An Analysis of Students' Reasoning Ability in the Differential Calculus Course

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Abstract. The objective of this research is to analyze the university students' reasoning abilities in the differential calculus course.The students' reasoning abilities should be developed in order to give them experience to solve problems which are often found during the lecture of the differential calculus course. It isa descriptive research which was conducted in semester 2 in Mathematics Education of State Islamic Institute (IAIN) of Tulungagung. The instruments usedwerereasoning problem test and interview. The respondents were6 university students consisting of 2 students with highabilities, 2 students with moderate abilities, and 2 students with low abilities. The research stages consist of planning, implementation, and reporting. The research results showedthe followings:(1) The respondents with high abilities tended to fulfill the reasoning process in the following indicators: making hypotheses, doing manipulation, giving reasons of the correctness of solutions, and drawing conclusions; (2) The respondents with moderate abilities merely met the reasoning processin the indicators of making hypotheses and doing the mathematical manipulation; (3) The respondents with low abilities just fulfilled the reasoning process under the indicator of making hypotheses.

Keywords: reasoning ability, differential calculus application

INTRODUCTION

Reasoning plays an important role in one's thinking process. To be able to solve mathematical problems, students should possess the following abilities: understanding the problem, planning the mathematical model, solving the mathematical model, and reviewing the result [1]. Calculus method is a method to solve problems in the daily life, and the differential calculusapplication will optimize the result. The best way will be obtained from the problem one encounters in the daily life. Therefore, students' reasoning ability plays a great role in solving problems in the differential calculus application.

From the results of observations and interviews obtained by the researcher as the lecturer teaching the differential calculus course, it showsthat students showed less abilities in solving differential application problems. In the learning process of the differential calculus course, the students did not understand the materials presented, and it took some sessions to explain the materials to the students well.Due to the fact, it is necessary to use alearning approach which is suitable with the students' characteristics to make the students' reasoning better.Based on the problem, the researcher intendsto understand the students' reasoning abilities: high, moderate and lowin the differential calculus course.

According to NCTM, the standard of school mathematics covers the material content and the obtaining process. The process standard includes problem solving, reasoning and proof, interrelatedness, communication, and representation. [2]. According to Rohana, reasoning is a process of drawing conclusion about a number of ideas based on the existing facts through logical and critical thinking in solving mathematical problems [3]. Moreover, Karin Brodie states that mathematical reasoning is reasoning about and with the object of mathematics[4]. According to the regulation of *Dirjen Dikdasmen Depdikna s*(General Directorate for Basic and Elementary Education, Department of National Education) No.506/C/PP/2004 regarding indicators of reasoning the students should reach, the indicators covermaking hypotheses, doing mathematical manipulation, giving reasons of some solutions, drawing a conclusion from a statement, reviewing the validity of an argument, and finding patterns of mathematical symptoms to make generalizations [5].From some of the opinions above, it can be concluded that reasoning is the conclusion drawing through the thinking process based on some statements that are known or considered to be correct.

Baroody defines deductive reasoning as an activity starting from premises leading to a conclusion [6]. Meanwhile, Lithner states that reasoning is a thought adopted to produce a statement and to reach a conclusion in a problem solving which is not always based on the formal logic so that it is not limited to the evidence [7]. Based on the descriptions above, it can be concluded that thinking process is anactivity to come to a conclusion or to make a new statement of which the truth has been proved.

Based on the descriptions of the reasoning above, the indicators to reveal the students' reasoningabilities in this present research are as follows: 1) making hypotheses; 2) doing manipulation; 3) giving reasons about the correctness of solutions; and 4) drawing conclusions.

METHOD

The method used is descriptive research [8], [9], a research that is intended to describe the students with high, moderate, and low reasoning abilities. The respondents of this present research are6 students

consisting of 2 students with highabilities, 2 students with moderate abilities, and 2 students with lowabilities. They studied in semester2 in the academic year of 2017/2017 inState Islamic Institute (IAIN) of Tulungagung.

The respondents with high reasoning abilities were given codes as S-1 and S-2, those with moderate reasoning abilities as S-3 and S-4, and those with low reasoning abilities as S-5 and S-6. The data were obtained through written test in the form of application problems intended to know the students' reasoning process in solvingdifferential calculus materials problems [10].Aninterview guide was used to interview the respondents after they did the reasoning tests.

