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The simultaneous equations models for the application in forestry

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Abstract: In accordance with weakness of traditional biomass models in which the sum of the above- and below-ground tree components was not equal to the whole tree, compatible biomass equations were developed for the above- and below-ground tree components of 11 kinds of tree species in Heilongjiang Province. The data used to develop biomass models are from 299 trees that were collected from 69 sample plots, and represented a wide range of stand and site conditions in Heilongjiang Province. Based on the total biomass model as restrictions, the compatible tree biomass equations for each component (stems, branches, foliages, and roots) were separately established by considering diameter at breast height (D) as independent variables in the form of simultaneous equations with measure error of independent variables. The evaluation and validation procedures for tree biomass models were performed by using the following statistical criterions: the coefficient of determination (R2), sum square of error (SEE), Mean Bias (MB), and Mean Absolute Bias (MAB). The results showed that although parameter estimation accuracy of compatible biomass equations was not better than traditional biomass models, the compatible biomass equations could effectively overcome the weakness of un-compatible of traditional biomass models.

1. Introduction

Forest biomass estimation is the groundwork of analyzing carbon cycle and its dynamics in terrestrial ecosystems. So there is a current need for reliable biomass models for total tree and individual tree components. First of all, which have to produce reliable biomass estimates applicable over the wide range; secondly, which should be based on the variables that are normally measured in forest inventories, or which can be estimated easily and reliably from inventory data; thirdly, one desirable feature of the tree components equations is biomass compatible, i.e. that the sum of the predictions for the tree component equals the prediction for the whole tree (Kozak 1974, Cunia and Briggs 1984, Parresol 1999, 2001). The estimation models for tree biomass had the problem of un-compatibility between total forest biomass and its components(stem wood, branches, foliages and roots) as follows Table (1), where for traditional model $w = aD^b$ as examples with Korean pine plantation, Mono maple and Manchurian ash, the total biomass is not equal to the sum of their components; even the aboveground biomass is equal to the sum of biomass of stem, branches and foliages. There are other two problems in current biomass models:(i) Forest biomass estimates mainly consider aboveground component(stem、branch、foliage), and underground biomass are estimated less (Williams, et al., 2005,Bond-Lamberty et al., 2002). Due to the difficult to determine below-ground biomass, the sample size is rarely, and consequently the accuracy of fit and predictor of the models with this small number of samples is unsatisfactory. (ii) Most of current researches are more concentrated on the plantation, furthermore tree species are relatively few.

Based on method of the simultaneous equations and measure error model, the compatible tree biomass models were developed for above- and below-ground tree components for 11 kinds of main species in Heilongjiang Province.

Table 1 An example of incompatibility of traditional models with equation (1)

Tree species	D(cm)	H(cm)	△1(kg)	△1(%)	△2(%)	△2(kg)
KP plantation	33	12.3	110.603	21.38618	-37	-111.97
	25.6	11.9	-12.785	-5.8998	-5.2	-75.72
	18.5	9.9	19.855	15.10898	-60	-33.65
	19.5	12.5	-9.186	-7.91405	-19.76	-56.92
Mono maple	32.5	15.3	-60.468	-18.8779	-45.67	-99.86
	23	13.4	13.320	67.23944	-37.25	-72.46
	16.1	12.8	30.948	36.06704	-58.65	-23.49
Manchurian ash	14.1	15.2	-63.713	-6.87664	-11	-59.942
	18.8	16.1	50.298	29.53442	-46	-49.57
	8.2	8.9	-3.083	-4.63676	-12	-10.45
	16.7	14.1	-37.551	-22.8649	-5.3	-27.78

Note: △1= total–(stem+branch+foliage+root), △1%=△1/total*100;
△2=total of aboveground-(stem+branch+foliage), △2%=△2/ total of aboveground *100

2. Materials and Method

The data were collected from 16 counties in Heilongjiang Province of Northeast China, ranging across about 1120 km in NS and 930km in WE (43°25′-53°33′N, 121°11′-135°05′ E) and the total land area is 454,000 km2.

2.1. Study areas

The climate features is continental monsoon climate with cold winters and warm summers. The average annual temperature is 4℃～4℃, the average annual rainfall is 450 mm to700 mm, and Frost-free period is between 100d to 160d. Main species are Korean Pine (*Pinus koraiensis*), Mongolian pine (*Pinus sylvestris var. mongolica*), Korean spruce (*Picea koraiensis*), Jezo spruce(*Picea jezoensis*), Amur fir (*Abies nephrolepis*), Dahurian Larch (*Larix gmelini*), Changbai Larch (*Larix olgensis*), Manchurian ash (*Fraxinus mandshurica*), Walnut (*Junglus mandshurica*), Cork tree (*Phellodendron amurense*), Basswood (*Tilia amurensis, T. mandshurica*), Mono maple (*Acer mono*), elm (*Ulmus propinqua*, *Ulmus laciniata*), Mongolian oak (*Quercus mongolica*), Costata birch (*Betula costata*), White birch (*Betula platyphlla*), Dahurian birch (*Betula davuria*), papen (*Populus davidiana, Populus ussuriensis*), and etc.

