

Hyperbolic Sliding Mode Trajectory Tracking Control of Mobile Robot

Yaogang Ding^{1,2*}, Chongxin Liu^{1,2}, Shengmin Lu³ and Ziwei Zhu^{1,2}

¹State Key Laboratory of Electrical Insulation and Power Equipment, Xi'an Jiaotong University, Xi'an 710049, China

²School of Electrical Engineering, Xi'an Jiaotong University, Xi'an 710049, China

³Shaanxi Provincial Electric Power Design and Research Institute, Xi'an 710065, China

*Corresponding author

Abstract—The trajectory tracking control is performed for the kinematics model of the mobile robot. The switching term of the traditional sliding mode control law function contains the symbolic function. Because of its discontinuity, it is not suitable to use the switching function in the derivation occasion. In this paper, the hyperbolic tangent function is introduced into the control switching term, and the entire closed-loop system is analyzed and controlled for stability, so as to realize the global asymptotic stability of the error tracking system. The error trajectory phase diagram and control input signal before and after improvement are compared by MATLAB simulation, it is found that the improved control function is more smooth, the control time is reduced from 10 seconds to 0.9 seconds, the chattering phenomenon is also greatly reduced. In this way, the kinematic trajectory tracking control of the mobile robot system is better achieved.

Keywords—trajectory tracking control; mobile robot; sliding mode control law function; symbolic function; hyperbolic tangent function

I. INTRODUCTION

The emergence of intelligent robots has been around 60 years ago. The world's first intelligent robot was developed by Joseph Engelberg in 1959. The introduction of robots has attracted worldwide attention. Scholars from all over the world have invested in robot research. The great tide. Today, robots have been widely viewed as a production tool that can be used in indoor transportation, indoor cleaning, tourist attractions, exhibition halls, and other venues, and can help or replace humans to perform some difficult or more dangerous actions. Although the robot can perform various difficult movements, it does not have all kinds of emotions like humans. Therefore, the control of robots has become a key research direction. The following focuses on a study of a mobile robot[1-4].

The mobile robot in the narrow sense refers to the ground mobile robot. It can accomplish some relatively dangerous actions through mobile, such as seabed detection, pilotless driving and so on. It has great practical value in various industries such as industry, national defense and so on. At present, there are many mature methods in robot control, such as PID control[5-6], adaptive control[7-8], fuzzy control[9], neural network control[10], genetic algorithm control[11], sliding mode variable structure control[12-14] and so on. Especially the sliding mode control, because of its simple control algorithm, good robustness and high reliability and is

widely used in motion control, especially for use in precise systems where mathematical models can be established. The literature [1] studied the impact of collision when capturing target with two-armed space robot system and the stability control problem of the closed chain system formed later. The literature [2] proposed a sliding mode control based on grey theory for apple picking robot. The literature [3] has proposed a sliding-mode variable structure control based on RBF neural network for joint flapping problem of brewing turn-up robots. The literature [4] used a saturation function method to reduce the chattering phenomenon of sliding mode for mobile welding robots. The symbolic function is often used in the control switching function of most of the existing literatures. Although the chattering can be overcome limitedly, its disadvantage is that it is a discontinuous function, and it is not suitable for the occasion of derivation of the switching function. In this paper, the hyperbolic tangent function is used to replace the symbolic function for this problem. Using the comparison and MATLAB simulation results, the latter is more effective than the former to reduce chattering.

II. KINEMATICS EQUATIONS AND CONTROL OF MOBILE ROBOTS

A. Kinematics Equation of Mobile Robot

$$\begin{cases} \dot{x} = v \cos \theta \\ \dot{y} = v \sin \theta \\ \dot{\theta} = \omega \end{cases} \quad (1)$$

Where x and y are the positions of the mobile robot, θ is the angle between the robot forward and the x -axis, and v and ω are the linear and angular speeds of the robot.

B. Positional Sliding Mode Variable Structure Control

By designing the control law function to achieve x tracking x_d , y tracking y_d , where $[x_d, y_d]$ is the ideal motion trajectory, the tracking error system equation is

$$\begin{cases} \dot{x}_e = v \cos \theta - \dot{x}_d \\ \dot{y}_e = v \sin \theta - \dot{y}_d \end{cases} \quad (2)$$

and

$$\begin{cases} v \cos \theta = u_1 \\ v \sin \theta = u_2 \end{cases} \quad (3)$$

For $\dot{x}_e = v \cos \theta - \dot{x}_d$, we can take $s_1 = x_e$, then

$$\dot{s}_1 = \dot{x}_e = u_1 - \dot{x}_d$$

So the controller can be designed as follows:

$$u_1 = \dot{x}_d - k_1 s_1, (k_1 > 0) \quad (4)$$

Then $\dot{s}_1 = -k_1 s_1$, constructs the Lyapunov function as

$$V_x = \frac{1}{2} s_1^2 \text{ then } \dot{V}_x = s_1 \dot{s}_1 = -k_1 s_1^2 \leq 0$$

Thus x_e quickly converges to zero according to the index. The same can be designed

$$u_2 = \dot{y}_d - k_2 s_2, (k_2 > 0) \quad (5)$$

y_e can also quickly converge to zero according to the index.

