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Permanence and almost periodic solution for a n-species competitive system

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Abstract: By applying the theory of inequality on time scales and the Lyapunov function method, we obtain some sufficient conditions which guarantee the permanence and existence of a unique uniformly asymptotically stable almost periodic sequence solution of a n-species Lotka-Volterra competitive system with infinite delay and feedback control.

1. Introduction

The traditional non-autonomous n-species Lotka-Volterra competitive system can be expressed as follows

$$\dot{x}_i(t) = x_i(t)\left[b_i(t) - \sum_{j=1}^{n} a_{ij}(t)x_j(t)\right], i = 1, 2, \ldots, n, \tag{1.1}$$

where $b_i(t)(i = 1, 2, \ldots, n)$ represents the intrinsic growth rate of the i th species at time t and $a_{ij}(t)$ is the coefficient of competitive between species the j th and the i th. Some sufficient conditions have been obtained for permanence, extinction and global stability for system (1.1). The authors of [1-3] established the conditions for the permanence and global asymptotic behaviour for the system (1.1).

However, in the real world, the growth rate of a natural species population would not respond immediately to changes in its own population or that of an interacting species, but rather will do so after a time lag. The authors of [4-7] have paid great attention to the dynamic behaviors for multi-species competitive systems with finite delay. In [8], the authors propose the concept of almost periodic time scales and the definition of almost periodic functions on almost periodic time scales. Based on these, our main aim in this paper is to study the almost periodic solutions of the following n-species Lotka-Volterra competitive system with infinite delay and feedback control on time scales.

$$\begin{cases} x_i^{\Delta}(t) = r_i(t) - a_i(t)\exp\{x_i(t)\} - \sum_{j=1}^{n} a_{ij}(t)\int_0^{\infty} K_{ij}(s)\exp\{x_j(t-s)\}\Delta s - b_i(t)\int_0^{\infty} H_i(s)u_i(t-s)\Delta s, \\ u_i^{\Delta}(t) = -c_i(t)u_i(t) + d_i(t)\int_0^{\infty} R_i(s)\exp\{x_i(t-s)\}\Delta s, i = 1, 2, \ldots, n, \end{cases} \tag{1.2}$$

where $x_i(t), u_i(t), r_i(t), a_i(t)(i = 1, 2, \ldots, n)$ stand for the i th species population density, the i th feedback control, the i th species birth rate and death rate, espectively. $b_i(t), a_{ij}(t)(i, j = 1, 2, \ldots, n-1)$ represent the feedback control rate and the j th species competition rate on the i th species. $\int_0^{\infty} K_{ij}(s)\exp\{x_j(t-s)\}\Delta s$ and $\int_0^{\infty} H_i(s)u_i(t-s)\Delta s$ denote the the effect of all the past life history of the species on its present birth rate and the restrain rate of feedback control on its present birth. We assume that $r_i(t), a_i(t), b_i(t), c_i(t)$, $d_i(t)$ and $a_{ij}(t)(i, j = 1, 2, \ldots, n)$ are all bounded non-negative almost periodic functions on $\mathbb{T}$.

Under the assumptions of almost periodicity of the coefficients of (1.2), we first discuss the permanence of (1.2) on time scales. Based on the permanence result, we establish sufficient conditions for the existence and uniformly asymptotical stability of a unique almost periodic solution of (1.2).

Throughout this paper, we assume that, for an almost periodic function $f : \mathbb{T} \to \mathbb{R}$, we take the following notes for convenience

$$f^M = \sup_{t\in\mathbb{T}} f(t),\ f^m = \inf_{t\in\mathbb{T}} f(t),\ f^{2M} = \sup_{t\in\mathbb{T}}(f(t))^2,\ f^{2m} = \inf_{t\in\mathbb{T}}(f(t))^2,$$

and we denote the solutions of system (1.2) by $X(t) = (x_1(t),\ldots,x_n(t),u_1(t),\ldots,u_n(t))^T$.

