Mathematical model of potato slices under process-based temperature and humidity integration control of tilted tray air impingement drying

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Abstract. Mathematical model of potato slices under process-based temperature and humidity integration control of tilted tray air impingement drying were investigated under different parameters. Results indicated that the Weibull distribution function simulated the drying curves of potato slices under PDTHIC drying well. The coefficient of determination ranged from 0.989 to 0.999, the root mean square error was in the range of 1.04×10^{-2} to 2.57×10^{-2}, and the derivate square ranged from 1.25×10^{-4} to 7.15×10^{-4}. The scale parameter α is related to both the humidity control parameters during the earlier stage and the medium stage. The shape parameter β decreased in the range of 0.811~0.964 with the increase of constant humidity, whereas fluctuated in the range of 0.836~0.987 and had no regulations with the changing of the values of humidity control time. The estimated moisture effective diffusivity of potato slices under different drying conditions calculated by scale parameter α ranged from $1.664\times10^{-9}m^2\cdot s^{-1}$ to $2.371\times10^{-9}m^2\cdot s^{-1}$, and the values of geometric parameters R_g was calculated in a fluctuant way in the range of 2.575~5.351.

Introduction

The drying model is a mathematical abstraction for describing the drying process and further revealing the regulations of drying parameters during the dehydration of material. The setup of drying model can provide the technical basis for describing, predicting, analyzing, controlling and optimizing of material drying process. Recently, some empirical and semi-empirical models have been already widely applied to the expression of drying process, such as Lewis, Page, Modified Page, Wang and Singh, Logarithmic, Two-order experimental model and Two-order exponential model, etc. Although these models can extremely simulate the curves of drying kinetics with a high accuracy, the practical guiding significance of parameters in the models are still indefinite, and both the materials properties and drying conditions are difficult to relate to the parameters of these models used. Hence, the whole process is still considered as a black box. These drying models can't be closely related to the drying technology, drying efficiency and heat transfer process, greatly limiting the application value and research significance.

Recently the Weibull model has been widely applied to the field of biological sterilization, pharmacology, thermodynamics and food engineering due to its application and compatibility. Rong-Rong Lu et al.[1] published spore death curves under ultra-high pressure sterilization condition using the Weibull model. Boekel[2] studied the Weibull model to describe the thermal inactivation characteristics of microbial cells. Recent years, some researchers have used the Weibull model for analyzing drying characteristics of agricultural products and obtained research progress. Corzo et al.[3] studied the Weibull distribution for fitting curves of hot air drying of mango slices. Bai et al.[4] used the Weibull distribution function to describe the drying kinetic characteristics of grapes. But as a whole, the relative reports about process-based drying temperature and humidity integration control air impingement model of high-starch materials have not been reported yet.

In this paper, the objectives of current work were to focus on microwave intermittent drying kinetics of banana slices. Then the quality of fitting was determined according to the statistical

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parameters such as the correlation coefficient (R^2), root mean square error (RMSE) and derivate square (χ^2). And the final objectives was to illustrate the physical significance of the parameters in drying process, to provide a kind of operational tool for predicting, controlling, analyzing high-starch materials in optimization of drying technology.

Materials and methods

Raw material

Fresh potatoes (Yanshu NO.4） were purchased from an agricultural wholesale market in Beijing. Potatoes of equal size were selected as raw materials. The initial moisture content of the potato samples was 3.9 g/g in dry basis. The prepared samples were wrapped with a plastic film and stored in a refrigerator at 5±1°C prior to the experiments.

Experimental set-up and procedure

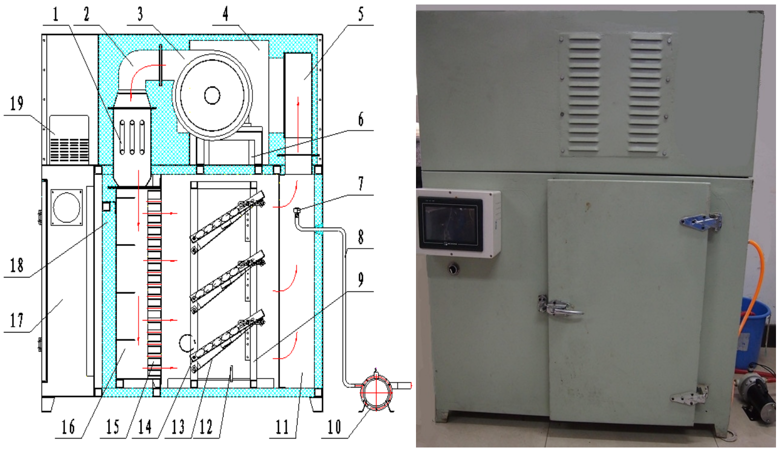

Fig. 1 Schematic diagram for process-based temperature and humidity integration control of tilted tray air impingement dryer

