Abstract—The Goldbach conjecture declares that any even number $2m=2n+2>4$ can be expressed as the sum of two prime numbers. The mathematical modeling of the conjecture is: any even number $2m=2n+2$ greater than 4 can be expressed as $2n+2=a+b$, $2\leq a\leq n+1$, $n+1\leq b\leq 2n$. With the modeling, let $c$ be a composite number in $2-2n$, a mapping number is $2m-c$ or $2n+2-c$. A complete composite pair is a pair $(c, 2m-c)$ that both $c$ and $2m-c$ are composite numbers. The composite numbers one-to-one correspond to the mapping numbers. Using an induction to absurdity, suppose the Goldbach conjecture is wrong, so that $2n+2$ cannot be expressed as the sum of two primes. With the mathematical modeling for the even number $2n+2$, numbers $2\leq n\leq 128$ are all composite numbers or mapping numbers. A false supposition does not stand when $n\geq 128$. Meanwhile, the Goldbach conjecture can be easily verified for the even numbers in $6\sim 256$. Hence, the Goldbach conjecture is proved.

Keywords—Goldbach conjecture; composite number pair; mapping numbers; induction to absurdity

NOTATION

- $e$: An even number greater than 4; $e = 2m = 2n + 2$.
- $(a, b)$: A decomposition pair. $a$ and $b$ are natural numbers, $e=xa+by$.
- $a\sim b$: The natural number range from $a$ to $b$. A natural number $x$ in the range satisfies $a\leq x\leq b$.
- $p$: A prime number.
- $[x]$: The largest integer less than or equal to $x$.
- $n!$: The factorial computation, meaning $n\times(n-1)\times\ldots\times 2\times 1$.
- $c_k^e$: $c_k^e = \frac{n!}{k!(n-k)!}$.
- $c_k^e$: $c_k^e = \frac{(2n)\times(2n-1)\times\ldots\times(n+1)}{2\times 3\times\ldots\times n}$.
- $\prod_{p\leq n}p$: The product of all the prime numbers less than or equal to $n$.
- a|b: Divide exactly. Integer $b$ is a multiple of integer $a$.
- a|b: Not divide exactly. Integer $b$ is not a multiple of integer $a$.
- $i_p$: The minimum mapping number in $2-n$ corresponding to the maximum composite number in $(n+1)-2n$ produced by prime $p$, $2 \leq i_p \leq p+1$.

I. INTRODUCTION

Three major mathematical conjectures include Fermat’s Conjecture, Four Color Conjecture and Goldbach Conjecture. The former two conjectures have been proved as Fermat’s Last Theorem and Four color theorem. The Goldbach conjecture (referred as the conjecture) is a bright pearl in the field of mathematics and is regarded as unproved till now. It was raised in 1742 in a letter from Goldbach to Euler. In 1900, D. Hilbert proposed the Goldbach conjecture as one of the twenty-three unsolved problems. The Goldbach’s conjecture is worth studying. Solving the conjecture promotes a breakthrough in related fields, especially in the field of number theory, leading to a series of related progress.

A common referred expression for the Goldbach conjecture is: any even number greater than or equal to 6 (or greater than 4) can be expressed as the sum of two odd prime numbers. The conjecture is also called as “1+1” or “1+1=2”. The problem is very difficult. In many journals, periodicals and Internet resources, enthusiasts have shared their "proofs" for the Goldbach’s conjecture. As far as I know, the problem has not been solved. So far, the best work about the conjecture is the proof of “1+2” by Chen Jingrun.

II. RELATED STUDIES ON THE GOLDBACH CONJECTURE

Geniuses have poured great enthusiasms and made great efforts on the proof of the Goldbach conjecture. As far as I know, in most existing literatures, researchers verify or prove rigorously the Goldbach conjecture, or research on similar propositions to the Goldbach’s conjecture. This means that the proof of the Goldbach conjecture has not been successful. The best contribution on Goldbach’s conjecture is made by China’s Chen Jingrun. Chen Jingrun proves that every sufficiently large even number can be written as the sum of either two primes, or a prime and a semiprime (the product of two primes). His contribution is called as “1+2”. There are also other researches [3]. However, these researches do not prove the conjecture itself.

The Goldbach conjecture brings other topics relevant with the Goldbach conjecture. Reference [4] discusses the sum of four prime numbers. Reference [5] verifies “ternary Goldbach conjecture”. Reference [6] further improves Hua Luogeng’s work. Some works are not the proofs of the Goldbach conjecture[7]. The study of the conjecture promotes progresses in the field of number theory. For example, prime distribution rule [8] is a harvest in the study of the conjecture.
III. RELATED PROPERTIES OF NUMBER THEORY

Definition 1 A prime number is a natural number that can only be divided exactly by 1 and itself. A composite number can be divided exactly by 1, itself, and other natural numbers.