(H_1) $r_i(t), a_i(t), b_i(t), c_i(t), d_i(t)$ and $a_{ij}(t)(i,j=1,2,\ldots,n)$ are all bounded non-negative almost periodic functions on $\mathbb{T}$ such that

$$0 < r_i^m \le r_i(t) \le r_i^M; 0 < a_i^m \le a_i(t) \le a_i^M;$$
$$0 < b_i^m \le b_i(t) \le b_i^M; 0 < c_i^m \le c_i(t) \le c_i^M;$$
$$0 < d_i^m \le d_i(t) \le d_i^M; 0 < a_{ij}^m \le a_{ij}(t) \le a_{ij}^M, i,j=1,2,\ldots,n;$$

(H_2) $-a_i^m, -c_i^M, c_i^m \in \Re^+, i=1,2,\ldots,n;$

(H_3) $K_{ij}:[0,\infty)\to[0,\infty), H_i:[0,\infty)\to[0,\infty), R_i:[0,\infty)\to[0,\infty), i,j=1,2,\ldots,n,$ are rd-continuous functions and satisfy $\int_0^\infty K_{ij}(s)\Delta s = 1; \int_0^\infty H_i(s)\Delta s = 1; \int_0^\infty R_i(s)\Delta s = 1;$

(H_4) $r_i^m - b_i^M u_i^* - \sum_{j=1}^n a_{ij}^M \exp\{x_i^*\} > a_i^M, i=1,2,\ldots,n.$

2. Preliminaries

Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma,\rho:\mathbb{T}\to\mathbb{T}$ and the graininess $\mu : \mathbb{T}\to\mathbb{R}^+$ are defined, respectively, by

$$\sigma(t) = \inf\{s\in\mathbb{T}: s>t\},\quad \rho(t) = \sup\{s\in\mathbb{T}: s<t\},\quad \mu(t) = \sigma(t) - t.$$

A point $t\in\mathbb{T}$ is called left-dense if $t > \inf\mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup\mathbb{T}$ and $\sigma(t) = t$, and right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T}\setminus\{m\}$; otherwise $\mathbb{T}^k = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}_k = \mathbb{T}\setminus\{m\}$; otherwise $\mathbb{T}_k = \mathbb{T}$.

A function $f:\mathbb{T}\to\mathbb{R}$ is right-dense continuous provided it is continuous at right-dense point in $\mathbb{T}$ and its left-side limits exist at left-dense points in $\mathbb{T}$. If f is continuous at each right-dense point and each left-dense point, then f is said to be a continuous function on $\mathbb{T}$.

Definition 2.1[8] Assume that $f:\mathbb{T}\to\mathbb{R}$ is a function and let $t\in\mathbb{T}$. Then we define $f^\Delta(t)$ to be the number (provided it exists) with the property that given any $\varepsilon>0$, there is a neighborhood U of t (i.e., $U=(t-\delta,t+\delta)\cap\mathbb{T}$ for some $\delta>0$) such that

$$\left|[f(\sigma(t)) - f(s)] - f^\Delta(t)[\sigma(t)-s]\right| \le \varepsilon\,|\sigma(t)-s| \quad \text{for all } s\in U.$$

we call $f^\Delta(t)$ the delta derivative of f at t. The function f is delta differentiable on $\mathbb{T}$ provided $f^\Delta(t)$ exists for all $t\in\mathbb{T}$. The set of functions $f:\mathbb{T}\to\mathbb{R}$ that are delta differentiable and whose delta derivative are rd-continuous functions is denoted by $C_{rd}^1 = C_{rd}^1(\mathbb{T}) = C_{rd}^1(\mathbb{T},\mathbb{R})$.

