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Determination of parameters and stability zones of pendulum auto-balancer of rotor, installed in housing on elastic supports

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***Abstract*—Using the methods of the small parameter of nonlinear mechanics, the formulas for determining the parameters of the pendulum auto-balancer were obtained in the article. Conditions for the stability of the auto-balancing process of the unbalanced rotor, fastened in the housing on the elastic supports, were also found. It is established that the stable operation of the auto-balancer can be implemented in the angular velocity ranges of the rotor due to the choice of the stiffness characteristics of the elastic supports.**

Keywords—Housing, Rotor, Auto-balancer, Pendulum, Stability, Elastic supports)

I. INTRODUCTION

The main reason for increasing the dynamic loading and increasing the vibration level of machines and mechanisms with rotating rotors is a change in the imbalance of the rotors during operation. The change in the imbalance is due to wear on the bearings and rotor assemblies, weakening of joints and fastenings, deformation of the rotor elements, adhesion of the processed material and other reasons. This operational change in the imbalance of the rotor is usually at a low speed and is random. In this connection, it appears expedient to use automatic balancing devices that compensate for the change in the imbalance of the rotor in operating conditions without interrupting the technological process and working according to the principles of self-centering.

The most complete description of automatic balancing devices, their advantages and disadvantages are given in the monograph by A.A. Gusarov [1]. Of great importance for the development of the theory of automatic balancing of rotors are the works of I.I. Blekhman [2], [3]. These works lay down the methodological bases for calculating automatic balancing devices and formulate a generalized principle of auto-balancing, based on the method of small parameter and the method of separation of motion of nonlinear mechanics. Methods for calculating autobalancing devices of rotor systems with a large number of degrees of freedom were developed in the works of V.P. Nesterenko [4] and A.I. Artyunin [5] on the basis of these representations.

A significant contribution to the solution of scientific and practical problems of automatic balancing of rotors was made by G.B. Filimonikhin [6]. Various problems of the dynamics of rotors with autobalancers have been studied in [7-15]. The current state of the problem of balancing rotors is presented quite fully in the article of A.N. Nikiforov [16]. In connection with the chosen direction of research in this article, one should mention the works [17], [18], related to the study of the auto-balancing process of rotors installed in the housing.

II. DESCRIPTION OF THE MOVEMENT OF THE DYNAMIC ROTOR MODEL IN THE HOUSING WITH PENDULUMS MOUNTED ON THE ROTOR FOR AUTO-BALANCING

For the study, let us select and use the dynamic model of the rotor system with an auto-balancer shown in Fig. 1. This model is a massive housing, fastened on a fixed base with weightless elastic supports. In the housing a rigid rotor is installed in its own bearings and rotates with an angular velocity ω. Two pendulums of the same mass m and length l are suspended on the rotor shaft in pairs on both sides of the rotor to compensate for the dynamic imbalance from possible free rotation.

The motion of the model will be considered with respect to the fixed coordinate system *Oxyz*. Since in most real constructions there is practically no motion of the housing along the axis of the rotor, then, to describe the motion of the rotor with the housing, we choose the following generalized coordinates: y, z are linear displacements of the point O_1 (O_1 is the point of intersection of the rotor axis with the plane passing through its center of mass perpendicularly axes of rotation, φ_x, θ, ψ are the angular movements of the housing along with the rotor around axes x_1, y_1, z_1. Axes x_1, y_1, z_1 are parallel to axes x, y, z at the initial moment of time. The positions of the pendulums are determined by angles φ_1, φ_2, φ_3, φ_4. The positive directions for reading these angles are shown in Fig. 1.

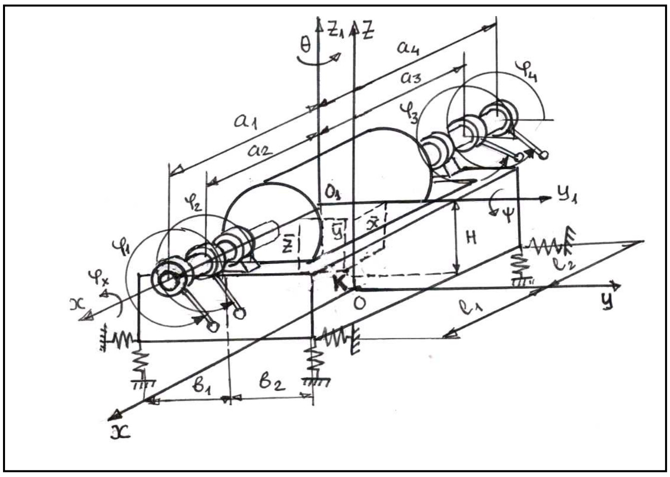

Fig. 1. A dynamic model of the rotor with pendulum auto-balancers, installed in the housing on elastic supports.