The fresh potatoes were washed to remove the dust, and then wiped with gauze to eliminate excess water on its surface. The samples were cut into slices with different thickness using a kitchen cutter. Each sample was located so as not to touch the adjacent ones. Weigh the samples on an electronic balance at 1 h intervals during drying. Drying process was continued until the potato slices reached the desired final moisture content of 14% (w.b.) or 0.16 kg/kg (d.b.). The temperature and velocity of airflow were kept at 60°C and 15 m/s, respectively. In addition, the samples were pretreated with hot water blanching under 90°C for 60s so as to keep partial enzyme activity, with the subsequent objective of actually finding the effects of the PDTHIC technology on color protection. The relative humidity of surroundings was measured with 17%RH. All experiments were performed in triplicate as shown in Table 1.

Table 1 Design for experiments with drying parameters included

NO.	Drying at earlier stage		Drying at medium stage		Drying at later stage	
	Humidity/%RH	Time/h	Humidity/%RH	Time/h	Humidity/%RH	Time/h
1	10(continuous dehumidification)	---	---	---	---	---
2	20	---	---	---	---	---
3	30	---	---	---	---	---
4	40	---	---	---	---	---
5	50	---	---	---	---	---
6	50	1	10	---	---	---
7	50	2	10	---	---	---
8	50	3	10	---	---	---

9	50	4	10	---	---	---
10	50	5	10	---	---	---
11	20	2	10	---	---	---
12	30	2	10	---	---	---
13	40	2	10	---	---	---
14	50	2	30	1	10	---
15	50	2	30	2	10	---
16	50	2	30	3	10	---
17	50	2	30	4	10	---

Calculation of moisture effective diffusivity Mathematical modeling of drying curves

Drying curves of potato slices were fitted with Weibull model (Eq. (1))[5].

$$MR = \exp\left[-\left(\frac{t}{b}\right)^{a}\right]. \tag{1}$$

Where t is the drying time, α is the shape parameter of the Weibull model, and β is the scale parameter of the model.

The moisture ratio (MR) of potato slices during drying experiments was calculated with a more simplified form as follows[6]:

$$MR = \frac{M_t}{M_o}. \tag{2}$$

Where M_0 is the initial moisture content of potato slices, M_t is the moisture content at time t.

The drying rate of sample slices during drying experiments was computed using Eq. (3)[7]:

$$DR = \frac{M_{t_1} - M_{t_2}}{t_2 - t_1}. \tag{3}$$

Where t_1 and t_2 is the drying time at different moment respectively during drying with expression in hours; M_{t1} and M_{t2} is the moisture content of samples at t_1 and t_2, $g \cdot g^{-1}$.

The natural logarithm form of moisture effective diffusivity was determined as shown in Eq. (4)[8]:

$$\ln(MR) = \ln\left(\frac{6}{p^2}\right) - \left(p^2 \frac{D_{eff} t}{r_o^2}\right). \tag{4}$$

There is a linear relationship between the natural logarithm of moisture ratio and drying time of sample slices according to Eq. (4). However, this equation does not apply to the products with acceleration sections in the drying process. Hence, it is significant for Weibull model to evaluate the effective moisture diffusivity without considering the moisture movement properties. In this model, D_{eff} can be determined as shown in Eq. (5)[9]:

$$D_{eff} = \frac{D_{cal}}{R_g} = \frac{r^2}{\beta R_g}. \tag{5}$$

Where D_{cal} is the estimate moisture diffusivity ($m^2 \cdot s^{-1}$); r is the volume equivalent radius of potato samples, with 1.59×10^{-2} m as its value; β is the scale parameter of Weibull model; R_g is the physical dimension constant with 18.6 $m^2 \cdot s^{-1}$ for sphere material[10].