Definition 2.2[8] A function $p:\mathbb{T}\to\mathbb{R}$ is called regressive provided $1+\mu(t)p(t)\ne 0$ for all $t\in\mathbb{T}^k$. The set of all regressive and rd-continuous functions $p:\mathbb{T}\to\mathbb{R}$ will be denoted by $\mathcal{R} = \mathcal{R}(\mathbb{T}) = \mathcal{R}(\mathbb{T},\mathbb{R})$. We define the set $\mathcal{R}^+ = \mathcal{R}^+(\mathbb{T},\mathbb{R}) = \{p\in\mathcal{R}: 1+\mu(t)p(t)>0, \forall t\in\mathbb{T}\}$.

Definition 2.3[8] A function $F:\mathbb{T}\to\mathbb{R}$ is called antiderivative of $f:\mathbb{T}\to\mathbb{R}$ provided $F^\Delta(t) = f(t)$ for all $t\in\mathbb{T}$. Then we write $\int_r^s f(t)\Delta t = F(s) - F(r)$ for $r,s\in\mathbb{T}$.

Definition 2.4[8] If $a\in\mathbb{T}$, $\sup\mathbb{T}=\infty$, and f is rd-continuous on $[a,\infty)$, then we define the improper integral by $\int_a^\infty f(t)\Delta t = \lim_{b\to\infty}\int_a^b f(t)\Delta t$ for $a,b\in\mathbb{T}$.

Definition 2.5[8] A time scale $\mathbb{T}$ is called an almost periodic time scale if $\Pi := \{\tau\in\mathbb{R}: t+\tau\in\mathbb{T}, \forall t\in\mathbb{T}\} \ne \{0\}$.

Throughout this paper, we restrict our discussion on almost periodic time scales.

Definition 2.6[8] Let $\mathbb{T}$ be an almost periodic time scale. A function $f:\mathbb{T}\to\mathbb{R}^n$ is said to be almost periodic on $\mathbb{T}$, if for any $\varepsilon>0$, the set $E(\varepsilon,f) = \{\tau\in\Pi: |f(t+\tau)-f(t)| < \varepsilon, \forall t\in\mathbb{T}\}$ is relatively

dense in $\mathbb{T}$, that is, for any $\varepsilon > 0$, there exists a constant $l(\varepsilon) > 0$ such that each interval of length $l(\varepsilon)$ contains at least one $\tau \in E(\varepsilon, f)$ such that $|f(t+\tau) - f(t)| < \varepsilon, \forall t \in \mathbb{T}$. The set $E(\varepsilon, f)$ is called the ε-translation set of $f(t)$, τ is called the ε-translation number of $f(t)$, and $l(\varepsilon)$ is called the inclusion of $E(\varepsilon, f)$.

Definition 2.7 System (1.2) is said to be permanent if there exist positive constants $x_{i*}, x_i^*, u_{i*}, u_i^*$, which are independent of the solutions of the system, such that any positive solution $X(t)$ of system (1.2) satisfies

$$x_{i_*} \le \liminf_{t\to+\infty} x_i(t) \le \limsup_{t\to+\infty} x_i(t) \le x_i^*, \quad u_{i_*} \le \liminf_{t\to+\infty} u_i(t) \le \limsup_{t\to+\infty} u_i(t) \le u_i^*, \quad i = 1, 2, \ldots, n.,$$

Lemma 2.1[8] Let $-a \in \mathcal{R}^+$.

(i) If $x^\Delta(t) \le b - ax(t)$, then for $t > t_0$, $x(t) \le x(t_0)e_{(-a)}(t, t_0) + \frac{b}{a}(1 - e_{(-a)}(t, t_0))$. In particular, if $a > 0$, we have $\limsup_{t\to+\infty} x(t) \le \frac{b}{a}$.

(ii) If $x^\Delta(t) \ge b - ax(t)$, then for $t > t_0$, $x(t) \ge x(t_0)e_{(-a)}(t, t_0) + \frac{b}{a}(1 - e_{(-a)}(t, t_0))$. In particular, if $a > 0$, we have $\liminf_{t\to+\infty} x(t) \ge \frac{b}{a}$.

