection range( $r_1$ ,km)	0.3938	0.0792	-0.0203
transmit power( $r_2$ ,kW)	0.3089	-0.0509	0.4698
endurance( $r_3$ ,h)	0.3850	0.1133	-0.1194
working frequency( $r_4$ ,MHz),	0.3946	-0.0648	0.0397
pulse width( $r_5$ , $\mu$ s)	0.3359	0.2244	-0.2814
pulse repetition rate( $r_6$ ,kHz)	0.3535	0.1662	-0.2706
antenna length( $r_7$ ,m)	0.3393	-0.1761	0.3208

Indicators	Principal Component		
	1	2	3
radome thickness( $r_8$ ,m),	-0.0249	0.5422	0.2806
azimuth beamwidth( $r_9$ , $^\circ$ )	0.0194	0.5014	-0.3874
elevation beamwidth( $r_{10}$ , $^\circ$ )	0.3013	-0.3776	-0.0759
antenna side lobe ( $r_{11}$ ,dB)	-0.0229	-0.4205	-0.5224

The coefficient of each indicator in the PCA composite score indicates the weight of the indicator, so we can get the first 5 indicators: pulse width, endurance, elevation beam width, antenna length, antenna side lobe.

In the same way, the evaluation indicators of the other five phases can be obtained, so that the hierarchy can be constructed. Then we can successively get the AHP weights, the EWM weights and the combined weights, which are shown in TABLE VI.

TABLE VI. HIERARCHY EVALUATED

Phase	Weights	Indicators	Weights		
			AHP	EWM	Combined weights
Find	0.0812	endurance	0.0983	0.0397	0.0104
		pulse width	0.5120	0.5605	0.7625
		antenna length	0.0388	0.0133	0.0014
		elevation beam width	0.2527	0.3042	0.2042
		antenna side lobe.	0.0983	0.0823	0.0215
Fix	0.1964	sensitivity	0.0775	0.0329	0.0064
		output power	0.5272	0.6597	0.8671
		MTBF	0.0775	0.0547	0.0106
		peak power	0.2403	0.1655	0.0991
		recognition range	0.0775	0.0873	0.0169
Track	0.0812	target tracking capacity	0.5021	0.7544	0.9177
		angular accuracy	0.1737	0.0748	0.0315
		velocity measurement range	0.0957	0.0842	0.0195
		clutter improvement factor	0.1737	0.0687	0.0289
		operating range	0.0547	0.0179	0.0024
Target	0.1964	navigation capability	0.2412	0.2386	0.1375
		target processing capacity	0.5652	0.6232	0.8415
		azimuth	0.0908	0.0428	0.0093
		data transfer rate	0.0514	0.0835	0.0103
		operating range	0.0514	0.0119	0.0015
Engage	0.049	service ceiling	0.0323	0.0440	0.0059
		maximum suspension weight	0.1613	0.1749	0.1167
		maximum level flight speed	0.2903	0.2700	0.3243

Phase	Weights	Indicators	Weights		
			AHP	EWM	Combined weights
Assess	0.3958	hitting probability	0.2258	0.2278	0.2128
		maximum missile speed	0.2903	0.2832	0.3402
		endurance	0.1717	0.0987	0.0291
		MTBF	0.6643	0.8452	0.9640
		transmit frequency	0.0440	0.0019	0.0001
		dynamic range	0.0440	0.0063	0.0005
		sensitivity	0.0760	0.0478	0.0062

We can calculate the effectiveness of the kill chains as shown in Table VII.

TABLE VII. KILL CHAIN EFFECTIVENESS

Serial Number	1	2	3	4
Effectiveness	0.3011	0.2062	0.2938	0.1989

The evaluation results show that the kill chain consisting of E-3 and F-15E has the best operational performance and is consistent with the actual situation.

## V. CONCLUSION

The effectiveness evaluation of kill chain is a complex problem. Therefore, the evaluation method must be objective and rigorous. In this paper, PCA is used to filtrate indicators to reduce the uncertainty of pairwise comparisons. Then combined weight method is proposed combining AHP and EWM to reflect the quantity of information and value that contained in original data at the same time. Finally, the effectiveness of kill chains formed by actual weapons was evaluated based on the evaluation model, and the results turned to be consistent with the expectations, which indicating that the model is effective and feasible.

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