2.2. Data

The whole data set (297 trees from 64 sample plots) was randomly divided into two data sets: fitting data set (254 samples) and validation data set (43 samples).

Table 2. Summary of sample trees

species	Attribute	Modeling data			Validation data		
		Minimum	Mean (s.d.)	Maximum	Minimum	Mean (s.d.)	Maximum
1 N=26; N^{+}=4	D (cm)	4.8	17.39 (1.30)	30.8	19	24.15 (3.34)	33
	H (m)	4.68	11.54 (0.58)	16.2	9.9	11.65 (0.59)	12.5
2 N=16; N^{+}=2	D(cm)	12.2	23.2 (2.0)	38.7	17.4	19 (1.6)	20.6
	H(m)	15.1	44.09 (0.31)	19.7	16	16.6 (0.6)	17
3 N=18; N^{+}=5	D (cm)	2.30	3.37 (0.86)	5.60	15.5	20.43 (4.15)	28.7
	H(m)	8	11.8 (2.2)	15	12.5	16.53 (3.54)	23.6
4 N=27;	D (cm)	4.8	15.3 (1.12)	27.0	16.1	23.86 (4.75)	32.5

N^+=3	H(m)	6.2	1.21 (0.50)	18.6 5	12.8	13.83 (0.75)	15.3
5	D (cm)	6.0	20.2 (1.86)	33.4	8.2	14.45 (2.29)	18.8
N=18; N^+=4	H(m)	9.20	16.97 (0.98)	22.5 0	8.9	13.57 (1.61)	16.1
6	D (cm)	8.1	19.5 (1.66)	28.8	14.7	16.15 (1.45)	17.6
N=16; N^+=2	H(m)	9.6	17.94 (0.96)	23.1	12.7	15.25 (2.55)	17.8
7	D (cm)	4.2	16.45 (1.68)	37.1	7.2	24.5 (8.65)	33.2
N=27; N^+=3	H(m)	5	11.5 (0.74)	18.7	6.3	13.76 (3.73)	17.8
8	D (cm)	10.0	18.7 (1.31)	28.7	12.1	20.6 (5.88)	31.9
N=20; N^+=3	H(m)	10.9	15.8 (0.75)	21.4	11.2	15.73 (2.57)	20.1
9	D (cm)	10.4	19.3 (1.28)	31.3	11	14.47 (4.50)	26.8
N=18; N^+=4	H(m)	9.80	16.22 (0.82)	22.7 0	10	13.4 (2.76)	19.9
10	D (cm)	7.3	18.9 (1.65)	47.0	19	24.15 (3.34)	33
N=29 ; N^+=4	H(m)	8.2	13.84 (0.65)	19.7	9.9	11.65 (0.59)	12.5
11	D (cm)	3.6	15.42 (1.21)	35.1	3	12.89 (0.37)	24.8
N=39; N^+=9	H(m)	3.6	13.09 (1.10)	46.8	4.8	11.73 (2.58)	36
Total	D (cm)	3.6	17.961 (0.47)	47.0	3	18.24 (1.28)	33
N=254; N^+=43	H(m)	3.6	14.37 (0.35)	74.0	5	13.7 (0.61)	24

Note: D and H stand separately for diameter at breast height and total tree height. S.d. is Std. Error of Mean. N^+ is the number of Validation data. Tree species: 1- Korean pine plantation; 2- Mongolian pine plantation; 3-Spruce; 4- Mono maple; 5- Manchurian ash; 6-aspen; 7-Mongolian oak; 8-Larch; 9-Fir; 10- Korean pine; 11- Dahurian birch

3. Compatible Tree Biomass Model

AffiliationThe traditional tree biomass equations were fitted separately for each component and total biomass by using of Equation (1). As a result, they were not compatible i.e. the sum of predicted for all components was not equal to the prediction for total tree biomass.

$$w = aD^b \; ; \tag{1}$$

Where w trees for biomass,a、b、and c are corresponding parameters.

Based on the total biomass model as restrictions,the compatible tree biomass equations for each component (stem, branches, foliages, and roots) were separately developed by using the theory of simultaneous equations and measure error mode with Equation(1).