Lemma 2.2[8] Suppose that there exists a Lyapunov function $V(t, x, y)$ satisfying the following conditions

(i) $a(|x - y|) \le V(t, x, y) \le b(|x - y|)$, where $a, b \in \mathcal{K}, \mathcal{K} = \{a \in C(R^+, R^+) : a(0) = 0$ and $a(x)$ is increasing in $x\}$

(ii) $|V(t, x_1, y_1) - V(t, x_2, y_2)| \le L(|x_1 - x_2| + |y_1 - y_2|)$, where $L > 0$ is a constant;

(iii) $D^+V^\Delta(t, x, y) \le -cV(t, x, y)$ where $-c \in \mathcal{R}^+$, and $c > 0$.

Moreover, if there exists a solution $x(t)$ of $x^\Delta = f(t, x)$ such that $x(t) \in \mathbb{S}$ for all $t \in \mathbb{T}^+$, where $S \subset D$ is a compact set. Then there exists a unique uniformly asymptotically stable almost periodic solution $p(t)$ in S.

3. Permanence

In this section, we establish some permanence results for system (1.2). From Lemma 2.1, one can obtain the following results.

Theorem 3.1 Assume that (H_1), (H_2) and (H_3) hold. Then every solution $X(t)$ of system (1.2) satisfies

$$\limsup_{t\to+\infty} x_i(t) \le x_i^*, \quad \limsup_{t\to+\infty} u_i(t) \le u_i^*, i = 1, 2, \ldots, n,$$

where

$$x_i^* = \frac{r_i^M - a_i^m}{a_i^m}, \quad u_i^* = \frac{d_i^M \exp\{x_i^*\}}{c_i^m}, i = 1, 2, \ldots, n.$$

Theorem 3.2 Assume that (H_1)-(H_4) hold, then every solution $X(t)$ of system (1.2) satisfies

$$\liminf_{t\to+\infty} x_i(t) \ge x_{i_*}, \quad \liminf_{t\to+\infty} u_i(t) \ge u_{i_*}, i = 1, 2, \ldots, n,$$

where

$$x_{i_*} = \ln \frac{r_i^m - b_i^M u_i^* - \sum_{j=1}^n a_{ij}^M \exp\{x_i^*\}}{a_i^M}, \quad u_{i_*} = \frac{d_i^m \exp\{x_{i_*}\}}{c_i^M}, i = 1, 2, \ldots, n.$$

Theorem 3.3 Assume that (H_1)-(H_4) hold. Then system (1.2) is permanent.

4. Existence of a unique almost periodic solution

For convenience, we denote by Ω the set of all solutions $X(t)$ of system (1.2) satisfying $x_{i_*} \le x_i(t) \le x_i^*, u_{i_*} \le u_i(t) \le u_i^*$, for all $t \in \mathbb{T}, i = 1, 2, \ldots, n.$

Theorem 4.1 Assume that (H_1) - (H_4) hold. Then $\Omega \neq \varnothing$.

Theorem 4.2 Assume (H_1) - (H_4) hold, then we have the following inequalities

$$\left|\int_0^\infty R_i(s)\big(x_i(t-s)-y_i(t-s)\big)\Delta s\right| \le 2\,|\,x_i(t)-y_i(t)\,|,$$

$$\left|\int_0^\infty H_i(s)\big(u_i(t-s)-v_i(t-s)\big)\Delta s\right| n \le 2\,|\,u_i(t)-v_i(t)\,|,$$

$$\left|\int_0^\infty K_{ij}(s)\big(x_j(t-s)-y_j(t-s)\big)\Delta s\right| n \le 2\,|\,x_i(t)-y_i(t)\,|, i,\ j=1,2,\ldots,n.$$

