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A Novel Multi-level Wavelet Image Coding Algorithm by Full Sub-band Compressed Sensing

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***Abstract*—The performance of wavelet transform based image compressed sensing coding algorithms severely rely on the level of wavelet transform. To this end, this paper investigated the combination of mutil-level wavelet full sub-band in compressed sensing framework. Firstly, we constructed a full sub-band coefficient sparse vector of mutil-level discrete wavelet transform. Secondly, we designed a weight matrix to improve measurement matrix. Finally, sparsity vector was processed by compressed sensing to get the measured value. Compared with the exiting algorithms, the experimental results of the proposed algorithm show that the PSNRs of reconstructed images is improve up to 1~2dB under the same objective quality.**

Keywords—image coding; wavelet transform; compressed sensing; full sub-band coefficient sparsity vector

I. INTRODUCTION

Storage and transmission of image are basis problems in communication field, traditional ways are base on the Nyquist sampling theorem, in which signal usually is sampled firstly and then compressed, and the signal sampling and processing rate must be higher than the Nyquist frequency. However, only partially sampled data are used in the compression process, this causes a waste of sampling resources. Considering the shortcomings of the traditional sampling theorem, Donoho proposed compressed sensing(CS) [1]. In CS, for sparse signal or signal which can be sparse represented, signal sampling and signal compression process can be combined into one process, and then the signal reconstruction process is a nonlinear algorithm. The theory of compressed sensing has made great breakthroughs since it was put forward. Lu Gan proposed block compressed sensing(BCS) algorithm at 2007[2], this algorithm used the block-by-block processing mechanism, the sampling algorithm has very low complexities compare to conventional compressed sensing. Recently, M.Kalra combined compressed sensing theory with wavelet and vector quantization[3]. Yanxia Rong[4] analyze the characteristics of image wavelet transform coefficients, proposed a new algorithm based on wavelet transform and image blocking, which successfully applied to address image block effect. Although complexity and image block effect of wavelet transform based image compressed sensing, which has been improved greatly, yet these algorithms severely rely on the level of wavelet transform are still a problem.

In this paper, we use distribution of quad-tree, which is a special characteristic of high frequency coefficients of mutil-level discrete wavelet transform, to establish a full sub-band coefficient sparse vector, designing weight matrix base on the different importance of frequency coefficients in different levels during reconstructed image, using random measurement matrix of compressed sensing point multiplication weight matrix to get the improved measurement matrix.

II. INTRODUCTION TO COMPRESSED SENSING THEORY

Compressed sensing contains three main technologies[5]: including sparse representation, non-related measurements, reconstructing signal with nonlinear optimization. The sparse representation of signal is the precondition of compressed sensing, and the non correlation random measurement is the key of compressed sensing. The nonlinear optimization reconstruction of the signal is a unique method for reconstructing signal. The framework of image compression sensing is shown in Figure I.

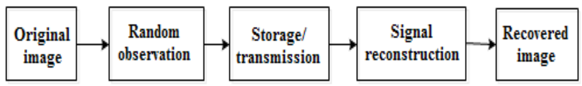

FIGURE I. FRAMEWORK OF IMAGE COMPRESSION SENSING

Considering a one dimensional signal **x**($x \in R^n$) with a length of n, and it is a sparse signal under an orthogonal basis ***Ψ***, orthogonal linear decomposition of signal is as follow:

$$x = \Psi a = \sum_{i=1}^{n} a_i \Psi_i \tag{1}$$

where $a = [a_1, a_2, \Lambda\ , a_{i\text{-}1}, a_i]$ is $n \times 1$ column vector ***Ψ***, $\boldsymbol{\Psi}_i$ is $n \times 1$ basis of orthogonal basis ***Ψ***, ***a*** is the conversion coefficient of the signal under basis ***Ψ***. If there is only a few of coefficients are non-zero in ***a***, other coefficients are zero or approximately zero, we call signal ***x*** is sparse. Using a measurement matrix **Φ**, which has no relativity to the transform basis ***Ψ***, to perform adaptive linear projection on signal ***x***, and get the measurement value **y**:

$$\mathbf{y} = \mathbf{\Phi}\, \mathbf{x} = \mathbf{\Phi \Psi\, a} \tag{2}$$

where **y** is $m \times 1$ vector, which is generally used for storage or transmission, its matrix description is illustrated in Figure II.

