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# Research of Improved Generalized Iteration Shrinkage Algorithm

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**Keywords:** generalized iterative shrinkage algorithm, $l_p$ regularization, image restoration, gradient acceleration operator.

**Abstract.** Generalized iterative shrinking algorithm combined with the contraction operator and the gradient operator of the fitting term can easily solve the $l_p$ regularized non-convex optimization problem and has a very good ability of image restoration. However, the gradient descent speed in the generalized iterative shrinking algorithm is slow, which limits the convergence rate of the algorithm. To solve this problem, a Nesterov gradient acceleration operator is introduced and an improved generalized iterative shrinkage algorithm is proposed. The improved algorithm accelerates the gradient descent in each iteration, speeding up the convergence of the algorithm. Experimental results show that compared with the generalized iterative contraction algorithm, the improved algorithm has a great improvement in visual effects and peak signal-to-noise ratio. And it has a faster convergence speed.

## 1. Introduction

Image restoration is an important branch of image processing. Many key technologies have been studied at home and abroad. In fact, image restoration involves three aspects of the image degradation model of the image, the image restoration algorithm and the recovery image evaluation criteria. Different imaging models, problem spaces, optimization rules and methods can lead to different recovery algorithms and apply to different application areas. The existing restoration methods can be summarized as the following types of deconvolution restoration algorithms, linear algebra recovery, image blind deconvolution algorithms, etc. Other restoration methods are mostly derivatives and improvements of these three types.

Compared to $l_1$ norm regularization, a sparser solution can be obtained by regularizing the $l_p$ norm. Therefore, the image de-restoration problem based on sparse representation is generally transformed into an optimization problem with a regularization term of $l_p$ norm. Although the $l_p$ norm has continuity, the non-convexity of $l_p$ regularization makes it difficult to solve. Based on the ISTA algorithm [1] for solving the $l_1$ regularization problem, Zuo extended the soft threshold operator and combined GST algorithm, and proposed the GISA algorithm [2,3] to solve the problem of $l_p$ non-convex regularization. The GISA algorithm [3] is theoretically more reliable than other algorithms, easier to understand and more efficient to implement, and it can converge to a more accurate solution.

For the image restoration problem, this paper improves the gradient descent method in the generalized iterative shrinkage algorithm, accelerates the convergence of the algorithm and reduces the computational complexity. Finally, using the improved algorithm for image restoration experiments, combined with the experimental results to summarize the superiority of the improved algorithm.

## 2. Generalized Iterative Shrinkage Algorithm

The goal of image restoration is to use observation images to estimate real images [4]. In this type of problem, the common image degradation process can be modeled as the following formula.

$$y = Ax + b \tag{1}$$

Where y is the observation image, x is the real image, and A is the degenerative function. When A is a unit matrix, the model becomes an image denoising problem; when A is a fuzzy matrix, the model becomes an image deblurring problem; when A is an under-sampled matrix, the model becomes a super-resolution problem of the image.

Image restoration is to estimate the real image x from the observation image y. This problem can be transformed into a non-convex optimization problem for solving $l_p$ regularization.

$$\min_x \frac{1}{2}\|Ax - y\|_2^2 + \lambda \|x\|_p^p \tag{2}$$

$\frac{1}{2}\|Ax - y\|_2^2$ is fidelity term that characterizes the fidelity between the observed image and the real image. $\|x\|_p^p$ is regularization term that characterizes the constraints on the real image. λ is regularization parameter.

For the non-convex nature of the $l_p$ norm, generalized iterative shrinking algorithm can solve the problem (2). The core of the algorithm is to find a soft threshold contraction operator similar to the ISTA algorithm. The generalized threshold contraction operator needed to solve the regularization of the $l_p$ norm is deduced by analytical method, and the corresponding algorithm GST [3] is given.