We accept some assumptions that negatively affect the final result. First, we will not take into account the damping due to its little influence on the choice of the parameters of the auto-balancer and on the position of the stability zones. The resistance to rotation of the pendulums will be proportional to their relative rotational velocities. Secondly, suppose that the main axes of inertia of the housing are parallel to the axes x_1, y_1, z_1. Third, we assume that the housing supports are isotropic, and the distance between the pendulum suspension points in each pair is small, compared to the distance from the center of the rotor masses to the pendulum suspension points. We take the following notation: M_p, A, C are mass, equatorial and polar moments of inertia of the rotor; M_k, I_x, I_y, I_z are mass and moments of inertia of the housing; e, δ, ε are parameters of the rotor imbalance: $c_{1y} = c_{2y} = c_{3y} = c_{4y} = c_y$; $c_{1z} = c_{2z} = c_{3z} = c_{4z} = c_z$ are stiffness coefficients of 4 horizontal and 4 vertical supports of the housing; $\bar{x}, \bar{y}, \bar{z}$ are constant coordinates of the center of mass of the hull in reference system $x_1y_1z_1$; $a_1 = a_2 = a_3 = a_4 = a$ are distances from the center of mass of the rotor to the pendulum suspension points; β_O is the coefficient of resistance to rotation of the pendulums. We proceed further from the assumption that the linear and angular displacements of the housing with the rotor are small in the sense that linear differential equations are sufficient for describing their motion without pendulums. Then the equations of motion of the model under study are divided into equations for the rotor with the housing:

$$[M]\{\ddot{q}\} + [G]\{\dot{q}\} + [K]\{q\} = \{F\} \tag{1}$$

and equations for the auto-balancer pendulums:

$$\ddot{\varphi}_k + \beta_O(\dot{\varphi}_k - \omega) = [\ddot{y}\sin\varphi_k - \ddot{z}\cos\varphi_k + \alpha\sigma_k(\ddot{\theta}\sin\varphi_k - \ddot{\psi}\cos\varphi_k) - g\cos\varphi_k]/l, \tag{2}$$

where $\{q\} = \{y, z, \theta, \psi, \varphi_x\}^T$; ; $k = \overline{1,4}$;

$$[M] = \begin{vmatrix} M^* & 0 & m_1 & 0 & m_2 \\ 0 & M^* & 0 & m_1 & m_3 \\ m_1 & 0 & I_z^* & m_4 & m_5 \\ 0 & m_1 & m_4 & I_y^* & m_6 \\ m_2 & m_3 & m_5 & m_6 & I_x^* \end{vmatrix};$$

$$[G] = \begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C\omega & 0 \\ 0 & 0 & -C\omega & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{vmatrix};$$

$$[K] = \begin{vmatrix} c_1 & 0 & c_2 & 0 & c_1H \\ 0 & c_3 & 0 & c_{14} & c_5 \\ c_2 & 0 & c_6 & 0 & c_2H \\ 0 & c_4 & 0 & c_7 & c_8 \\ c_1H & c_5 & c_2H & c_8 & c_9 \end{vmatrix}.$$

When compiling the matrices [M], [K] the following notation was used:

$$M^* = M_p + M_k + 4m;$$

$$I_z^* = A + M_k(\bar{x}^2 + \bar{y}^2) + I_z + m\sum_{k=1}^{4} a_k^2;$$

$$I_y^* = A + M_k(\bar{x}^2 + \bar{z}^2) + I_y + m\sum_{k=1}^{4} a_k^2;$$

$$I_x^* = C + M_k(\bar{y}^2 + \bar{z}^2) + I_x;$$

$$m_1 = \bar{x}M_k; \quad m_2 = -\bar{z}M_k; \quad m_3 = \bar{y}M_k; \quad m_4 = \overline{yz}M_k;$$