Definition 2 Let p be a prime number, k be a natural number and k>1, we call kp or k×p as p’s composite number, or the composite number kp is produced by p. When k≥1, kp is p’s multiple, or kp is a multiple produced by p. The multiples produced by the prime p are p, 2p, 3p, 4p…

The composite numbers produced by p are 2p, 3p, 4p…, so 2=2. Hence 2≥2, 2=2, and 2=2, “ we get 2≥2. Because the smallest prime number is 2, a composite number g is divisible exactly by a prime number that is less than or equal to 2.

When n=10, the number of prime numbers is not greater than 2. When n≥2, any even number is not prime. Therefore, the number of prime numbers not greater than n is 2.

Definition 3 Let x be a real number, [x] is equal to the largest integer less than or equal to x.

Property 1 Let x≥0, then [x]≤x. 2[x]+1 ≥ [2x] ≥ 2[x].

Proof: When x≥0, let x=a+b, a=[x], 0≤b<1; then [x]≤x, 2x=2a+2b, so [2x] = [2a+2b]=\(2[a]+1,1+b\geq0.5\).

Hence 2[2x]+1 ≥ [2x] ≥ 2[x].

An equivalent expression of property 1 is 0 ≤ [2x] – 2[x] ≤ 1.

Property 2 Let a, b, i be integers,

(i) a>b>0, i>0, then \(a+i \leq b+i \leq a\).

(ii) a≥2, a=0, then \(a+1 \leq b\).

Proof: Let a, b, i be integers

(i) When a>b>0, i>0 then \(a+i \leq b+i \leq a\).

From \(\frac{a+i}{b+i} \leq \frac{a}{b}\), we get \(a+i = \frac{a+i}{b+i} \leq \frac{a}{b}\), for i≥0.

Hence we get \(a+i \leq a\).

(ii) When a≥2, a>0, then \(a+1 \leq b\).

\(\frac{a+i}{b+i-i} = \frac{a+i}{b+i-i} \leq \frac{a+1}{b+1-i}<1\). Therefore, \(a+i \leq a\).

Theorem 1 Let n be a positive integer, then \(C^k_n \geq C^k_m\) when k≥n.

Proof: Let k be a positive integer, \(C^k_n \geq C^k_m\) when k≥n.

Meanwhile, \(C^k_n \geq C^k_m\).

Hence \(2nC^k_n+C^k_m = 2n = 4^n\).

Therefore, \(C^k_n \geq 4^n\).

Corollary 1.1 Let n be a positive integer, then \(C^k_n \geq C^k_m\) when k|n and k≤2.

Proof: Let k be a positive integer, \(C^k_n \geq C^k_m\) when k|n and k≤2.

Meanwhile, \(C^k_n \geq C^k_m\).

Hence, \(2nC^k_n+C^k_m = 2n = 4^n\). Therefore, \(C^k_n \geq 4^n\).

Corollary 1.2 n≥2. When n is an even number, \(C^n_n \geq 4^n\).

When n is an odd number, \(C^n_n \geq 4^n\).

Proof: n is a positive number, n≥2. From theorem 1, when n is a positive even number, \(C^n_n \geq 4^n\).

From Corollary 1.1, when n is a positive odd number, \(C^n_n \geq 4^n\).

Notice that the right side is \(4^n\) whether n is an even number or an odd number.

Lemma 1 Let p be a prime number, k be a natural number, then \(\prod_{k+1 < p \leq k+1} p < 4^k\).

Proof: Let p be a prime number, k be a natural number. All the prime numbers satisfying k+1<p≤2k+1 are in (2k+1)! not in (k+1)! or k!. Hence \(\prod_{k+1 < p \leq k+1} P\) is a factor of \(C^{2k+1}_{k+1}\). Thus \(\prod_{k+1 < p \leq k+1} P \leq C^{2k+1}_{k+1}\). And \(C^{2k+1}_{k+1} = C^{2k+1}_{k+1} + C^{2k+1}_{k+1} < 2^{2k+1} = 2^k\).

Therefore, \(\prod_{k+1 < p \leq k+1} P < C^{4k+1}_{k+1} \times 4^k\). Proved.

Theorem 2 Let n be a positive integer, p be a prime number, then \(\prod_{p \leq n} p < 4^n\).

Proof: Using mathematical induction. When n=2, 2<16, it is satisfied.

Suppose the theorem is satisfied for all the positive integer numbers less than n.

If n=2 and n is an even number, \(\prod_{p \leq n} p = \prod_{p \leq n-1} p\).