Algorithm GST

Input: y, p,J, $\lambda$

$\tau_p^{GST}(\lambda) = (2\lambda(1-\mathrm{p}))^{\frac{1}{2-p}} + \lambda p(2\lambda(1-\mathrm{p}))^{\frac{p-1}{2-p}}$
if $|y| \le \tau_p^{GST}(\lambda)$
  $\mathrm{T}_p^{GST}(\mathrm{y};\lambda) = 0$
else
  $k = 0, x_k = |y|$
  Iterate on k=0, 1…, J
    $x_{k+1} = |y| - \lambda p(\mathrm{x}_k)^{p-1}$
    $k \leftarrow k+1$
  $\mathrm{T}_p^{GST}(\mathrm{y};\lambda) = \mathrm{sgn}(\mathrm{y})\,\mathrm{x}_k$
  end.

Output: $\mathrm{T}_p^{GST}(\mathrm{y};\lambda)$

The GISA algorithm is an iterative algorithm that uses a gradient descent algorithm for each iteration, followed by a generalized threshold contraction step. The gradient descent algorithm used is the following formula.

$$x_k = x_{k-1} - \xi \nabla f(\mathrm{x}_{k-1}) \tag{3}$$

Thus, the GISA algorithm for solving problem (2) is:

$$\begin{cases} x_{k+0.5} = x_k - \xi A^T(\mathrm{Ax}_k - \mathrm{y}) \\ x_{k+1} = GST(\mathrm{x}_{k+0.5}, \xi\lambda, \mathrm{p}, \mathrm{J}) \end{cases} \tag{4}$$

Where the initial value of $\xi$ is $\|A\|^{-2}$. From the experimental results, J=2 or J=3 is appropriate.

## 3. Decrease of Acceleration Gradient

Although the GSA algorithm uses a new soft-threshold operator, it is a generalization of the ISTA algorithm. Because the traditional Nesterov gradient [5] descent method is used in the algorithm, the convergence speed of the algorithm is slow. Based on the GISA algorithm, the acceleration operator in the FPGM-OCG algorithm can accelerate the gradient descent and improve the convergence speed of the algorithm.

Compared with the FISTA algorithm, the FPGM-OCG algorithm [6] can achieve the $O(1/\mathrm{N}^{\frac{3}{2}})$ gradient descent rate, while the FISTA algorithm can only achieve the $O(1/\mathrm{N})$ gradient descent rate. First, this paper gives the definition of $\mathrm{t}_i$ and $T_k$.

$$t_i = \begin{cases} 1, & \mathrm{i}=0, \\ \dfrac{1+\sqrt{1+4t_{i-1}^2}}{2}, & \mathrm{i}=1,\cdots,\left\lfloor \dfrac{N}{2} \right\rfloor -1, \\ \dfrac{N-i+1}{2}, & \mathrm{i}=\left\lfloor \dfrac{N}{2} \right\rfloor,\cdots,N-1, \end{cases} \tag{5}$$

$$T_k = \sum_{i=0}^{k} t_i \tag{6}$$

Then, an iterative form of the gradient acceleration factor in the FPGM-OCG algorithm is given.

$$x_{i+1} = p_L(\mathrm{y}_i)$$

$$\mathrm{t}_{i+1} = \begin{cases} \dfrac{1+\sqrt{1+4t_i^2}}{2}, & i=1,\ldots,\left\lfloor \dfrac{N}{2} \right\rfloor -2 \\ \dfrac{N-i}{2}, & i=\left\lfloor \dfrac{N}{2} \right\rfloor -1,\ldots,N-2 \end{cases} \tag{7}$$

$$y_{i+1} = x_{i+1} + \frac{(\mathrm{T}_i-\mathrm{t}_i)\,\mathrm{t}_{i+1}}{t_i T_{i+1}}(\mathrm{x}_{i+1}-\mathrm{x}_i) + \frac{(\mathrm{t}_i^2-\mathrm{T}_i)\,\mathrm{t}_{i+1}}{t_i T_{i+1}}(\mathrm{x}_{i+1}-y_i), \quad \mathrm{i}<\mathrm{N}-1$$

Therefore, we propose an improved GISA algorithm, which is denoted as IGISA algorithm.