Proof According to Theorem 4.1, similarly, for $t > t_+ + s, s \to \infty$, we have

$$x_i(t-s) \to y_i(t-s), u_i(t-s) \to v_i(t-s), i=1,2\ldots,n, \text{ for } s \to +\infty,$$

which mean that

$$\left|\,x_i(t-s)-y_i(t-s)\,\right| \to 0, \left|\,u_i(t-s)-v_i(t-s)\,\right| \to 0,\ i=1,2\ldots,n, \text{for } s \to +\infty.$$

Then for $t > t_+ + s, s \to \infty,$ we get

$$\left|\int_{t-s}^t \big(x_i^\Delta(\tau)-y_i^\Delta(\tau)\big)\Delta\tau\right| = \left|\,x_i(t)-y_i(t)\,\right| + \left|\,x_i(t-s)-y_i(t-s)\,\right| \le |\,x_i(t)-y_i(t)\,|,$$

$$\left|\int_{t-s}^t \big(x_i^\Delta(\tau)-y_i^\Delta(\tau)\big)\Delta\tau\right| n = \left|\,x_i(t)-y_i(t)\,\right| + \left|\,x_i(t-s)-y_i(t-s)\,\right| n \le |\,x_i(t)-y_i(t)\,|, i=1,2,\ldots,n.$$

Then

$$\begin{aligned}\left|\int_0^\infty R_i(s)\big(x_i(t-s)-y_i(t-s)\big)\Delta s\right| &= \left|\int_0^\infty R_i(s)\big(x_i(t-s)-x_i(t)+y_i(t)-y_i(t-s)+x_i(t)-y_i(t)\big)\Delta s\right| \\ &\le \int_0^\infty R_i(s)\left|\int_{t-s}^t \big(y_i^\Delta(\tau)-x_i^\Delta(\tau)\big)\Delta\tau\right|\Delta s + \left|\,x_i(t)-y_i(t)\,\right| \\ &\le 2\left|\,x_i(t)-y_i(t)\,\right|, i=1,2,\ldots,n.\end{aligned}$$

Similarly, we can prove the other two inequalities.

Theorem 4.3 Assume that (H_1) - (H_4) hold. And furthermore assume that (H_5) $\Theta > 0$ and $-\Theta \in \mathcal{R}^+$, where $\Theta = \min\{A_i - B_i, P_i - Q_i, i=1,2,\ldots,n\}$, and

$$\begin{aligned}A_i &= 2a_i^m \exp\{x_{i_*}\},\\ B_i &= 2b_i^M + 2d_i^M \exp\{x_i^*\} + \mu^M a_i^M \exp\{2x_i^*\}\big(a_i^M + 2na_{ij}^M\big)\\ &\quad + 2n\mu^M a_{ij}^M \exp\{2x_i^*\}\big(a_i^M + 2a_{ij}^M\big) + 2\exp\{x_i^*\}\big(na_{ij}^M + \mu^M a_i^M b_i^M\big)\\ &\quad + 2na_{ij}^M \exp\{x_i^*\}\big(1+2\mu^M b_i^M\big) + 2\mu^M d_i^M \exp\{x_i^*\}\big(c_i^M + 2d_i^M \exp\{x_i^*\}\big),\\ Q_i &= 2b_i^M + \mu^M c_i^{2M} + 2d_i^M \exp\{x_i^*\}\big(1+\mu^M c_i^M\big)n + 4\mu^M b_i^{2M} + 2\mu^M b_i^M \exp\{x_i^*\}\big(a_i^M + 2na_{ij}^M\big),\\ P_i &= 2c_i^m.\end{aligned}$$

Then there exists a unique uniformly asymptotically stable almost periodic solution $X(t)$ of system (1.2), and $X(t) \in \Omega$..