In case of Equation (1), compatible biomass model was constructed as follows:

$$\begin{cases} w_s = a_1(x)D^{b_1} \\ w_b = a_2(x)D^{b_2} \\ w_l = a_3(x)D^{b_3} \\ w_r = a_4(x)D^{b_4} \\ w_t = c_0 D^{b_0} = a_1(x)D^{b_1} + a_2(x)D^{b_2} + a_3(x)D^{b_3} + a_4(x)D^{b_4} \end{cases} \tag{2}$$

Where w_s, w_b, w_l, w_r and w_t are biomass of stem, branch, foliage, root, and the total tree,separately;

$a_1(D), b_1, a_2(D), b_2$, $a_3(D), b_3$, $a_4(D)$, b_4, c_0 and b_0 are parameters to be estimated.

In Equation(3), $a_1(D), a_2(D), a_3(D), a_4(D)$ should be introduced as follows:

$a_1(D) = c_0 c_1 D^{b0} / (c_1 D^{b1} + c_2 D^{b2} + c_3 D^{b_3} + c_4 D^{b_4})$, $a_2(D) = c_0 c_2 D^{b0} / (c_1 D^{b1} + c_2 D^{b2} + c_3 D^{b_3} + c_4 D^{b_4})$

$a_3(D) = c_0 c_3 D^{b0} / (c_1 D^{b1} + c_2 D^{b2} + c_3 D^{b_3} + c_4 D^{b_4})$, $a_4(D) = c_0 c_4 D^{b0} / (c_1 D^{b1} + c_2 D^{b2} + c_3 D^{b_3} + c_4 D^{b_4})$

Let $r_1 = c_2 / c_1$, $r_2 = c_3 / c_1$; $r_3 = c_4 / c_1$; $k_1 = b_2 - b_1$; $k_2 = b_3 - b_1$; $k_3 = b_4 - b_1$

Then Equation (2) is transformed into as follows:

$$\begin{cases} w_s = c_0 D^{b_0} / (1 + r_1 D^{k_1} + r_2 D^{k_2} + r_3 D^{k_3}) \\ w_b = c_0 c_1 D^{k_1 + b_0} / (1 + r_1 D^{k_1} + r_2 D^{k_2} + r_3 D^{k_3}) \\ w_l = c_0 c_2 D^{k_2 + b_0} / (1 + r_1 D^{k_1} + r_2 D^{k_2} + r_3 D^{k_3}) \\ w_r = c_0 c_3 D^{k_3 + b_0} / (1 + r_1 D^{k_1} + r_2 D^{k_2} + r_3 D^{k_3}) \end{cases} \quad (3)$$

The parameter estimates of nonlinear simultaneous equations with measure error (Equation (3)) were performed using Newton-tang Algorithm implemented in ForStat 2.1 software (CAF, 2009). Goodness-of-fit of the alternative models were assessed on the basis of the statistics of the residual sum of squares, RSS, and the coefficient of determination (R2).

The validation procedures for traditional biomass models and the compatible biomass models developed in this paper were performed by using the following statistical criterions: the coefficient of determination (R2), sum square of error (SSE), Mean Bias (MB), and Mean Absolute Bias (MAB).

$$\mathrm{R}^2 = 1 - \sum (y_i - \hat{y}_i)^2 / \sum (y_i - \bar{y})^2 \;;\; \mathrm{MAB} = \frac{1}{n}\sum_{i=1}^{n} |y_i - \hat{y}_i| \,,\; \mathrm{MB} = \frac{1}{n}\sum_{i=1}^{n} (y_i - \hat{y}_i) \,;\; \mathrm{SSE} = \sum (y_i - \hat{y}_i)^2$$

4. Results and Discussion

The following is the results of fitting, estimating compatible biomass models.

Table 3 The Results of compatible biomass models fitting with Equation (1)

Species	Parameters estimated							
	c_0	b_0	r_2	k_2	r_3	k_3	r_4	k_4
KP plantation	24.14086	1.03140	0.000840	0.64832	0.21412	-0.042039	0.48287272	-0.0230
MP plantation	25.17351	1.00058	0.031350	0.07854	0.03978	-0.056128	0.14073115	0.10040
Spruce	17.38124	1.07325	0.005069	0.42985	0.03599	0.0487775	0.02062650	0.34267
Mono maple	19.12495	1.04872	0.002434	0.52693	0.00263	0.3412480	0.544307	-0.052
Manchurian ash	36.8943	0.95334	1.398060	-0.2455	3.336398	-0.41341	11.12461	-0.41102
Poplar	46.0008	0.94360	0.002397	0.41533	0.024524	0.02983	0.157821	0.09695
Mongolian oak	83.6952	0.91440	6.148357	-0.3639	3.102146	-0.48562	0.25736	0.06634
Larch	84.1796	0.88250	0.116760	0.07156	6.116512	-0.45477	0.43551	-0.02923
Fir	54.0066	0.91293	0.131415	0.01541	0.05126	0.100863	0.20059	0.07070
Korean pine	66.2385	0.90011	1.529737	-0.2375	0.04931	-0.011538	0.02810	0.24183
Dahurian Birch	33.28051	1.00933	0.017472	0.34220	0.01654	0.150447	2.94250	-0.20181