The formula(2) combines signal sampling and compression together when use to calculated measured value **y**.

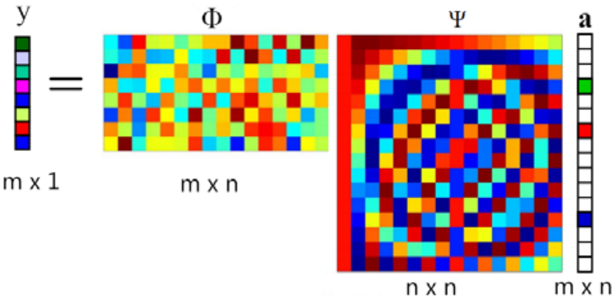

FIGURE II. MATRIX DESCRIPTION OF MEASUREMENT VALUE ***Y***

According to Figure II, measurement value ***y*** is the result of signal ***x*** linearly measure in the low dimensional space. During reconstruction, conversion coefficient ***a*** is reconstructed from measurement value ***y***, calculating sparse vector ***a*** by equation(2) becomes an optimal linear programming problem. Since the dimension of ***y*** is much smaller than the dimension of ***a***, the number of equations is much less than the number of unknown variables. So the equation(2) is an ill-condition equation, there will be have infinite number of solutions. In order to solve this problem, measurement matrix ***Φ*** and sparse basis ***Ψ*** should satisfy restricted isometry property(RIP)[6], the equation of RIP is as follows:

$$1-\varepsilon \leqslant \frac{\| \Phi \Psi a \|_2}{\| a \|_2} \leqslant 1+\varepsilon \tag{3}$$

where ε is a positive number less than 1. The process of rebuilding sparse coefficient ***a*** from measured value ***y*** is the optimization problem under l_0 norm:

$$\begin{aligned} &\hat{a} = \arg\min \| a \|_0 \\ &s.t. y = \Phi \Psi a \end{aligned} \tag{4}$$

where $\| a \|_0 = \sum_{i=1}^{n} | \operatorname{sgn}(a_i) |$, representing the non-zero component of **a**. Solving l_0 norm is an NP-hard problem and can be transformed a optimization problem under l_1 norm. For solving the problem of l_1 norm optimization, generally use the matching pursuit(MP), orthogonal matching pursuit (OMP)[7], basic pursuit(BP) and project onto convex sets(POCS)[8].

III. CONSTRUCTION OF SPARSE VECTOR AND IMPROVED MEASUREMENT MATRIX

A. Construction of Sparse Vectors

Wavelet transform is a localized time-frequency analysis method with many advantages, such as time-frequency analysis, direction selection and muti-resolution analysis. In case of the image process with wavelet transform, image is decomposed into a low-frequency sub-band LL and three high-frequency sub-band. Three high frequency sub-band are vertical sub-band LH, horizontal sub-band HL, and the diagonal sub-band HH.

High-frequency sub-bands contain details of edges, contours and textures in the image. The coefficients of High-frequency are equal to zero or close to zero, which show remarkable sparse. The low frequency sub-band is the approximation of the original image, and it can be decomposed by wavelet transform again.

In the mutil-level wavelet transform, SPIHT[9] algorithm indicate that the coefficients of high-frequency sub-band correspond to low frequency coefficients of the low frequency sub-band. The sparseness of the high frequency sub-band coefficients in high-level wavelet transform is lower than in low-level wavelet transform. The parent coefficients of the high frequency sub-band include the descendant coefficients of the higher frequency sub-band of the next level wavelet transform, this relationship is show in Figure III.