$$m_5 = -\bar{x}\cdot\bar{z}M_k; \; m_6 = \bar{x}\cdot\bar{y}M_k;$$

$$c_1 = 4c_y; \quad c_2 = 2c_y(l_1 - l_2); \quad c_3 = 4c_z; \quad c_4 = 2c_z(l_1 - l_2);$$

$$c_5 = 2c_z(b_2 - b_1); \; c_6 = 2c_y(l_1^2 + l_2^2);$$

$$c_7 = 2c_z(l_1^2 + l_2^2); \; c_8 = c_z(l_1b_2 - l_2b_2 + l_2b_1 - l_1b_1);$$

$$c_9 = 4c_yH^2 + 2c_z(b_1^2 + b_2^2).$$

The vector of disturbing forces from the imbalance and pendulums $\{F\}$ with the selected generalized coordinates is:

$$\{F\}=\begin{Bmatrix} M_p e\omega^2\cos\omega t+ml\sum_{k=1}^{4}(\ddot{\varphi}_k\sin\varphi_k+\dot{\varphi}_k^2\cos\varphi_k);\\ M_p e\omega^2\sin\omega t+ml\sum_{k=1}^{4}(-\ddot{\varphi}_k\cos\varphi_k+\dot{\varphi}_k^2\sin\varphi_k);\\ (A-C)\delta\omega^2\cos(\omega t-\varepsilon)+\\ +mla\sum_{k=1}^{4}\sigma_k(\ddot{\varphi}_k\sin\varphi_k+\dot{\varphi}_k^2\cos\varphi_k);\\ (A-C)\delta\omega^2\sin(\omega t-\varepsilon)+\\ mla\sum_{k=1}^{4}\sigma_k(-\ddot{\varphi}_k\cos\varphi_k+\dot{\varphi}_k^2\sin\varphi_k);\\ 0 \end{Bmatrix}$$

where $\sigma_k = 1$ for $k = 1,2$ and $\sigma_k = -1$ or $k = 3,4$.

III. DERIVATION OF FORMULAS FOR SELECTING THE PARAMETERS OF THE PENDULUM AUTO-BALANCER

Proceeding from the fact that the forces resistant to the motion of the pendulums are small and their movement is close to the uniform rotational motion, we introduce small parameter μ in the equations (2):

$$\ddot{\varphi}_k=\mu\phi_k(\varphi_k,\dot{\varphi}_k,\ddot{y},\ddot{z},\ddot{\theta},\ddot{\psi}),\ (k=\overline{1,4}), \tag{3}$$

where

$$\phi_k(\varphi_k,\dot{\varphi}_k,\ddot{y},\ddot{z},\ddot{\theta},\ddot{\psi})=[\ddot{y}\sin\varphi_k-\ddot{z}\cos\varphi_k+ +a\sigma_k(\ddot{\theta}\sin\varphi_k-\ddot{\psi}\cos\varphi_k)-g\cos\varphi_k]/l-\beta_O(\dot{\varphi}_k-\omega).$$

Assuming that the change in the generalized coordinates of the rotor and the machine is periodic, and the motion of the pendulums, as mentioned above, is close to the uniform rotational motion, we seek the solution of equations (1), (2) in the form:

$$y=y(\omega t);z=z(\omega t);\varphi_x=\varphi_x(\omega t);\theta=\theta(\omega t);\psi=\psi(\omega t);$$
$$\varphi_k=\omega t+\alpha_k+\mu\phi_k^*(\omega t),\ k=\overline{1,4} \tag{4}$$

Here, $q(\omega t)$ is periodic functions of time t with period $2\pi/\omega$, and α_k is unknown constants. In accordance with the method of a small parameter, supplying φ_k from (4) for $\mu = 0$ in (1), we obtain the generating system of equations:

$$[M]\{\ddot{q}_o\}+[G]\{\dot{q}_o\}+[K]\{q_o\}= =\{F_c\}\cos\omega t+\{F_s\}\sin\omega t. \tag{5}$$