If n is an odd number, let n=2k+1. From Lemma 1 and the inductive supposition, we get

\(\prod_{p \leq n} p = \prod_{p \leq 2k+1} p \times \prod_{p \leq k} p < 4^{2k+1} \times 4^k = 4^n\).

proved.
IV. MATHEMATICAL MODELINGS OF THE GOldbach CONJECTURE

A. Primitive Mathematical Modelings of the Goldbach Conjecture

Any even number \( e \) not less than 6 can be expressed as the sum of two natural numbers; i.e., \( e=a+b \), \( 0<a, b<e \). The \( a \) is located in the left column, \( b \) in the right column, and we name \((a, b)\) as a decomposition pair. For short, \( a \) is in the left, \( b \) is in the right. The corresponding decomposition of \( a \) and \( b \) is listed in the left and right columns of Table 1.

Table I: The Entire Left-Right Decompositions of an Even Number \( e \), \( e \geq 6 \), \( e=A+B \)

<table>
<thead>
<tr>
<th>left column ( a )</th>
<th>right column ( b )</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>( e-1 )</td>
</tr>
<tr>
<td>2</td>
<td>( e-2 )</td>
</tr>
<tr>
<td>3</td>
<td>( e-3 )</td>
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<td>( e-3 )</td>
<td>3</td>
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<tr>
<td>( e-2 )</td>
<td>2</td>
</tr>
<tr>
<td>( e-1 )</td>
<td>1</td>
</tr>
</tbody>
</table>

In Table I, \( e=a+b \), \( 0<a, b<e \), \( e \) is an even number. It is natural that we list the numbers from small to large in the left column, and from large to small in the right column accordingly. Each decomposition pair \((a, b)\) has the relation \( a+b=e \). Any number in the left is one-to-one corresponding to another number in the right when \( e \) is expressed as two addend numbers \( a \) and \( b \) according to the Goldbach conjecture.

**Definition 4** \((a, b)\) is an even pair when \( a \) and \( b \) are even numbers; \((a, b)\) is an odd pair when \( a \) and \( b \) are odd numbers; \((a, b)\) is a prime pair when \( a \) and \( b \) are prime numbers; \((a, b)\) is a composite pair when at least one number of \( a \) and \( b \) is a composite number.

There are duplicate decomposition pairs in Table I; i.e., there exist decomposition pairs which have the same numbers only with a different order. For example, \((3, 5)\) and \((5, 3)\) have the same numbers and with only different orders. Table 2 lists out the left-right decomposition of the Goldbach conjecture without duplicate pairs.

Table II: The Left-Right Decomposition of \( e \), \( e \) is an Even Number, \( e \geq 6 \), \( e=A+B \)

<table>
<thead>
<tr>
<th>left column ( a )</th>
<th>right column ( b )</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>( e-1 )</td>
</tr>
<tr>
<td>2</td>
<td>( e-2 )</td>
</tr>
<tr>
<td>3</td>
<td>( e-3 )</td>
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<td>...</td>
<td>...</td>
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<td>( e/2-2 )</td>
<td>( e/2+2 )</td>
</tr>
<tr>
<td>( e/2-1 )</td>
<td>( e/2+1 )</td>
</tr>
<tr>
<td>( e/2 )</td>
<td>( e/2 )</td>
</tr>
</tbody>
</table>

According to the addition exchange law, Table 2 can represent all the decomposition pairs in Table 1. In order to express and observe intuitively the inherent properties of the Goldbach conjecture, we replace the even number \( e \) with \( 2m \), and get Table 3. \( m \) is the shared number in the left column and right column. \( 2m=a+b \), \( 1 \leq a, b \leq 2m-1 \).

Table III: The Left-Right Decomposition of the Goldbach Conjecture, \( 1 \leq a, b \leq 2m-1 \)

<table>
<thead>
<tr>
<th>left column ( a )</th>
<th>right column ( b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 2m-1 )</td>
</tr>
<tr>
<td>2</td>
<td>( 2m-2 )</td>
</tr>
<tr>
<td>3</td>
<td>( 2m-3 )</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>( m-2 )</td>
<td>( m+2 )</td>
</tr>
<tr>
<td>( m-1 )</td>
<td>( m+1 )</td>
</tr>
<tr>
<td>( m )</td>
<td>( m )</td>
</tr>
</tbody>
</table>

The left column of Table 3 includes all the natural numbers from 1 to \( m \), the right column includes all the natural numbers from \( 2m-1 \) to \( m \) accordingly. According to the addition exchange law \( a+b=b+a \), The decomposition pairs in Table 3 represent all the decomposition pairs in Table 1.