C. Angle Sliding Mode Variable Structure Control

It is known from formula (3)

$$\theta = \arctan \frac{u_2}{u_1} \quad (6)$$

If θ and θ_d in equation (6) are exactly equal, then the control method of equation (4) and (5) is sufficient. However, the actual models are not equal, in the initial stage of control, this type of error may cause instability of the closed-loop system (2). So consider x as an ideal value, so

$$\theta_d = \arctan \frac{u_2}{u_1} \quad (7)$$

Where $\theta_d \in (-\frac{\pi}{2}, \frac{\pi}{2})$

The following design controller ω to achieve θ fast

track θ_d . Take $\theta_e = \theta - \theta_d$ and the sliding mode function is $s_3 = \theta_e$ thus $\dot{s}_3 = \dot{\theta}_e = \omega - \dot{\theta}_d$ so the controller can be designed as follows:

$$\omega = \dot{\theta}_d - k_3 s_3 - \eta_3 \operatorname{sgn} s_3 \quad (8)$$

Where $k_3 > 0, \eta_3 > 0$ so $\dot{s}_3 = -k_3 s_3 - \eta_3 \operatorname{sgn} s_3$

The Lyapunov function is $V_\theta = \frac{1}{2} s_3^2$ so

$$\begin{aligned} \dot{V}_\theta &= s_3 \dot{s}_3 = -k_3 s_3^2 - \eta_3 |s_3| \\ &\leq -k_3 s_3^2 = -2k_3 V_\theta \end{aligned}$$

So that θ converges exponentially to θ_d . It must be noted that since $\dot{\theta}_d$ is required to design the inner loop control, so θ_d must be a continuous value, which requires that the design of u_1 and u_2 cannot contain discontinuous switching functions.

D. Sliding Mode Control of Hyperbolic Tangent Function

Because the switching function term of the general sliding mode controller contains discontinuity symbol function and is not suitable for derivation, this paper considers the use of hyperbolic tangent function instead of the symbol function, which can effectively reduce chattering. Hyperbolic functions are defined as follows:

$$\tanh(kx) = \frac{e^{kx} - e^{-kx}}{e^{kx} + e^{-kx}} \quad (9)$$

Lemma 1 For any given x , there exists $k > 0$,

which makes the following inequalities set up:

$$x \tan(kx) \geq 0 \quad (10)$$

Lemma 2 (The Lagrange mean value theorem) let f be a function that satisfy the following: f is continuous on the closed interval $[a, b]$; f is differentiable on the open interval (a, b) . Then there is a number ξ in (a, b) such that $f(b) - f(a) = f'(\xi)(b - a)$.

Let the switching function $f(s) = \eta \operatorname{sgn}(s)$, the hyperbolic tangent function $g(s) = \eta \tanh(s)$, where

$s \in [-20, 20]$. The two contrast function images shown in Figure I.

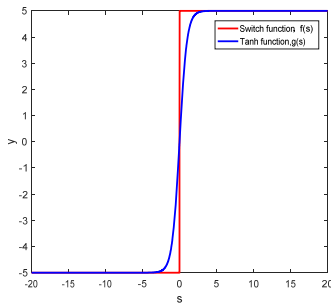


FIGURE 1. SWITCHING FUNCTION AND HYPERBOLIC TANGENT FUNCTION

It can be seen from the figure that the hyperbolic tangent function is smoother than the switching function because the hyperbolic tangent function is a continuously differentiable function. Then, it is introduced into the switching item of the switching controller. Equations (4) and (5) can be designed as the following controllers:

$$u_1 = \dot{x}_d - a \tanh(\varepsilon_1 x_e) \quad (11)$$

$$u_2 = \dot{x}_d - b \tanh(\varepsilon_2 x_e) \quad (12)$$

The following analysis of the closed-loop stability of the entire system, assuming that there is an ideal angle θ_d , the system (1) can be equivalently deformed as follows:

$$\begin{cases} \dot{x} = v \cos \theta_d + v(\cos \theta - \cos \theta_d) \\ \dot{y} = v \sin \theta_d + v(\sin \theta - \sin \theta_d) \\ \dot{\theta} = \omega \end{cases} \quad (13)$$