Here $\{q_0\} = \{y_0, z_0, \varphi_{x0}, \theta_0, \psi_0\}^T$;

$$\{F_c\}=\{D,N,S,W,0\}^T;$$
$$\{F_s\}=\{-N,D,-W,S,0\}^T;$$
$$D=Me\omega^2+ml\omega^2(\cos\alpha_1+\cos\alpha_2+\cos\alpha_3+\cos\alpha_4);$$
$$N=ml\omega^2(\sin\alpha_1+\sin\alpha_2+\sin\alpha_3+\sin\alpha_4);$$
$$S=(A-C)\omega^2\delta\cos\varepsilon+ +mla\omega^2(\cos\alpha_1+\cos\alpha_2-\cos\alpha_3-\cos\alpha_4);$$
$$N=-(A-C)\omega^2\delta\sin\varepsilon+ +ml\omega^2(\sin\alpha_1+\sin\alpha_2-\sin\alpha_3-\sin\alpha_4) \tag{6}$$

It is seen that the equilibration (5) will take place if $D = N = S = W = 0$. Then equating the right-hand sides of (6) to zero and introducing the notation

$$\eta=\frac{Me}{ml};\quad \xi=\frac{(A-C)\delta\cos\varepsilon}{ml};\quad \zeta=\frac{(A-C)\delta\sin\varepsilon}{ml},$$

we obtain the following system of equations for determining the unknown constants α_k:

$$\cos\alpha_1+\cos\alpha_2+\cos\alpha_3+\cos\alpha_4=-\eta;$$
$$\sin\alpha_1+\sin\alpha_2+\sin\alpha_3+\sin\alpha_4=0;$$
$$\cos\alpha_1+\cos\alpha_2-\cos\alpha_3-\cos\alpha_4=-\xi/a;$$
$$\sin\alpha_1+\sin\alpha_2-\sin\alpha_3-\sin\alpha_4=\zeta/a\ . \tag{7}$$

Among the set of solutions of trigonometric equations (7) the following will be the main ones:

$$\alpha_1=\pi-arctg\left[\frac{\zeta}{\eta a+\xi}\right]-\arccos\left[\frac{1}{4a}\sqrt{\zeta^2+(\eta a+\xi)^2}\right];$$
$$\alpha_2=-\pi-arctg\left[\frac{\zeta}{\eta a+\xi}\right]+\arccos\left[\frac{1}{4a}\sqrt{\zeta^2+(\eta a+\xi)^2}\right];$$
$$\alpha_3=\pi+arctg\left[\frac{\zeta}{\eta a-\xi}\right]-\arccos\left[\frac{1}{4a}\sqrt{\zeta^2+(\eta a-\xi)^2}\right];$$
$$\alpha_4=-\pi+arctg\left[\frac{\zeta}{\eta a-\xi}\right]+\arccos\left[\frac{1}{4a}\sqrt{\zeta^2+(\eta a-\xi)^2}\right]. \tag{8}$$

From (8), we can obtain conditions for choosing the parameters of the auto- balancer, starting from the fact that the cosine of any angle can only be less than or equal to one.

$$[(A-C)\delta\sin\varepsilon]^2+[M_p ea+(A-C)\delta\cos\varepsilon]^2\le(4mla)^2;$$

$$[(A-C)\delta\sin\varepsilon]^2+[M_p ea-(A-C)\delta\cos\varepsilon]^2\le(4mla)^2\cdot \quad (9)$$

IV. DETERMINATION OF THE STABILITY CONDITIONS OF THE ROTOR AUTO-BALANCER INSTALLED IN THE CASING

Finding periodic solutions of the generating equations in the form:

$$q_{oi}=a_i\cos\omega t+b_i\sin\omega t,\ i=\overline{1,5}\ , \quad (10)$$

we arrive at a system of ten linear algebraic equations with respect to a_i, b_i. In the matrix form, these equations are written as follows:

$$[A]\{q^*\}=\{F_o\},$$

where $\{q^*\}=\{a_1,b_1,...,a_5,b_5\}^T$;

$\{F_o\}=\{D,-N,N,D,S,-W,W,S,0,0\}^T$.