**Lemma 2** Equivalent proposition No.1 of the Goldbach conjecture: There is at least one prime pair in Table 3; i.e., at least one row in Table 3 contains only prime numbers.

Proof: When the Goldbach conjecture is right, there are two prime numbers \( a \) and \( b \) whose sum is \( 2m \). Suppose \( a \) is less than or equal to \( b \), then \( 1 \leq a \leq m \), \( m \leq b \leq 2m-1 \), so the prime pair \((a, b)\) must exist in Table 3.

When there is one prime pair \((a, b)\) in Table 3, then \( a+b=2m \), the Goldbach conjecture is right.

Therefore, “There is at least one prime pair in Table 3” is both necessary condition and sufficient condition of the Goldbach conjecture. Proved.

Lemma 2 shows that the Goldbach conjecture implies that there must be a prime pair for an even number \( 2m \) in Table 3.

B. Formal Left-right Decomposition of the Conjecture

**Lemma 3** Equivalent proposition No.2 of the Goldbach conjecture: except \((1, 2m-1)\), at least one prime pair \((a, b)\) exists among the other decomposition pairs in Table 3.

Proof: Number \( 1 \) is neither a prime number, nor a composite number. The only difference between Lemma 3 and Lemma 2 is that Lemma 3 excludes one decomposition pair \((1, 2m-1)\), so the equivalent proposition No.2 is also right.

According to Lemma 3, we only need to study the decomposition pairs when \( a \geq 1 \). Let \( 2n=2m-2 \), we can get Table 4 that excludes the decomposition pair \((1, 2m-1)\). \( m \) is in the left, and is also in the right; \( m \) is not in \( 1-n \), but in \((n+1)-2n\).
TABLE IV. THE LEFT-RIGHT DECOMPOSITION OF THE GOLDBACH CONJECTURE. 2≤a≤2n+1, N+1≤b≤2n, A=B=2n+2, M=N+1, 2M=6

<table>
<thead>
<tr>
<th>left column</th>
<th>right column</th>
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</thead>
<tbody>
<tr>
<td>a</td>
<td>b</td>
</tr>
<tr>
<td>2</td>
<td>2n</td>
</tr>
<tr>
<td>3</td>
<td>2n-1</td>
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<td>...</td>
<td>...</td>
</tr>
<tr>
<td>m-2</td>
<td>m-2</td>
</tr>
<tr>
<td>m</td>
<td>m</td>
</tr>
</tbody>
</table>

According to Lemma 3, we get:

**Lemma 4** Equivalent proposition No.3 of the Goldbach conjecture: there is at least one prime pair (a, b) in table 4.

Lemma 4 and Lemma 3 have the same meaning. Lemma 4 uses 2n to substitute 2m-2.

The numbers in the left and right column in table 4 includes all the composite and prime numbers from 2 to 2n. Any composite number in the left or right column in table 4 can be expressed as p^k (short for p×k), where p is a prime number not greater than n, and k is a natural number between 2 and n. The multiples of prime p are p, 2p, 3p,...,kp,...The first number p is the only prime number; the other numbers are composite.

C. Mapping Numbers

Let p be a prime number not greater than n. We focus on the corresponding numbers to p’s composite numbers in table 4, where a+b=2n+2. When b is p’s maximum composite number in the right, and let i_p be its corresponding number in the left, then 2≤i_p≤p+1. p’s maximal composite number is \( \left\lfloor \frac{2n}{p} \right\rfloor \), hence 2≤i_p=2n+2. Any corresponding number in the left to p’s composite number in the right also has the form of kp+i_p, k is a non-negative integer. Any corresponding number in the right to p’s composite number in the left also has the form of kp+i_p.

Suppose a is a composite number in 2~2n, then b=2n+2-a is a’s **mapping number**. Suppose a is p’s composite number in 2~2n, then b=2n+2-a is also called as p’s **mapping number**. p’s mapping numbers have the form of kp+i_p corresponding to prime p. The mapping number kp+i_p is also called as a mapping number of prime number p. For example, when 2n=34, p=5, the mapping number in the left is 6 corresponding to 30 in the right; the mapping number in the right is 21 corresponding to 15 in the left; i_p=6. Notice that when p∤m, m is a special number. The p’s composite number and mapping number is m itself.

i_p is the **minimum mapping number** in the left corresponding to the maximum composite number in the right produced by p, 2≤i_p≤p+1. 2n+2-2p is the maximum mapping number in the right corresponding to the minimum composite produced by p in the left, m≤2p.