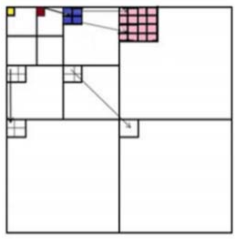

FIGURE III. THE RELATIONSHI BETWEEN HIGH FREQUENCY COEFFICIENTS OF WAVELET TRANSFORM IN ADJACENT LEVEL

In order to present the method of constructing sparse vector based on the above theory, selecting a matrix with a size of 8×8 to carry out the three-level wavelet transform the coefficients is show in Figure IV, then use the coefficients of the wavelet transform to construct sparse vector. Three sparse vectors with dimension of 22 are constructed. Three sparse vectors are:

{1,2,5,17,18,19,20,6,21,22,23,24,7,25,26,27,28,8,29,30,31,32},

{1,3,9,33,34,35,36,10,37,38,39,40,11,41,42,43,44,12,45,46,47,48},

{1,4,13,49,50,51,52,14,53,54,55,56,15,57,58,59,60,16,61,62,63,64}.

The coefficient 1 at low frequency position appears three times in sparse vector.

1	2	5	6	17	18	21	22
3	4	7	8	19	20	23	24
9	10	13	14	25	26	29	30
11	12	15	16	27	28	31	32
33	34	37	38	49	50	53	54
35	36	39	40	51	52	55	56
41	42	45	46	57	58	61	62
43	44	47	48	59	60	63	64

FIGURE IV. 8×8 MARTIX

According to the above analysis, for the image with the size of $m\times n$ pixels, and the level of the wavelet transform is B, the steps of constructing sparse vectors in this paper are as follows:

1) Performing B-level wavelet transform on the original image with $m\times n$ pixels.

2) Getting Sparse coefficient matrix by preorder traversal a quad-tree structure, which consists of a coefficient of high-frequency sub-band and its all descendants' coefficient in highest level wavelet transform.

3) The low frequency coefficients in low-level wavelet transform, which is correspond to the coefficients of the high frequency sub-band in the highest-level wavelet transform. The low frequency coefficients in low-level wavelet transform are inserted to first position of sparse coefficient vector separately, that is constructed by three high frequency sub-band, and the sparse coefficient vector is converted to sparse vector. The dimension of each sparse vector is:

$$n=1+\sum_{i=1}^{B}(4^{i-1}) \tag{5}$$

The total number of the sparse vector is:

$$V=3\times(\frac{m}{2^B})\times(\frac{n}{2^B}) \tag{6}$$

B. Design of Weight Matrix and Measurement Matrix

The function of a measurement matrix is equivalent to a sensor in compress sensing. The main function of measurement matrix is compress sparse vectors to get the observation data.

Figure V is the original Lena image with a size of 512×512 pixels. The transformation coefficient matrix of the five level wavelet transform of the Lena image is shown in Figure VI, the size of the low frequency region is 1/1024 of the original image, and can acknowledge that the content of the low frequency sub-band is very similar to the original image. So the low frequency sub-band is very important while rebuilding the original image. Meanwhile, the importance of sub-band of high level wavelet transform is higher than that in low level wavelet transform when rebuild the original image.

FIGURE V. LENA IMAGE

FIGURE VI.COEFFICIENT MATRIX OF FIVE-LEVEL WAVELET TRANSFORM

For the two adjacent high frequency sub-band, the energy of the high frequency sub-band in high-level wavelet transform is about $\sqrt{2}$ as much as that in the low-level wavelet transform.

Therefore, we can assign different weights to the coefficients of each level, the highest sub-band coefficient is set to 1.The weights of the sub-band in each level are $1,\frac{1}{\sqrt{2}},\frac{1}{2},\frac{1}{2\sqrt{2}},\frac{1}{4}$ when the level of wavelet transform is 5.Designing the weight matrix’s steps are as same as constructing a sparse vector. The steps of designing measurement matrix are as follows:

1) Generating a random measurement matrix($m\times n$);

2) Using weight matrix point multiplication random measurement matrix to get a new matrix.

3) Normalizing matrix obtained in step2, and the improved measurement matrix is obtained.

In the encode, we transform the image to the sparse domain by mutil-level wavelet transform, and get the sparse coefficient. Sparse vectors are constructed by the structure characteristics of wavelet transform, and the measured values are obtained after measurement data is compressed by improved measurement matrix.