The non-zero elements of matrix A have the form:

$a_{11}=a_{22}=c_1-\omega^2M^*$; $\quad a_{33}=a_{44}=c_3-\omega^2M^*$;

$a_{55}=a_{66}=c_6-\omega^2I_z^*$; $\quad a_{77}=a_{88}=c_7-\omega^2I_y^*$;

$a_{99}=a_{10,10}=c_9-\omega^2I_x^*$;

$a_{15}=a_{26}=a_{51}=a_{62}=c_2-\omega^2m_1$;

$a_{58}=a_{85}=C\omega^2$; $a_{67}=a_{76}=-C\omega^2$;

$a_{37}=a_{48}=a_{73}=a_{84}=c_4-\omega^2m_1$;

$a_{19}=a_{2,10}=a_{91}=a_{10,2}=c_1H-\omega^2m_2$;

$a_{39}=a_{4,10}=a_{93}=a_{10,4}=c_5-\omega^2m_3$;

$a_{57}=a_{68}=a_{75}=a_{86}=-\omega^2m_4$;

$a_{59}=a_{6,10}=a_{95}=a_{10,6}=c_2H-\omega^2m_5$;

$a_{79}=a_{8,10}=a_{97}=a_{10,8}=c_8-\omega^2m_6$.

For sufficiently small μ, periodic solutions of the generating system of equations depending on constants α_k only correspond to asymptotically stable periodic solutions of the original equations if for constants α_k, that satisfy the equations:

$$P_s(\alpha_1,...,\alpha_k)=0,\ s=\overline{1,k} \quad (11)$$

the condition is fulfilled, which consists in the requirement that the real parts of all roots of the algebraic equation are to be negative:

$$\left|\frac{\partial P_s}{\partial\alpha_j}-\sigma_{sj}\lambda\right|=0, \quad (12)$$

$s,j=\overline{1,k}$.

Here $P_s(\alpha_1, \alpha_2, ..., \alpha_k)$ are generating functions, and σ_{sj} is the Kronecker symbol. Generating functions $P_s(\alpha_1, \alpha_2, ..., \alpha_k)$ are found after substituting q_{oi} and $\varphi_{ko}=\omega t+\alpha_k$ into the right-hand sides of (3) and averaging the obtained expressions:

$$P_s(\alpha_1,...,\alpha_k)=\frac{\omega}{2\pi}\int_0^{\frac{2\pi}{\omega}}\phi_k(\varphi_{ko},\ddot{y}_o,\ddot{z}_o,\ddot{\theta}_o,\ddot{\psi}_o)dt.$$

In our case:

$$P_s(\alpha_1,\alpha_2,\alpha_3,\alpha_4)=-\frac{\omega^2}{2l}[(\tilde{A}D+\tilde{C}N+\tilde{E}S+\tilde{G}W)\sin\alpha_k+$$
$$+(\tilde{B}D+\tilde{K}N+\tilde{F}S+\tilde{H}W)\cos\alpha_k],$$

where

$\tilde{A}=a_{11}+a_{14}+a_{41}+a_{44}+a(a_{51}+a_{54}+a_{81}+a_{84})$;

$\tilde{B}=a_{21}+a_{24}-a_{31}-a_{34}+a(a_{61}+a_{64}-a_{71}-a_{74})$;

$\tilde{C}=-a_{12}+a_{13}-a_{42}+a_{43}+a(-a_{52}+a_{53}-a_{82}+a_{83})$;

$\tilde{K}=-a_{22}+a_{23}+a_{32}-a_{33}+a(-a_{62}+a_{63}-a_{72}+a_{73})$;

$\tilde{E}=a_{15}+a_{18}+a_{45}+a_{48}+a(a_{55}+a_{58}+a_{85}+a_{88})$;

$\tilde{F}=a_{25}+a_{28}-a_{35}-a_{38}+a(a_{65}+a_{68}-a_{75}-a_{78})$;

$\tilde{G}=-a_{16}+a_{17}-a_{46}+a_{47}+a(-a_{56}+a_{57}-a_{86}+a_{87})$;

$\tilde{H}=-a_{26}+a_{27}+a_{36}-a_{37}+a(-a_{66}+a_{67}+a_{76}-a_{77})$.