Results and discussion

Mathematical modeling of drying kinetics with Weibull distribution function

Table 2 Design for experiments with drying parameters included

Drying method	Humidity parameter	Scale parameter β/min	Shape parameter α	coefficient of determinati R^2	root mean square error RMSE	derivate square χ^2
Different constant humidity	10%RH constant humidity	175.722	0.884	0.996	1.69×10^{-2}	3.28×10^{-4}
	20%RH constant humidity	210.528	0.964	0.997	1.59×10^{-2}	2.86×10^{-4}
	30%RH constant humidity	197.040	0.942	0.999	1.04×10^{-2}	1.25×10^{-4}
	40%RH constant humidity	219.882	0.875	0.996	1.67×10^{-2}	3.14×10^{-4}
	50%RH constant humidity	240.186	0.811	0.989	2.57×10^{-2}	7.15×10^{-4}
Different humidity control time at initial stage	10%RH constant humidity	175.722	0.884	0.996	1.69×10^{-2}	3.28×10^{-4}
	Initial stage 50%RH 1h	215.682	0.885	0.998	1.14×10^{-2}	1.49×10^{-4}
	Initial stage 50%RH 2h	200.280	0.987	0.998	1.42×10^{-2}	2.35×10^{-4}
	Initial stage 50%RH 3h	204.336	0.912	0.995	1.98×10^{-2}	4.46×10^{-4}
	Initial stage 50%RH 4h	213.744	0.836	0.996	1.68×10^{-2}	3.15×10^{-4}
	Initial stage 50%RH 5h	223.158	0.897	0.994	2.07×10^{-2}	4.74×10^{-4}
Different humidity control humidity at initial stage	10%RH constant humidity	175.722	0.884	0.996	1.69×10^{-2}	3.28×10^{-4}
	Initial stage 20%RH 2h	250.374	0.876	0.998	1.26×10^{-2}	1.81×10^{-4}
	Initial stage 30%RH 2h	197.856	0.860	0.996	1.60×10^{-2}	2.87×10^{-4}
	Initial stage 40%RH 2h	188.994	0.966	0.997	1.51×10^{-2}	2.63×10^{-4}
	Initial stage 50%RH 2h	200.280	0.987	0.998	1.42×10^{-2}	2.35×10^{-4}
Different humidity control time at medium stage	Initial stage 50%RH 2h	200.280	0.987	0.998	1.42×10^{-2}	2.35×10^{-4}
	Medium stage 30%RH 1h	219.282	0.887	0.996	1.63×10^{-2}	3.00×10^{-4}
	Medium stage 30%RH 2h	217.812	0.932	0.997	1.53×10^{-2}	2.74×10^{-4}
	Medium stage 30%RH 3h	218.946	0.956	0.996	1.69×10^{-2}	3.32×10^{-4}
	Medium stage 30%RH 4h	241.368	0.909	0.996	1.68×10^{-2}	3.17×10^{-4}

The results of potato slices under different drying conditions were simulated by Weibull function and presented in Tab.2. From Tab.2, it can be seen that the coefficient of determination R^2 ranged from 0.989 to 0.999, the root mean square error (RMSE) was in the range of 1.04×10^{-2} to 2.57×10^{-2}, and the derivate square (χ^2) changed from 1.25×10^{-4} to 7.15×10^{-4}. Hence, the simulation results showed that the Weibull distribution function was suitable for matching the drying kinetic curves of potato slices under different drying conditions, which provided the subsequent basis for analyzing the drying process with mathematical model.

(1) Physical significance and influential factors of scale parameter α

As for the whole drying process of potato slices, the scale parameter α in the Weibull model meant the rate constant of the drying process. And its value concretely represented the time needed for removal approximately 63% of the samples moisture content during the entire dehydration process. According to Tab.2, when the air humidity in the drying chamber rose from 10%RH (continuous dehumidification) to 50%RH with the hot-air velocity of $15\text{m}\cdot\text{s}^{-1}$ at 60°C, the scale parameter gradually increased from 175.722 min to 240.186 min in a fluctuant way. It illustrated that the drying efficiency gradually slowed down with the rose of drying humidity. At the same time, the value of α accounted for 19.52%, 21.93%, 24.33%, 21.56% and 16.01% of the total drying time of potato slices, respectively. The results indicated that the dehydration process during the later stage had taken up most of the whole drying time. Therefore, it's very important to adjust the process conditions according to the value of scale parameter. And the obtain of scale parameter α had practical guiding significance on the optimization of drying process, which can be calculated by fitting analysis on partial experiment data in a short time.

As the humidity control time of 50%RH in the earlier stage increased from 0h (continuous dehumidification) to 5h, the value of α rose oscillatorily from 175.722 to 223.158min. The minimum value during the humidity control trials was got under the earlier stage with control time at 2h, and its time for removal 63% of the samples moisture content also accounted for the highest level of the whole drying time. This illustrated that the drying time under the above drying condition was much shorter than others. Further, as the value of control humidity increased from 20%RH to 50%RH during the earlier stage for both 2h, the scale parameter fluctuated from 175.7 to 250.4min. Although the time for removal 63% of moisture content accounted for the highest level of the whole drying time at 20%RH for 2h during the earlier stage, the scale parameter α still got the maximum value. As a result, the total drying time under this condition was not the shortest one due to the above comprehensive influences.

(2) Physical significance and influencing factors of shape parameter β

The shape parameter β is a dimensionless constant closely related to kinetic curves during material drying process. Combining graphical distribution characteristic and changing regulations, the effect of the shape parameter β on the material drying condition and analysis of changing trend can be intuitively understood.