Algorithm IGISA

**input** : y, $\lambda$, p , J

**1.** Initialize $x_0$, $\xi = \|A\|^{-2}$

2. for k=0:N-1

**3.** $x_{k+0.5} = x_k - \xi A^T(\mathrm{A}x_k - \mathrm{y})$

**4.** $\mathrm{x}_{k+1} = GST(\mathrm{x}_{k+0.5}, \xi\lambda, \mathrm{p}, \mathrm{J})$

**5.** $$\mathrm{t}_{k+1} = \begin{cases} \dfrac{1+\sqrt{1+4(\mathrm{t}_k)^2}}{2}, & k=1,\ldots,\left\lfloor \dfrac{N}{2} \right\rfloor -2 \\ \dfrac{N-i}{2}, & k=\left\lfloor \dfrac{N}{2} \right\rfloor -1,\ldots,N-2 \end{cases}$$

**6.** $$z_{k+1} = x_{k+1} + \frac{(\mathrm{T}_k-\mathrm{t}_k)\,\mathrm{t}_{k+1}}{\mathrm{t}_k\,T_{k+1}}(\mathrm{x}_{k+1}-\mathrm{x}_k) + \frac{(\mathrm{t}_k^2-\mathrm{T}_k)\,\mathrm{t}_{k+1}}{\mathrm{t}_k\,T_{k+1}}(\mathrm{x}_{k+1}-z_k), \quad \mathrm{k}<\mathrm{N}-1$$

**End**

**Output:** $x_N$

## 4. Numerical Experiments

Gaussian blur, uniform blur, and motion blur were separately performed on the test images, and Gaussian noise with a standard deviation of 0.6 was added to generate the observed image y required for the experiment. The Gaussian fuzzy kernel has a support area of 9 × 9 and a standard deviation of 9; the support domain size of the uniform fuzzy kernel is 9 × 9; the length of the motion blur kernel is 15, and this angle is 9°.The initial input to the IGISA algorithm is as follows: p=1, J=3, $\lambda = 0.001$ .During the experiment, the value of the parameter $\lambda$ is manually adjusted according to the experimental results so that the image restoration can achieve the best effect.

In order to illustrate the acceleration effect of IGISA algorithm on GISA algorithm, the experimental image is restored by IGISA, FISTA (Fast Iterative Shrinkage-Thresholding Algorithm) [7] and GISA, and the recovery results of the three algorithms are compared.

(a)Ideal image (b) Blurred image

(c) FISTA (d) GISA (e) IGISA

Fig 1. Lena's original image, blurred image, and three algorithms to restore the image

This article compares time, SNR improvement, and mean square error. ISNR and MSE are defined as:

$$\begin{cases} ISNR = 10\lg\dfrac{\|y-x\|^2}{MSE \times N} \\ MSE = \dfrac{\|x - x_{k+1}\|_2^2}{N} \end{cases} \tag{8}$$

The number of iterations of the three algorithms was set to 100, and the image was run 20 times. The average value of the resulting data was used as the final experimental result.

Table 1. Observed image (gaussian blur) restoration results

| Algorithm | Time | MSE | ISNR |
|---|---|---|---|
| GISA | 14.38 | 114.32 | 5.78 |
| FISTA | 6.88 | 89.14 | 4.93 |
| IGISA | 4.53 | 74.95 | 6.63 |

Table 2. Observed image (uniform blur) restoration results

| Algorithm | Time | MSE | ISNR |
|---|---|---|---|
| GISA | 15.20 | 113.07 | 6.67 |
| FISTA | 7.33 | 86.10 | 5.83 |
| IGISA | 4.43 | 72.64 | 6.06 |

Table 3. Observed image (motion blur) restoration results

| Algorithm | Time | MSE | ISNR |
|---|---|---|---|
| GISA | 7.27 | 58.63 | 9.27 |
| FISTA | 15.22 | 74.69 | 8.43 |
| IGISA | 4.58 | 45.12 | 10.46 |

## 5. Summary

This paper uses the $l_p$ regularization method to solve the image deblurring problem and presents a fast algorithm for solving the $l_p$ regularized image deblurring model. Based on the generalized iterative shrinkage algorithm, this algorithm improves the gradient descent method, accelerates the gradient descent by the acceleration factor, and accelerates the convergence of the algorithm. And through the numerical experiments, different types of fuzzy images were tested. Through the restoration of the observed images, the effectiveness and superiority of the IGISA algorithm were verified.

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