Proof From Theorem 4.1, there exists $X(t)$ such that

$$x_{i_*} \le x_i(t) \le x_i^*, u_{i_*} \le u_i(t) \le u_i^*, i=1,2,\ldots,n,\ \ t \in \mathbb{T}.$$

We denote

$$E_i = \max\{|\,x_{i_*}\,|, |\,x_i^*\,|\}, F_i = \max\{|\,u_{i_*}\,|, |\,u_i^*\,|\}, i=1,2,\ldots,n,$$

then,

$$|\,x_i(t)\,| \le E_i, |\,u_i(t)\,| \le F_i, i=1,2,\ldots,n.$$

Define the norm

$$\|\,X\,\| = \sup_{t\in\mathbb{T}} \sum_{i=1}^n |x_i(t)| + \sup_{t\in\mathbb{T}} \sum_{i=1}^n |u_i(t)|, X(t) \in \mathbb{R}^{2n}. \tag{4.1}$$

In view of Theorem 4.1, we can suppose that

$$X_1 = (x_1(t),\ldots,x_n(t),u_1(t),\ldots,u_n(t))^T,\ \ Y_1 = (y_1(t),\ldots,y_n(t),v_1(t),\ldots,v_n(t))^T, \tag{4.2}$$

be any two positive solutions of system (1.2).

Substituting (4.2) into the system of (1.2), we arrive to

$$\begin{cases} x_i^{\Delta}(t) = r_i(t) - a_i(t)\exp\{x_i(t)\} - \sum_{j=1}^{n} a_{ij}(t)\int_0^{\infty} K_{ij}(s)\exp\{x_j(t-s)\}\Delta s - b_i(t)\int_0^{\infty} H_i(s)u_i(t-s)\Delta s, \\ u_i^{\Delta}(t) = -c_i(t)u_i(t) + d_i(t)\int_0^{\infty} R_i(s)\exp\{x_i(t-s)\}\Delta s, \\ y_i^{\Delta}(t) = r_i(t) - a_i(t)\exp\{y_i(t)\} - \sum_{j=1}^{n} a_{ij}(t)\int_0^{\infty} K_{ij}(s)\exp\{y_j(t-s)\}\Delta s - b_i(t)\int_0^{\infty} H_i(s)v_i(t-s)\Delta s, \\ v_i^{\Delta}(t) = -c_i(t)v_i(t) + d_i(t)\int_0^{\infty} R_i(s)\exp\{y_i(t-s)\}\Delta s, i = 1,2,\ldots,n. \end{cases} \tag{4.3}$$

Considering Lyapunov function $V(t, X_1, Y_1)$ on $\mathbb{T}^+ \times \Omega \times \Omega$ defined by

$$V(t, X_1, Y_1) = \sum_{i=1}^{n}(x_i(t) - y_i(t))^2 + \sum_{i=1}^{n}(u_i(t) - v_i(t))^2. \tag{4.4}$$

Obviously, the norm of (4.1) is equated with the following form of norm

$$\| X \|_* = \sup_{t\in\mathbb{T}}\Big[\sum_{i=1}^{n}(x_i(t))^2 + \sum_{i=1}^{n}(u_i(t))^2\Big]^{\frac{1}{2}}.$$

Then, $\| X_1 - Y_1 \|_*$ and $\| X_1 - Y_1 \|$ are equivalent. Namely, there exist two constants $C_1 > 0, C_2 > 0$ such that

$$(C_1\| X_1 - Y_1 \|)^2 \le V(t, X_1, Y_1) \le (C_2 \| X_1 - Y_1 \|)^2.$$

Take $a, b \in C(\mathbb{R}^+, \mathbb{R}^+), a(x) = C_1^2 x^2, b(x) = C_2^2 x^2,$ thus the condition (i) of Lemma 2.2 is satisfied. In addition,

$$\begin{aligned} |V(t, X_1, Y_1) - V(t, X_1', Y_1')| &= \Big|\Big[\sum_{i=1}^{n}(x_i(t) - y_i(t))^2 + \sum_{i=1}^{n}(u_i(t) - v_i(t))^2\Big]n - \Big[\sum_{i=1}^{b}(x_i'(t) - y_i'(t))^2 + \sum_{i=1}^{b}(u_i'(t) - v_i'(t))^2\Big]\Big| \\ &\le L\Big[\sum_{i=1}^{n}(|x_i(t) - x_i'(t)| + |y_i(t) - y_i'(t)|) + \sum_{i=1}^{n}(|u_i(t) - u_i'(t)| + |v_i(t) - v_i'(t)|)\Big] \\ &\le L(\| X_1 - X_1' \| + \| Y_1 - Y_1' \|), \end{aligned}$$