According to Tab.2, the values of shape parameter β constantly decreased in the range of 0.811~0.964 with the increase of constant humidity (except constant dehumidification controlled trial), whereas the values of shape parameter β fluctuated in the range of 0.836~0.987 and had no regulations with the changing of the values of humidity control time and humidity at earlier stage and humidity control time at medium stage. This probably due to the fact that the shape parameter β is a parameter related to drying methods, thus the effect of drying temperature and humidity on β was slight for the same drying materials.

Analysis of moisture effective diffusivity

Table 3 Moisture effective diffusivity of potato slices during drying process

Drying method	Humidity parameter	Estimated moisture effective diffusivity D_{cal} （$10^{-9}m^2/s$）	Moisture effective diffusivity D_{eff} （$10^{-10}m^2/s$）	Geometric parameter Rg
Different constant humidity	10%RH constant humidity （continuous dehumidification）	2.371	6.120	3.874
	20%RH constant humidity	1.979	5.816	3.403
	30%RH constant humidity	2.115	6.717	3.148
	40%RH constant humidity	1.895	5.157	3.675
	50%RH constant humidity	1.735	3.242	5.351
Different humidity control time at initial stage	10%RH constant humidity （continuous dehumidification）	2.371	6.120	3.874
	Initial stage 50%RH 1h	1.932	5.876	3.288
	Initial stage 50%RH 2h	2.080	7.751	2.684
	Initial stage 50%RH 3h	2.039	5.664	3.600
	Initial stage 50%RH 4h	1.949	4.934	3.951
	Initial stage 50%RH 5h	1.867	4. 488	4.160
Different humidity control humidity at initial stage	10%RH constant humidity （continuous dehumidification）	2.371	6.120	3.874
	Initial stage 20%RH 2h	1.664	6.464	2.575
	Initial stage 30%RH 2h	2.106	5.289	3.982
	Initial stage 40%RH 2h	2.205	6.829	3.228
	Initial stage 50%RH 2h	2.080	7. 751	2.684
Different humidity control time at medium stage	Earlier stage 50%RH 2h	2.080	7. 751	2.684
	Medium stag 30%RH 1h	1.900	5.420	3.506
	Medium stage 30%RH 2h	1.913	6.960	2.749
	Medium stage 30%RH 3h	1.903	6.190	3.074
	Medium stage 30%RH 4h	1.726	5. 167	3.340

Geometric parameter R_g is a constant having nothing to do with the moisture effective diffusivity. Potato slices having the thickness of 10mm were selected in the experiments; thereby they can be regarded as flat materials. During the temperature and humidity integration control experiments, R_g values under different constant humidity fluctuated in the range of 3.148~5.35; R_g values ranged from2.684 to 4.160 with the increase of humidity control time with 50%RH at earlier stage; R_g values changed slightly in the range of 2.575~3.982 with the increase of humidity control humidity during humidity control of 2h at earlier stage; after humidity control for 2h with 50%RH at earlier stage, R_g values changes slightly in the range of 2.684~3.506 with the increase of humidity control time with 30%RH at medium stage.

Conclusions

The Weibull distribution function simulated the drying curves of potato slices PDTHIC air impingement drying well. The coefficient of determination ranged from 0.989 to 0.999, the root mean square error was in the range of 1.04×10^{-2} to 2.57×10^{-2}, and the derivate square ranged from 1.25×10^{-4} to 7.15×10^{-4}.

The scale parameter α is related to both the humidity control parameters during the earlier stage and the medium stage. In the condition of the drying temperature of 60°C and the wind speed of 15m/s, with the increase of constant humidity with 10%RH to 50%RH and the humidity control holding time with 50%RH in the earlier stage of the drying process from 0h to 5h, the values of the scale parameter α increased in a fluctuant way within the range of 175.722min to 240.186min; whereas the effect of humidity control of 2h under different humidity at earlier stage and humidity control time at medium stage on the values of α was of no significance, presenting a fluctuant changing rule. The shape parameter β constantly decreased (except constant dehumidification controlled trial) in the range of 0.811~0.964 with the increase of constant humidity, whereas the values of shape parameter β fluctuated in the range of 0.836~0.987 and had no regulations with the changing of the values of humidity control time.

The estimated moisture effective diffusivity of potato slices under different drying conditions calculated by scale parameter α ranged from 1.664×10^{-9} $m^2\cdot s^{-1}$ to 2.371×10^{-9} $m^2\cdot s^{-1}$ coinciding with the changing trend of D_{eff}, and the values of geometric parameters R_g was calculated in a fluctuant way in the range of 2.575~5.351 during drying based on humidity and temperature integration control.

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