The indicators of reasoning ability employed in this present research are presented in Table 1.
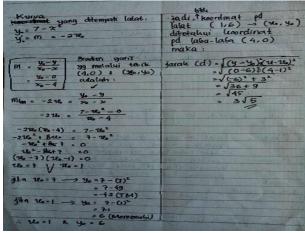
Table 1. Indicators of Reasoning Ability

No	Reasoning Indicator	Description
1	Making hypotheses	Mentioning what is to know and to ask.
2	Doing manipulation	Stating a problem into a mathematical model and determining the solving strategy.
3	Giving reasons about the correctness of solutions	Using a mathematical concept to solve the problem and explain in interrelation between the concept and what to ask.
4	Drawing conclusions	Finding the results from the problem solving strategies through interviews.

RESULTS

Reasoning Ability of Respondents S-1 and S-2

The answersfrom the respondent S-1 to the reasoning ability are presented in Picture 1. In the first indicator,it showsthat the respondent S-1 was able to understand the problem well so that he was able to fulfill the indicator of making a hypothesis by explaining what to know and what to ask. In the second indicator, the respondent S-1 was able to make a mathematical model and to determine the solving strategy. Inthe third indicator, the respondent was able to use a mathematical concept and explain the relationship among concepts.
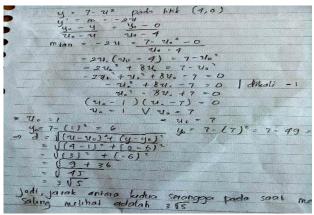

Picture 1

Based on the results of the reasoning test, the researcher interviewed the respondent S-1 to support the results of the test.The following is the transcript of the results of the interview:

R   Have you understood the intention of the problem?

S-1   Yes, I have.

R   What is known and asked in the problem ?

S-1   Curve $y = 7$ x^2 andpoint (4,0). What is asked is the distance between the two.

R   Why is this mathematical model used?

S-1   Because a derivation concept and a gradient through the two points are used.

R   What is the next step to solve it ?

S-1   First, the derivation of $y = 7 - x^2$ is found, the result of the derivation equals with the gradient through the two points, it is factorized and the value of x_0 is found.

R   What is the conclusion ofthe problem you have solved?

S-1   So, from the concepts of derivation and gradient, the distance between the two points can be found.

Based on the results of the analysis of the answers and interviews with the respondent S-1, it can be concluded that S-1 in doing the reasoning test had fulfilled all indicators of reasoning ability.

Picture 2

The answer of the respondent S-2 is shown in Picture 2. It showsthat the respondent S-2was able to fulfill some indicators but some others have no explanation. In the indicator of making the hypothesis, the respondent was able to explain what to know and what to ask, but theyhad not written it onthe answer sheet. The answer of the Respondent S-2 is presented in Picture 2 showingthe indicator of giving reasons about the correctness of solution. Here,the respondent did not give any reasons in using the concepts of derivation and distance through the two points. It is shown from the interview with the respondents S-2 below:
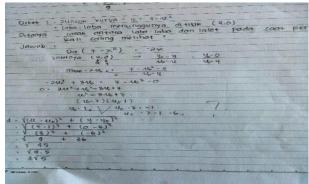
R   Is this problem difficult for you?
S-2   No, mam.
R   Try to explain it!
S-2   From this problem, it is known that the fly was crawling along the curve $y=7-x^2$and the spider was waiting for in the point (4,0).What is asked is the distance between the two insects when the two saw one another for the first time.
R   Then, how do you do it ?
S-2   The curve where the fly is located is at $y_o = 7 - x^2$, and the derivation of $y_o$was found first, then the gradient through the two points should also be found.
R   Why didn't you write the reason?
S-2   I forgot.

Respondents with high reasoning abilities in this research were able to solve the reasoning problems well.They made some mistakes, but they were still able to explain them. It is in line with Mansur's idea that students with high reasoning ability might not have problems in understanding lessons so that their learning achievement is good [11]. This finding is also supported by the results of Utami's research that students with high mathematical ability are able to give proofs which are in line with the reasoning indicators [12].