where

$$L = 4\max\{E_i, F_i : i = 1,2,\ldots,n\}, X_1' = (x_1'(t),\ldots,x_n'((t), u_1'(t),\ldots,u_n'(t))^T, Y_1' = (y_1'(t),\ldots,y_n'(t), v_1'(t),\ldots,v_n'(t))^T.$$

Thus the condition (ii) of Lemma 2.2 is also satisfied. Finally, we will prove the condition (iii) of Lemma 2.2 is satisfied, calculating the right derivative D^+V^{Δ} of V along the solution of (4.4).

$$\begin{aligned} D^+V^{\Delta}(t, X_1, Y_1) &= \sum_{i=1}^{n}\Big[2(x_i(t) - y_i(t)) + \mu(t)(x_i(t) - y_i(t))^{\Delta}\Big](x_i(t) - y_i(t))^{\Delta} \\ &\quad + \sum_{i=1}^{n}\Big[2(u_i(t) - v_i(t)) + \mu(t)(u_i(t) - v_i(t))^{\Delta}\Big](u_i(t) - v_i(t))^{\Delta} \\ &= V_1 + V_2. \end{aligned} \tag{4.5}$$

According to (4.3), we have

$$\begin{aligned} (x_i(t) - y_i(t))^{\Delta} &= -a_i(t)\big(\exp\{x_i(t)\} - \exp\{y_i(t)\}\big) - b_i(t)\int_0^{\infty} H_i(s)\big(u_i(t-s) - v_i(t-s)\big)\Delta s \\ &\quad - \sum_{j=1}^{n} a_{ij}(t)\int_0^{\infty} K_{ij}(s)\big(\exp\{x_j(t-s)\} - \exp\{y_j(t-s)\}\big)\Delta s, i = 1,2,\ldots,n, \\ (u_i(t) - v_i(t))^{\Delta} &= -c_i(t)(u_i(t) - v_i(t)) + d_i(t)\int_0^{\infty} R_i(s)\big(\exp\{x_i(t-s)\} - \exp\{y_i(t-s)\}\big), i = 1,2,\ldots,n. \end{aligned} \tag{4.6}$$

By the mean value theorem, we get $\exp\{x_i(t)\} - \exp\{y_i(t)\} = \exp\{\xi_i(t)\}(x_i(t) - y_i(t)),$ where $\xi_i(t)$ lie between $x_i(t)$ and $y_i(t), i = 1,2,\ldots,n.$ Then, (4.6) can be rewritten as

$$\begin{aligned} (x_i(t) - y_i(t))^{\Delta} &= -\Big[a_i(t)\exp\{\xi_i(t)\}\big(x_i(t) - y_i(t)\big) + b_i(t)\int_0^{\infty} H_i(s)\big(u_i(t-s) - v_i(t-s)\big)\Delta s \\ &\quad + \sum_{j=1}^{n} a_{ij}(t)\int_0^{\infty} K_{ij}(s)\exp\{\xi_j(t-s)\}\big(x_j(t-s) - y_j(t-s)\big)\Delta s\Big], i = 1,2,\ldots,n, \\ (u_i(t) - v_i(t))^{\Delta} &= -c_i(t)\big(u_i(t) - v_i(t)\big) + d_i(t)\int_0^{\infty} R_i(s)\exp\{\xi_i(t-s)\}\big(x_i(t-s) - y_i(t-s)\big)\Delta s, i = 1,2,\ldots,n. \end{aligned}$$