Table 4 the Results of compatible biomass models fitting with Equation (1)

species	statistical criterions									
	R^2				MAB	MB	SSE			
	WS	WB	WL	WR			WS	WB	WL	WR
KP plantation	.9871	.9250	.7748	.9463	5.95 E-11	4.046E-16	0.4890	0.1116	0.1140	0.1027
MP plantation	.8120	.6781	.8088	.8169	1.357E-10	3.155E-11	5.4137	1.1056	0.1214	0.8233
Spruce	.9923	.5997	.7060	.8981	3.037E-10	3.078E-11	0.1459	0.0394	0.0034	0.2728
Mono maple	.9966	.9698	.9594	.9806	5.001E-11	6.188E-12	0.0252	0.0194	0.0099	0.0392
Manchurian ash	.9918	.9350	.8052	.9130	4.119E-11	1.002E-11	0.1504	0.1113	0.0143	0.1878
Poplar	.8489	.6930	.7180	.7415	3.436E-11	1.447E-11	1.3944	0.1961	0.0447	0.4681
Mongolian oak	.9249	.9774	.772	.9422	8.328E-11	1.306E-11	0.2710	0.8691	0.0323	1.8315
Larch	.9259	.8685	.3467	.9266	1.926E-09	2.934E-12	1.9696	0.0150	0.0513	0.1033
Fir	.8933	.6868	.8782	.8317	1.515E-10	2.551E-11	3.1645	0.3131	0.3372	0.3409
Korean pine	.9723	.6930	.7499	.8264	9.317E-11	1.221E-11	0.1870	0.2560	0.0683	0.3155
Dahurian Birch	.9259	.8685	.6467	.92662	1.515E-10	2.551E-11	3.1645	0.3131	0.3372	0.3409

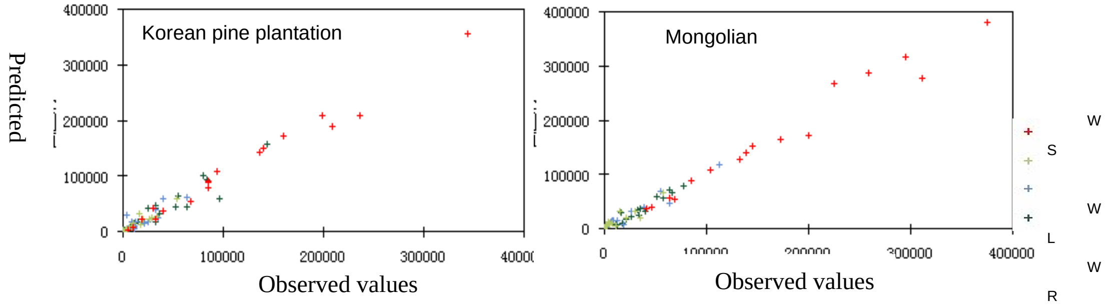

Figure 1 the comparison of predicted valuses-observed values

In Figure 1, Korean pine plantation and Mongolian pine are as delegates to show the predicted

The results of the compatible biomass developed above are as follows:

From Table 4, it was seen that the parameter estimation accuracy of compatible biomass equations (average R2=0.90, average SSE=0.0578) are relative ideal. Mean Bias (MB), and Mean Absolute Bias (MAB) of compatible biomass equations are much less than allowable deviation 0.00001. Moreover the compatible biomass equations could effectively overcome the defect of un-compatible of traditional biomass models, which achieved the purpose of the paper. As was shown in Table 1, traditional biomass models are un-compatible.

From Figure 1, the result can be received that the predicted effect of the compatible biomass model developed in this paper is very reasonable. So the compatible biomass model can be used to predicted forest biomass.

Considering the difficulty of collecting the biomass data and the biomass equation on D explained more than 87.88% of the biomass variance, this model uses only a single predictor of D and produces a range of prediction values closer to the upper and lower limits of the observed mean, and could be applied to forest biomass estimation in large-scale region.

The effects of fitting the models were better for stem (average R2=0.90) and root (average R2=0.88) and worst for branches (average R2=0.79) and foliage (average R2=0.71), whether traditional biomass models or the compatible biomass developed in the paper.

5. Conclusion

The conclusion is that the compatible biomass equations developed in the paper solved the un-compatible of traditional biomass models, which can be widely used in large-scale region of Heilongjiang Province, and the model is reliable to estimate forest biomass, so the research findings can be extrapolated for managing forests related to carbon balance; furthermore the paper provided the constructive method of establishing compatible biomass models for other region or nation. ubmission.

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