A composite pair (a, b) is a decomposition pair in which at least one addend of a and b is a composite. This means: (i) The number a in the left is either a composite number, or a mapping number corresponding to another composite number in the right; (ii) The number b in the right is either a composite number, or a mapping number corresponding to another composite number in the left.

A **complete composite pair** (a, b) is a composite pair when both a and b are composite numbers; an **incomplete composite pair** (a, b) is a composite pair when only one number of a and b is composite. A composite pair is either a complete composite pair or an incomplete composite pair.

The Goldbach conjecture means that there exists at least one prime pair (a, b) to an even number 2n+2, so we get:

**Theorem 3** A sufficient condition of the Goldbach conjecture: there exists at least one number in 2~2n that is neither a composite number nor a mapping number.

Proof: Suppose there exist a natural number g, 2≤g≤2n, and g is neither a composite number nor a mapping number. Then the decomposition (g, 2n - g) is not a composite pair. It is a prime pair. The Goldbach conjecture is right then. Proved.

D. The Numerator and denominator of \( C_{2n}^{p} \)

The numerator factors of \( C_{2n}^{p} \) include the natural numbers (n+1)-2n; the denominator factors of \( C_{2n}^{p} \) include the natural numbers 2~n. The multiplicative factor 1 can be excluded from the denominator, for it does not affect the value of \( C_{2n}^{p} \).

**Property 3** Let p be a prime number, n be a positive integer. Suppose s is the maximum integer satisfying \( p^s \mid n! \), then \( \sum_{i=1}^{s} i \frac{1}{p} \).

Proof: As n! includes all the product factors \( p^i, 2p^i, 3p^i, \ldots \) which are less than or equal to n, so the number of product factors of the multiples of \( p^i \) is \( \frac{n}{p^i} \). Suppose s is the maximum integer satisfying \( p^s \mid n! \), s is the sum of the exponents of all the multiples of p, then \( s = \sum_{i=1}^{s} i \frac{1}{p} \).

**Property 4** For any prime p, let s be the maximum number satisfying \( p^s \mid n! \), then \( n! = \prod_{p \mid n!} p^s \).

Proof: Let p be a prime. Any prime factor in n! is in 1~n, thus (n!) \( \prod_{p \mid n!} p^s \). For any prime p∤n, s is the maximal number satisfying \( p^s \mid n! \), so \( \prod_{p \mid n!} p^s \mid (n!) \). Hence \( n! = \prod_{p \mid n!} p^s \).

According to property 3 and property 4, we can get

**Corollary 4.1** For any prime \( p \leq n \), let \( s_p \) be the maximal number satisfying \( p^s \mid C_{2n}^{p} \), then \( s_p = \sum_{i=1}^{s} (\left\lfloor \frac{n}{p^i} \right\rfloor - 2\left\lfloor \frac{n}{p^{i-1}} \right\rfloor) \).

As we know, \( C_{2n}^{p} = \frac{(2n)!}{(2n-2p-1)! \cdot 2^p \cdot 5^{s_p}} \).

Define **composite expression** A1 as the expression with all the composite factors in \( C_{2n}^{p} \).

\[ A1 = \frac{(2n)!}{(2n-2p-1)! \cdot 2^p \cdot 5^{s_p}} \]

Define **mapping expression** A2 as the expression with all the mapping factors in \( C_{2n}^{p} \).

\[ A2 = \frac{(2n)!}{(2n-2p-1)! \cdot 2^p \cdot 5^{s_p}} \]

Define **p’s multiple expression** C1(p) as the expression with all of the p’s multiple numbers in \( C_{2n}^{p} \).
C1(p) = \frac{(\frac{2n-2}{p}) \times (\frac{n}{2p} \times p - p)}{2p \times x_{n-1}}

Define p's composite expression C2(p) as the expression with all of the p's composite numbers in \(C_{2n}^n\)

C2(p) = \frac{(\frac{2n-2}{p}) \times (\frac{n}{2p} \times p - p)}{2p \times x_{n-1}}

Define p's mapping expression C3(p) as the expression with all of the p's mapping numbers in \(C_{2n}^n\)

C3(p) = \frac{(2m-2n+2) \times (2m-2n+2)}{(2m-2n) \times (2m-2n)}

Theorem 4

\[ C_{2n}^n = \frac{2n \times (2n-1) \times (n+1)}{2 \times 3 \times n-1}. \]

For an even number \(2n+2\) with the mathematical modeling of table 4,

the multiplicative factors with only the factors of complete composite pairs in the numerator and denominator of \(C_{2n}^n\)

\[ \geq \frac{(n-1) \times (n-2) \times (n-3) \times \ldots \times 2 \times 3 \times n-1}{2 \times 3 \times n-1}. \]

\( (CM) \) (When n-1 is odd, it is “=”; when n-1 is even, it is “>”.)