In the decoder, reconstruction of sparse vectors from measured values by orthogonal matching pursuit (OMP). In section 3.1, low frequency sub-band coefficients are repeatedly inserted into the first position of the sparse vector constructed by different high frequency sub-band, so extracting the first value of all the sparse vectors is the result of low frequency coefficient value. Each low-frequency coefficient has three values, which from the sparse vectors of HL, LH, and HH sub-band. Reconstructed low frequency sub-band coefficients is average of the three values. High-frequency sub-band coefficients are obtained by the inverse operation of sparse vectors. Then using the high and low frequency coefficient to generate coefficient matrix for inverse wavelet transform to obtain a reconstructed image. The proposed algorithm frame is shown in Figure VII.

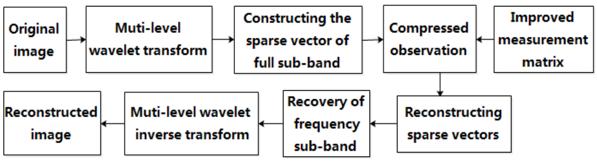

FIGURE VII. PROPOSED ALGORITHM FRAME

IV. EXPERIMENTS

The experiment is carried out using MATLAB 2013a on a 2.5GHz laptop computer. In this experiment, CDF9/7 wavelet is selected and the levels of transform is 5. The original image of the experiment are Lena, Plane, Woman, showing in Figure VIII, all the pictures are 512×512 pixels gray-level image.

FIGURE. VIII. ORIGINAL IMAGES OF THE EXPERIMENT

The measurement matrix uses a random Gauss matrix with zero mean and $1/n$ variance (n is the length of one-dimensional signal), and the reconstruction image is orthogonal matching tracking (OMP) algorithm. Using mean square error MSE, and peak signal-to-noise ratio PSNR as evaluation criteria for this experiment, The formula for calculating MSE, PSNR are as follows:

$$MSE = \frac{1}{m \times n} \sum_{i=1}^{m} \sum_{i=1}^{n} (f(i,j) - f'(i,j))^2 \tag{7}$$

$$PSNR = 10 \times \log_{10} \frac{255^2}{MSE} \tag{8}$$

The PSNR of reconstructed image of the proposed algorithm at different measurement rates(MR) is shown in Table I.

TABLE I. RESULTS OF PSNR IN DIFFERENT MEASUREMENT RATES

MR / PSNR / IMG	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45
Lena	26.88	27.75	28.40	29.18	29.97	30.67	31.48	32.75
Plane	23.12	24.33	25.29	26.47	27.18	28.05	28.89	29.73
Woman	29.43	30.47	31.26	32.44	33.41	34.66	35.81	36.74
Crowd	22.15	23.32	24.37	25.33	26.47	27.51	28.86	30.25

With the increase of measurement rate, the quality of image reconstruction is increasing. When the measurement rate is 0.3, the growth tends to be stable. So we chose measurement rate 0.3 for showing reconstruction effect of the proposed algorithm. Figure IX is the constructed images, and the reconstructed image with good visual effect.

FIGURE IX. CONSTRUCTED IMAGES OF THE ORIGINAL IMAGES

Table II is the PSNR comparison between the proposed algorithm and the algorithms in [3] and [4] at the same measurement rate. Compared with contrast algorithm, the proposed algorithm improves 1~2dB at the same measurement rate.

TABLE II PSNR VALUES FOR DIFFERENT ALGORITHMS AT THE SAME MEASUREMENT RATE

MR / PSNR / Algorithm	0.1	0.2	0.3	0.4	0.5
Algorithm in this paper	26.88	28.40	29.97	31.48	33.94
Kalra's algorithm[3]	24.21	25.71	28.14	29.45	31.81
Yanxia Rong'algorithm[4]	26.18	27.43	28.83	30.38	32.76

V. CONCLUSION

In order to improve performance of image compressed sensing coding algorithms, image compressed sensing coding algorithm based on mutil-level wavelet full sub-band is proposed. Compared with the traditional method of constructing sparse vectors, this algorithm does not need to preserve the low-frequency part of wavelet transform, and compresses all the transform coefficients. Designing the weight matrix to distinguish the importance of different coefficients. Through MATLAB simulation, the wavelet transform used by this algorithm has a lower number of levels, and the reconstructed image has a better peak-to-noise ratio under the same conditions.

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