In these formulas, a_{jk} is the elements of matrix $[A]^{-1}$, and D, N, S, W are determined from (6). It is not difficult to verify that the α_k found by (8) satisfy the equations (11). The expressions obtained in the analytic form for the generating functions make it possible to find partial derivatives $\partial P_k/\partial\alpha_i$, that is, the elements of the determinant (12):

$$\frac{\partial P_k}{\partial\alpha_i}=\frac{m\omega^2}{2}[(\tilde{A}+a\sigma_k\tilde{E})\sin\alpha_k\sin\alpha_i-$$
$$-(\tilde{C}+a\sigma_k\tilde{G})\sin\alpha_k\cos\alpha_i+(\tilde{B}+a\sigma_k\tilde{F})\cos\alpha_k\sin\alpha_i- \quad (13)$$
$$-(\tilde{K}+a\sigma_k\tilde{H})\cos\alpha_k\cos\alpha_i].$$

Expanding the determinant (12), we obtain an algebraic equation of the fourth order with respect to λ. It is advisable to verify the negativity of the real parts of the roots of the equation by means of the Routh-Hurwitz criterion. Carrying out such check within the chosen range of rotor angular velocities, it is possible to determine the zones of stable operation of the auto-balancing device.

V. CONCLUSIONS AND RESULTS OF CALCULATION

The calculation of the stability zones was carried out with the initial data corresponding to the data of the experimental stand: $M_p = 11{,}3$ *kg*; $M_k = 57{,}3$ *kg*; $A = 0{,}285$ *kgm*^{2}; $C = 0{,}007$ *kgm*^{2}; $I_x = 2{,}287$ *kgm*^{2}; $I_y = 6{,}33$ *kgm*^{2}; $I_z = 8{,}353$ *kgm*^{2}; $m = 0{,}05$ *m*; $c_y = c_z = 1{,}85 \cdot 10^4$ *H/m*; $e = 0{,}1$ *mm*; $\delta = 0{,}0005$ *rad*; $\varepsilon = 0$; $l = 0{,}08$ *m*; $b_1 = b_2 = 0{,}37$ *m*; $l_1 = 0{,}54$ *m*; $l_2 = 0{,}36$ *m*; $H = 0{,}2$ *m;* $\bar{x} = 0{,}2$ *m*; $\bar{y} = 0$*;* $\bar{z} = -0{,}1$ *m*; $a_1 = a_4 = 0{,}28$ *m*; $a_2 = a_3 = 0{,}25$ *m*, ($a = 0{,}265$ *m*;). Based on the results of the calculation, the resonant frequencies of the selected dynamic model are found: $\omega_1 = 30{,}2$ *rad/s*; $\omega_2 = 32{,}5$ *rad/s*; $\omega_3 = 41{,}8$ *rad/s*; $\omega_4 = 58{,}8$ *rad/s*; $\omega_5 = 59{,}4$ *rad/s*.

The analysis of frequencies and vibration modes showed that linear oscillations in horizontal and vertical directions predominate at frequencies ω_1 and ω_2. At frequency $\omega_3 = 41.8$ *rad/s*, angular oscillations around the *z* axis occur, at a frequency $\omega_4 = 58.8$ *rad/s*, oscillations about the *x* axis, and at $\omega_5 = 59.4$ *rad/s* – around the *y* axis. Fig. 2 shows the zones of stability of the auto-balancer, which are shaded. It can be seen that the stable operation of the auto-balancer is possible in two bands at $41.8 < \Omega < 49.4$ *rad/s* and at $\Omega > 59.4$ *rad/s*. And the last zone for this dynamic model is not bound from above. The calculations also showed that there is one more stability zone ($58.8 < \Omega < 59.1$ *rad/s*), but this zone is so small that it is difficult to depict it in the figure and it has no practical value.

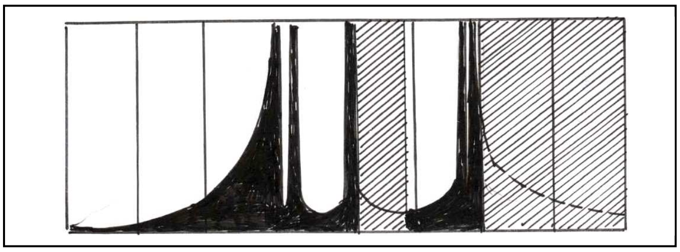

Fig. 2. Resonant zones of the rotor in the housing on elastic supports and zones of stable operation of auto-balancers (shaded).

The results of calculating the auto-balancer for the dynamic rotor model installed in the housing (Figure 1) show that the stable operation of auto-balancing devices can be achieved by the shock absorption of the housing, which does not contradict the generalized principle of auto-balancing [3].
Another important conclusion that can be drawn from the results of the research is that, by introducing the shock absorption of the housing, it is possible to increase the number of zones for the stable operation of the auto-balancer.

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