(CM) includes all the continuous numbers in 2–(n-1). When (n-1) is odd, there is an equal number of factors in the numerator and denominator of (CM). When (n-1) is even, there is one more factor in the numerator of (CM), and it is the minimal number \(- \frac{n-1}{2} \), which is less than \(n'' = n+1\). To sum up, there are two situations:

(i) if n is odd, (CM) = \(C_{n-1}^{n-1}\);

(ii) if n is even, (CM) = \(C_{n-1}^{n-1} = C_{n-1}^{n-1}\).

According to corollary 1.2, (CM) \(\geq \frac{n-1}{n-1}\).

Therefore, theorem 4 is proved.

E. Mapping Transformation Numbers

Let \(L = \frac{2n}{p}\), then \(2m = \frac{2n}{p} \times p + ip = L \times p + ip\).

A mapping number corresponding to a composite number \(k'p\) is \(2m - k'p = (L - k') \times p + ip = k\times p + ip\). If we express a mapping number as \(kp+ip\), \(0 \leq k \leq \lfloor 2n/p \rfloor - 2\), define the corresponding mapping transformation number of \(kp+ip\) as \(< kp+ip >\):

\[ k \leq p \quad \text{and} \quad 2 \leq i_p < p \]

\[ < kp + i_p > = \begin{cases} 1, & k = 0 \quad \text{and} \quad 2 \leq i_p < p \\ k \neq 0 \quad \text{and} \quad 2 \leq i_p < p \\ (k+1)p, & p \leq i_p < p + 1 \end{cases} \]  \( (PK) \)

Mapping transformation number (or shortened as mapping transformation) is a characteristic number induced from mapping numbers and convenient for analyzing mapping numbers. For example, as to \(2m - k'p = (L - k') \times p + ip\), the left expression \(2m - k'p\) is in mapping number formation; the right expression \(k\times p + ip\) is in mapping transformation formation. Mapping transformation is a bridge to analyze mapping numbers. Mapping transformation number is not a real factor in multiple expression, composite expression, mapping expression or \(C_{2n}^n\), but a factor existing in mapping transformation expression.

\[ p\text{'s mapping expression } C3(p) \text{ has the following relation:} \]

\[ C3(p) = \frac{(2n-2) \times (2n-2) \times (2n-2) \times \ldots \times (2n-2)}{2 \times 3 \times n-1} \]

The right expression is the mapping transformation expression \(C4(p)\) which satisfies

\[ C3(p) \leq C4(p) \]  \( (A) \)

And
Let $p$ be a prime less than or equal to $n$, $p$'s composite numbers and mapping numbers satisfying $C_4(p) \leq C_4(p)$.

Theorem 5 For an even number $2n + 2 \geq 6$ with the modeling of table 4, $p$ is a prime less than or equal to $n$,

$$\begin{align*}
\text{p's composite numbers in } (n+1)-2n & \text{ and } \text{p's composite numbers in } (n+1)-2n \\
\text{p's mapping numbers in } 1-n & \text{ and } \text{p's mapping numbers in } 1-n \\
\text{p's multiplicative factors in } 1-n & \text{ and } \text{p's mapping transformation numbers in } 1-n \\
\end{align*}$$

Let $s_p$ be the maximal integer satisfying $p \mid p_j (J)$, then $s_p \leq ((2n-1) \max (p') \leq 2n)$. $\max (p') \leq 2n$ means $r$ is the maximal value when $p' \leq 2n$.

Proof: The theorem is supported by the following table. In this table, $BS$ = the number of $p$'s multiples in $(n+1)-2n$ - the number of $p$'s composite numbers in $(n+1)-2n$, $HS$ = the number of $p$'s composite numbers in $(n+1)-2n$ - the number of $p$'s composite numbers in $1-n$, $YS$ = the number of $p$'s mapping numbers in $(n+1)-2n$ - the number of $p$'s mapping numbers in $1-n$, $ZH$ = the number of $p$'s mapping transformation numbers in $(n+1)-2n$ - the number of $p$'s mapping transformation numbers in $1-n$. Their corresponding values are shown in table 6.

As a result, we have

$$\begin{align*}
\text{HS} + ZH & \leq 1. \\
\text{(Notice that in the third row of the table when } p \mid m, \text{ HS} + ZH \text{ duplicates the } \sim m, \text{ their result should be } 2-0=1\leq 1.\)
\end{align*}$$

Let $s_p$ be the maximal integer satisfying $p \mid p_j (J)$, according to corollary 4.1 and property 1, then $s_p \leq ((2n-1) \max (p') \leq 2n)$. $\max (p') \leq 2n$ means $r$ is the maximal value when $p' \leq 2n$.