From equation (13), we can see that when θ and θ_d are not equal, the stability of the closed-loop system is bound to be greatly affected. Therefore, under ideal conditions, the control law is adopted. At this time, take $v \cos \theta_d = u_1$, $v \sin \theta_d = u_2$ and bring this with equation (11) and equation (12) into equation (13), so it can be obtained as follows:

$$\begin{cases} \dot{x} = \dot{x}_d - a \tanh(\varepsilon_1 x_e) + v(\cos \theta - \cos \theta_d) \\ \dot{y} = \dot{x}_d - b \tanh(\varepsilon_2 x_e) + v(\sin \theta - \sin \theta_d) \\ \dot{\theta} = \omega \end{cases} \quad (14)$$

Since $x_e = x - x_d$, $y_e = y - y_d$, further formula (14) can be simplified as follows:

$$\begin{cases} \dot{x}_e = -a \tanh(\varepsilon_1 x_e) + v(\cos \theta - \cos \theta_d) \\ \dot{y}_e = -b \tanh(\varepsilon_2 x_e) + v(\sin \theta - \sin \theta_d) \\ \dot{\theta} = \omega \end{cases} \quad (15)$$

E. Stability Analysis

First analyze the stability of x_e and consider the hyperbolic

cotangent function $\cosh(x) = \frac{e^{-x} + e^x}{2} \geq 1$, so

there is $\ln(\cosh(x)) \geq 0$, The Lyapunov function can be defined as follows:

$$V = a \ln(\cosh(\varepsilon_1 x_e)) + \frac{1}{2} \varepsilon_1 x_e^2 \quad (16)$$

Where $a, \varepsilon_1 > 0$, so that

$$\begin{aligned} \dot{V} &= a \frac{\sinh(\varepsilon_1 x_e)}{\cosh(\varepsilon_1 x_e)} \varepsilon_1 \dot{x}_e + \varepsilon_1 x_e \dot{x}_e \\ &= a \tanh(\varepsilon_1 x_e) \varepsilon_1 \dot{x}_e + \varepsilon_1 x_e \dot{x}_e \end{aligned}$$

And because of

$$\dot{x}_e = -a \tanh(\varepsilon_1 x_e) + v(\cos \theta - \cos \theta_d)$$

we can define $\xi_1 = a \tanh(\varepsilon_1 x_e)$, $\xi_2 = v(\cos \theta - \cos \theta_d)$

so $\dot{x}_e = -\xi_1 + \xi_2$ and the derivative of V is as follows:

$$\begin{aligned} \dot{V} &= \varepsilon_1 (-\xi_1 + \xi_2) \xi_1 + \varepsilon_1 x_e (-\xi_1 + \xi_2) \\ &= -\varepsilon_1 (\xi_1 - \frac{1}{2} \xi_2)^2 - a \varepsilon_1 x_e \tanh(\varepsilon_1 x_e) \\ &\quad + \frac{1}{4} \varepsilon_1 \xi_2 (\xi_2 + 4 x_e) \end{aligned}$$

From Lemma 1 $x_e \tanh(\varepsilon_1 x_e) > 0$, so that

$$\dot{V} \leq -\varepsilon_1 (\xi_1 - \frac{1}{2} \xi_2)^2 + \frac{1}{4} \varepsilon_1 \xi_2 (\xi_2 + 4 x_e)$$

From Lemma 2

$$\begin{aligned} |\cos \theta - \cos \theta_d| &= |\sin(\xi)(\theta - \theta_d)| \\ &= |\sin(\xi)| \cdot |\theta - \theta_d| \leq |\theta - \theta_d| \end{aligned}$$

It has also been proved that θ converges exponentially to θ_d , $\cos \theta$ also converges to $\cos \theta_d$, so that the

$\xi_2 = v(\cos \theta - \cos \theta_d)$ converges to zero. Therefore, there must exist $\delta > 0$ when $t \rightarrow \infty$, $\xi_2(\xi_2 + 4x_e) \rightarrow 0^-$

($\xi_2(\xi_2 + 4x_e) < 0$), so that

$$\lim_{t \rightarrow \infty} \dot{v} \leq -\varepsilon_1(\xi_1 - \frac{1}{2}\xi_2)^2 + \frac{1}{4}\varepsilon_1\xi_2(\xi_2 + 4x_e) < 0$$

Since when $t \rightarrow \infty$, $x_e \rightarrow 0$. With the same reason for $|\sin \theta - \sin \theta_d| \leq |\theta - \theta_d|$, $\sin \theta$ also converges exponentially to $\sin \theta_d$, when $t \rightarrow \infty$, we can obtain $y_e \rightarrow 0$, so the overall system is asymptotically stable.