According to Theorem 4.2 and (4.5), and combing with the inequality of $2|a||b| \le a^2 + a^2$ then we have

$$
\begin{aligned}
V_1 = & -\sum_{i=1}^{n}\Big\{2[x_i(t)-y_i(t)]-\mu(t)\Big[a_i(t)\exp\{\xi_i(t)\}[x_i(t)-y_i(t)]+b_i(t)\int_0^{\infty}H_i(s)[u_i(t-s)-v_i(t-s)]\Delta s \\
& +\sum_{j=1}^{n}a_{ij}(t)\int_0^{\infty}K_{ij}(s)\exp\{\xi_i(t-s)\}[x_j(t-s)-y_j(t-s)]\Delta s\Big]\Big\} \\
& \times\Big[a_i(t)\exp\{\xi_i(t)\}[x_i(t)-y_i(t)]+b_i(t)\int_0^{\infty}H_i(s)[u_i(t-s)-v_i(t-s)]\Delta s \\
& +\sum_{j=1}^{n}a_{ij}(t)\int_0^{\infty}K_{ij}(s)\exp\{\xi_i(t-s)\}[x_j(t-s)-y_j(t-s)]\Delta s\Big] \qquad (4.7) \\
\le & \sum_{i=1}^{n}\Big[-2a_i^m\exp\{x_{i*}\}+2b_i^M+2\exp\{x_i^*\}(na_{ij}^M+\mu^M a_i^M b_i^M)+\mu^M a_i^M\exp\{2x_i^*\}(a_i^M+2na_{ij}^M) \\
& +2na_{ij}^M\exp\{x_i^*\}(1+2\mu^M b_i^M)+2n\mu^M a_{ij}^M\exp\{2x_i^*\}(a_i^M+2a_{ij}^M)\Big][x_i(t)-y_i(t)]^2 \\
& +\sum_{i=1}^{n}\Big[2b_i^M+4\mu^M b_i^{2M}+2\mu^M b_i^M\exp\{x_i^*\}(a_i^M+2na_{ij}^M)\Big][u_i(t)-v_i(t)]^2,
\end{aligned}
$$

$$
\begin{aligned}
V_2 = & \sum_{i=1}^{n}\Big\{2(u_i(t)-v_i(t))+\mu(t)\Big[-c_i(t)[u_i(t)-v_i(t)]+d_i(t)\int_0^{\infty}R_i(s)\exp\{\xi_i(t-s)\}[x_i(t-s)-y_i(t-s)]\Delta s\Big]\Big\} \\
& \times\Big[d_i(t)\int_0^{\infty}R_i(s)\exp\{\xi_i(t-s)\}[x_i(t-s)-y_i(t-s)]\Delta s-c_i(t)[u_i(t)-v_i(t)]\Big] \\
\le & \sum_{i=1}^{n}\Big[2d_i^M\exp\{x_i^*\}+2\mu^M d_i^M\exp\{x_i^*\}(c_i^M+2d_i^M\exp\{x_i^*\})\Big][x_i(t)-y_i(t)]^2 \qquad (4.8) \\
& +\sum_{i=1}^{n}\Big[-2c_i^m+\mu^M c_i^{2M}+2d_i^M\exp\{x_i^*\}(1+\mu^M c_i^M)\Big](u_i(t)-v_i(t))^2.
\end{aligned}
$$

In view of (4.7) and (4.8), we obtain

$$
D^+V^{\Delta}(t,X_1,Y_1)=V_1+V_2\le\sum_{i=1}^{n}(A_i-B_i)[x_i(t)-y_i(t)]^2+\sum_{i=1}^{n}(P_i-Q_i)[u_i(t)-v_i(t)]^2\le-\Theta V(t,X_1,Y_1).
$$

Therefore, condition (H_5) is satisfied. Hence, according to Lemma 2.2, there exists a unique uniformly asymptotically stable almost periodic solution $X(t)$ of system (1.2), and $X(t)\in\Omega$..

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