Thus Theorem 5 is right.

Corollary 5.1 Let $s_p$ be the maximal integer satisfying $p \mid p_j (J)$, then

$$\begin{align*}
\text{p's composite numbers in } 1-n & \text{ and } \text{p's mapping numbers in } 1-n \\
\text{p's multiplicative factors in } 1-n & \text{ and } \text{p's mapping transformation numbers in } 1-n \\
\end{align*}$$

then

$$\begin{align*}
\text{p's composite numbers in } 1-n & \text{ and } \text{p's mapping numbers in } 1-n \\
\text{p's mapping transformation numbers in } 1-n & \end{align*}$$

Proof: There is a one-to-one mapping relation between the composite numbers and mapping numbers.
Let $A$ be the set of all the composite numbers in $2 \sim 2n$, $B$ be the set of all the mapping numbers in $2 \sim 2n$. The element number in $A$ equals to that in $B$. The elements in $A$ one-to-one correspond to those in $B$. Let $a$ be any element in $A$, $a \in A$. Its corresponding element in $B$ is $b$, $b \in B$, and $b = 2m - a$.

Consider any prime number $p \leq n$. Let $A(p)$ be the set of all $p$'s composite numbers, $B(p)$ be the set of all $p$'s mapping numbers. The element number in $A(p)$ equals to that in $B(p)$. The elements in $A(p)$ one-to-one correspond to those in $B(p)$. Suppose $a$ is an element in $A(p)$, $a \in A(p)$, and $a = kp$. Its correspondent element in $B(p)$ is $b$, $b \in B(p)$, and $b = 2m - a = (L-k)p + ip$. The difference between any adjacent elements in $A(p)$ and $B(p)$ is $p$.

Consider the numerator and denominator of $C_{2n}^n$. All the composite numbers in the numerator and denominator are produced by all the prime numbers not greater than $n$. All the mapping numbers in the numerator and denominator are also produced by all the prime numbers not greater than $n$ correspondingly.

Consider $p$'s composite numbers in the numerator and denominator of $C_{2n}^n$. Let $p$ be the maximal integer satisfying $p^{|p|} \left( \frac{p's\ composite\ numbers\ in\ (n+1)-2n}{p's\ composite\ numbers\ in\ 1-n} \right)$

Suppose $p$'s composite numbers in $1-2n$ are $m_1, m_2, \ldots, m_i, \ldots$. So, in the numerator and denominator of $C_{2n}^n$ are

$\frac{\left( \frac{1}{m_1} \times \frac{2}{m_2} \times \frac{3}{m_3} \times \ldots \times \frac{(n+1)}{m_i} \times \ldots \right)}{\left( \frac{1}{m_1} \times \frac{2}{m_2} \times \frac{3}{m_3} \times \ldots \times \frac{(n+1)}{m_i} \times \ldots \right)}$

Correspondently, $p$'s mapping numbers in the numerator and denominator of $C_{2n}^n$ are

$\frac{(2m - m_1) \times (2m - m_2) \times \ldots \times (2m - m_i) \times \ldots}{(2m - m_1) \times (2m - m_2) \times \ldots \times (2m - m_i) \times \ldots}$

Suppose $p$'s mapping transformation numbers are $M_1, M_2, \ldots$ to $p$'s mapping numbers $2m_1, 2m_2, \ldots$. According to the upper possible values of $p$'s multiples, we get

$\frac{2m - m_1 \times 2m - m_2 \times \ldots \times 2m - m_i \times \ldots}{(2m - m_1) \times (2m - m_2) \times \ldots \times (2m - m_i) \times \ldots}$

Let $S_p$ be the maximal integer satisfying $p^{|p|} \left( \frac{\left( \frac{1}{m_1} \times \frac{2}{m_2} \times \frac{3}{m_3} \times \ldots \times \frac{(n+1)}{m_i} \times \ldots \right)}{\left( \frac{1}{m_1} \times \frac{2}{m_2} \times \frac{3}{m_3} \times \ldots \times \frac{(n+1)}{m_i} \times \ldots \right)} \times M_1 \times M_2 \times \ldots \times M_i \times \ldots \right)$

Consider all the prime numbers less than or equal to $n$, we have

$\prod_{p \leq n} p^{|p|} \left( \frac{\left( \frac{1}{m_1} \times \frac{2}{m_2} \times \frac{3}{m_3} \times \ldots \times \frac{(n+1)}{m_i} \times \ldots \right)}{\left( \frac{1}{m_1} \times \frac{2}{m_2} \times \frac{3}{m_3} \times \ldots \times \frac{(n+1)}{m_i} \times \ldots \right)} \times M_1 \times M_2 \times \ldots \times M_i \times \ldots \right)$