III. NUMERICAL SIMULATION

The controlled object is formula (1), the initial position is taken as $(x_0, y_0, \theta_0) = (-2, 2, 0)$, and the standard instruction is $x_d = t$, $y_d = \sin(0.5x_d) + 1 + 0.5x_d$. Take $a = 3$

$\varepsilon_1 = 10$, $b = 3$, $\varepsilon_2 = 10$, $k_3 = 3$, $\eta_3 = 0.5$. The sliding mode control including the symbolic function and the sliding mode control including the hyperbolic tangent function are respectively adopted. Through the simulation of MATLAB, it can be seen that it takes about 10 seconds to achieve the synchronization using the former position tracking control, and only 0.9 seconds to achieve the synchronization using the latter position tracking control. It takes approximately 0.8 seconds for the tracking control of both control modes to achieve synchronization. It can be seen that the sliding mode control using the hyperbolic tangent function is much better than the sliding mode control of the symbol function when the angle tracking is in an ideal state, and the control effect is quite satisfactory. The simulation results are shown in Figure II and Figure III.

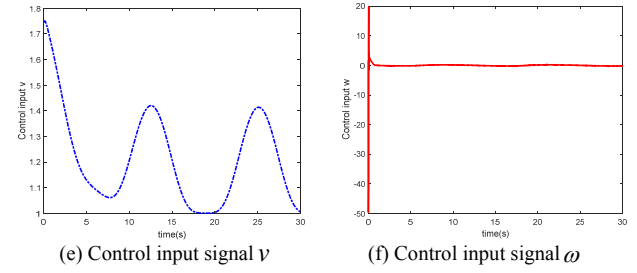


FIGURE II. TRAJECTORY TRACKING WITH SYMBOLIC FUNCTION

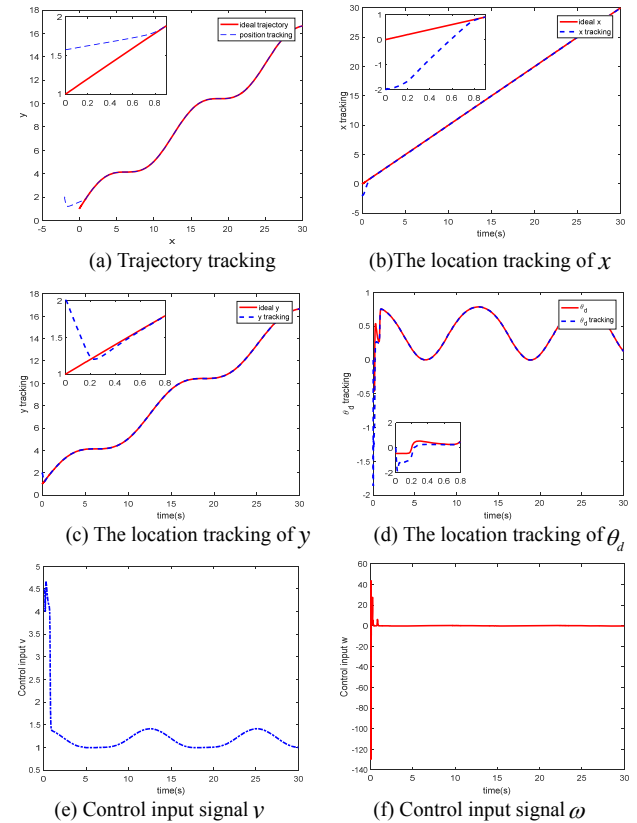


FIGURE III. TRAJECTORY TRACKING WITH HYPERBOLIC TANGENT FUNCTION

IV. CONCLUSION

In this paper, the sliding mode trajectory tracking control is performed for the kinematics model of mobile robots. The traditional sliding mode control switching function contains the symbolic function. Because of its discontinuity, it can't be practical in many occasions where it is necessary for the switching function to be differentiated. This paper replaces the switching function term with a continuous hyperbolic tangent function, combines the traditional sliding mode variable structure control to redesign the controller, and analyzes and demonstrates the stability of the error tracking system. Through MATLAB numerical simulation, it takes about 10 seconds for the traditional sliding mode control position tracking system to achieve synchronization, and the controller with the hyperbolic tangent function design is more superior. The position tracking system only needs 0.9 seconds to achieve synchronization. It

takes approximately 0.8 seconds for the tracking control of both control modes to achieve synchronization. It can be seen that the sliding mode control using the hyperbolic tangent function is much better than the sliding mode control of the symbolic function when the angle tracking is in an ideal state, and the control effect is quite satisfactory. Due to the continuity of the hyperbolic tangent function, the use of this switching function can form an ideal sliding mode on the switching surface, making it effectively weaken the chattering, and can be widely used in practice.

ACKNOWLEDGMENT

First of all, I would like to thank my tutor, Chongxin Liu. Without he's teachings, I could not complete the writing of this essay. Secondly, I would like to thank the senior members of the teaching and research section for their encouragement and support. Finally, we would like to thank the National Natural Science Foundation of China for grants from the Innovative Research Group Fund.

*Project supported by the Science Fund for Creative Research Groups of the National Natural Science Foundation of China(Grant No.51521065)

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