Reasoning Ability of the Respondents S-3 and S-4

Picture 3 shows the answer given by the respondent S-3. It showsthat the respondent was able to understandthe reasoning problem and two indicators: making hypotheses and doing manipulation. For the third indicator, the respondent was able to explain the

relationship among the concepts, so in finding y_o he was not able to substitute x_o into the known curve. The fourth indicator, namely drawing a conclusion, the respondent was able to do it correctly that the value of y_o is 6, but a misconception was still found in the process in the third indicator.
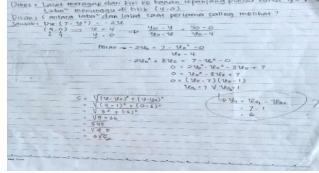

Picture 3

It can be identified from the interview with the respondent S-3 as follows:
R   :   What is the step to do this?
S-3   :   First, the concepts of derivation and the gradient through two points were used, then the point x_o was found. But I was confused how to find the y_o.

Based on the interview with respondent S-3, it is known that the end result iscorrect, but a misconception still occurred.

Picture 4

From the Picture 4, it is known that the answer given by therespondent S-4 isalmost the same asthose given by the respondent S-3, in whichthey were merely able to understand two reasoning indicators. For the third indicator, the process of finding y_o using the concept of x_{o1} minus x_{o2} iscorrect since the value of y_o is 6, but the process is incorrect. Based on the results of the answer and the interview, the respondent S-4 did not understand how to use the mathematical concept in finding the value of y_o. It is in line with the interview with the respondent S-4 below:
R   :   How is the process to do it?
S4   :   By determining the solving strategy using the derivation and the gradient through the two points, but I was confused how to find the y_o.

The research results from the respondents with moderate ability show that they were able to do two indicators,but they still made a mistake in determining the concept in finding the y_o. It is in line with the Retno's research result that students with moderate ability, when they had not been able to understand problems, they would repeat reading the problem and try to understand the problem again [13].

Reasoning Ability of the Respondents S-5 and S-6
Picture 5 presents the descriptions of the answers given by the respondent S-5 who wasable to write what to know and what to ask in the problem. It means that he was able to fulfill the indicator of making hypotheses. For further indicators, the respondent S-5 was not able to make the mathematical model and to determine the solving strategy. The respondent S-5 employed the concept of the derivation definition, but he was not able to give reasons for the correctness of the solution. Since the solution process was incorrect, the conclusion was also wrong.
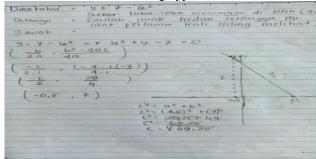

Picture 5

It is shown in the result of the following interview:

R   What is the step to do this?

S-5   First, I understood the problem and then determined what to know and what to ask. Then, I stated the problem into a mathematical model, and then I determined the strategy I would use. Since what is known was the peak of the curve and one point below it, I used a derivationdefinition. Firstly,it was correct, but the next step I did was wrong.

Based on the results of the answer and the interview, it can be concluded that the respondent S-5 merely fulfilled one indicator: making hypotheses.

Picture 6

The answers given by the respondents S-6 and S-5 merely fulfilled one indicator: making hypotheses, as presented in Picture 6. But in the next step, the respondents were not able to state the problem into a mathematical model to use the concept of the turning point to find x_o and y_o. Based on the result of the interview, the respondent S-6 wasdifficult to understandthe reasoning problem, so he was not able to state the mathematical concept and to determine the correct mathematical concept to do the problem. It can be known from t.

The results of the interview with the respondent S-6 are as follows:

R   : What is your opinion about the problem?

S-6   : It is very difficult Mam, I did not understand it.

R   : Try to read it again and understand it!

S-609 : I didn't understand it.

From the interview with the respondentswith low ability, it showsthat they were very difficult to understand the problem so that they were merely able to fulfill one indicator: making hypotheses. It is in line with Fahmi's research result that the weakness possessed by the students is among others that they had less ability in understanding and in being familiar with the mathematical basic concepts [14].

CONCLUSION

The followings are the results of the findings in the analysis of the students with high, moderate, and low reasoning abilities. First, the respondents with high reasoning abilities tendto fulfill the reasoning process in the following indicators: making hypotheses, doing manipulation, giving reasons about the correctness of solutions,and drawing conclusions.Second, the respondents with moderate reasoning abilities merely fulfillthe reasoning process in the indicators of making hypotheses and doing mathematical manipulation. Third, the respondents with low abilities merely fulfillthe reasoning process in the indicator of making hypotheses.It is recommended that to develop students' reasoning abilities, applicationproblems and extra-time for lecturing should be given to give students more experiences in solving differential applications problems.

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