Consider all the prime numbers less than or equal to $n$, we also have

$\frac{\left( \frac{1}{m_1} \times \frac{2}{m_2} \times \frac{3}{m_3} \times \ldots \times \frac{(n+1)}{m_i} \times \ldots \right)}{\left( \frac{1}{m_1} \times \frac{2}{m_2} \times \frac{3}{m_3} \times \ldots \times \frac{(n+1)}{m_i} \times \ldots \right)} \times M_1 \times M_2 \times \ldots \times M_i \times \ldots \right)$

Provided.
(B) ≤ \prod_{p \leq n} p^{x_p} \div \frac{n-1}{n}^{\frac{n}{n-1}}
= \prod_{\sqrt{2n} < p \leq n} p^{x_p} \times \prod_{p \leq \sqrt{2n}} p^{x_p} \div \frac{n-1}{n}^{\frac{n}{n-1}}
≤ \prod_{\sqrt{2n} < p \leq n} p \times \prod_{p \leq \sqrt{2n}} p^{x_p} \div \frac{n-1}{n}^{\frac{n}{n-1}}
= \prod_{p \leq n} p \times \prod_{p \leq \sqrt{2n}} p^{x_p-1} \div \frac{n-1}{n}^{\frac{n}{n-1}}

(According to theorem 2)

\[ \frac{\sqrt{2n}}{n} \text{-1 is the maximal number of prime numbers } \leq \sqrt{2n} \text{ when } \sqrt{2n} \geq 10 \text{ or } n \geq 50 \]

\[ \frac{\sqrt{2n}}{n} \times (n-1) \times (2n) \sqrt{2n-2} \]
\[ \frac{\sqrt{2n}}{n} \times n \times (2n) \sqrt{2n-2} \]
\[ = \frac{\sqrt{2n}}{n} \times (2n) \sqrt{2n-1} \]
Thus we get \( C_{2n}^n < \frac{\sqrt{2n}}{n} \times (2n) \sqrt{2n-1} \), \( n \geq 50 \).

Further, according to the lower bound of \( C_{2n}^n \) by theorem 1, we have

\[ \frac{\sqrt{2n}}{n} \leq C_{2n}^n < \frac{\sqrt{2n}}{n} \times (2n) \sqrt{2n-1}, n \geq 50 \]
\[ \frac{\sqrt{2n}}{n} \leq \frac{\sqrt{2n}}{n} \times (2n) \sqrt{2n-1}, n \geq 50 \]
Let \( x = \sqrt{2n} > 1 \), we get \( 2x^2 < x^{2x} \), so

\[ \frac{x^2}{2} < 2x \log_2 x \]
\[ x < 4 \log_2 x \]
\[ x < \log_2 x^4 \]
\[ 2^x < x^4, x > 1 \]

(C)

When \( n \geq 16 \), or when \( n \geq 128 \), the inequality \( 2^x < x^4 \) is not right. At last, an absurdity is induced when \( n \geq 258 \). Hence, as for even numbers greater than or equal to 2n+2=258, the supposition is not right. The induced benchmark event number 258 is a very small event number.

Contributions of the study include mainly two aspects.

(i) The mathematical modeling of the Goldbach conjecture expressed by table 4. Any even number greater than four can be expressed as 2n+2, and 2n+2=n+a+b, \( n \geq 4 \), \( 2 \leq n+1; n+1 \leq 2n \). a is in the denominator of \( C_{2n}^n \) when \( a+b=n+1 \), a and b are in the numerator of \( C_{2n}^n \) when \( b=n+1 \).

(ii) The proof of the Goldbach conjecture in section 5. Suppose the Goldbach conjecture is wrong. An inequation can be obtained \( 4^x < 4^x \times (2n) \sqrt{2n-1}, n \geq 50 \). This inequation is not true when \( n \geq 128 \). This means that the supposition “the Goldbach conjecture is wrong” does not stand when \( n \geq 128 \).

VI. CONCLUSION

We prove the Goldbach conjecture by an induction to absurdity. Suppose any even number e=2n+2 greater than or equal to 6 does not have a prime pair, but has only incomplete composite pairs or complete composite pairs. All the numbers in 2–2n can be then expressed by composite numbers or mapping numbers of a prime number less than or equal to n. Under such supposition, an inequality \( 2^x < x^4 \times (x = \sqrt{2n} > 1) \) is induced. The inequality \( 2^x < x^4 \) is not right when \( n \geq 128 \). Hence, as for even numbers greater than or equal to 2n+2=258,