Theoretical Investigation of the Superiority of Dropwise over Film Condensation

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Abstract—In this paper, we want to research on the superiority of dropwise condensation. First, the concept of interlayer convection was presented. Then, a mathematical model about the transition from film condensation to dropwise condensation was established. It is confirmed in theory that heat transfer coefficients for dropwise condensation are larger than those for film condensation and that the ratio can even be more than an order of magnitude. The effects of the change of drop quantity on the advantage of dropwise convective heat transfer are shown by drawing an instance curve.

Keywords—interlayer convection; dropwise condensation; film condensation; mathematical model

I. INTRODUCTION

Heat transfer enhancement has always been a key presence in refrigeration, aerospace, electronics and other fields. More and more researchers have kept an eye on the dropwise condensation because of the high heat transfer rate. According to the book *Fundamentals of Heat and Mass Transfer*, "In terms of maintaining high condensation and heat transfer rates, droplet formation is superior to film formation. In dropwise condensation, transfer rates that are more than an order of magnitude larger than those associated with film condensation may be achieved."[1] However, there are only physic explanations for this phenomenon. No quantitative analysis was given on the superiority of droplet formation convection.

In the existing researches, researchers have mainly studied about the methods to realize droplet formation and heat transfer for dropwise condensation [2-3]. For example, various physical or chemical methods were discovered to change the characteristics of the surface to realize droplet formation [4]. In addition, spray cooling, as a way of droplet formation convection, having high heat dissipation capacity, is now a popular research object [5-7]. When studying the dropwise condensation, researchers used to establish a single droplet model and consider the conductive performance of a single droplet [8-9]. However, few researchers established the model of the relationship between dropwise condensation and the film condensation. There is also little theoretical investigation of the superiority of dropwise over film condensation.

In this paper, the superiority of dropwise over film condensation is investigated in term of convective heat transfer. The concept of interlayer convection is presented. Based on it, a mathematical model about the transition from film condensation to dropwise condensation was established. In addition, the quantitative analysis on the superiority of droplet formation convection is presented. The droplet formation convection change with the droplet quantities is also presented in a diagram.

II. INTERLAYER CONVECTION

A. Concept and Significance

Convective heat transfer refers to the heat transfer process between the fluid and the surface when the fluid flows through the surface of an object [10]. When a glass of water is sprinkled on the table, the air flow drives the water film layer. The heat transfer process between the flowing water film layer and the table is convective heat transfer. However, the air flow above the water film layer cannot be neglected. In order to analyze the heat transfer performance of the overall phenomenon, the concept of **interlayer convection** is introduced, that is, a liquid layer is set between the air flow and the solid layer. The liquid layer can be continuous or discrete. Thus, the interlayer convection is divided into two forms: the film interlayer convection and the dropwise interlayer convection.

Researchers used to make some assumptions to investigate film condensation [1]. For example, the shear stress at the liquid–vapor interface is assumed to be negligible, and momentum and energy transfer by advection in the condensate film are also assumed to be negligible. It follows that heat transfer across the film occurs only by conduction. However, in this paper, by introducing the concept of interlayer convection, heat transfer across the air flow and the film can both be explained by convection.

B. Heat Transfer Rate in the Interlayer Convection

As shown in Figure 1, a layer of liquid film is laid on the plate, and gas flows over the liquid film, which is so called interlayer convection. Laminar flow and constant properties are assumed for the liquid film because of the low velocities associated with the film compared with the gas flow. Temperature of the liquid film is assumed to be stable, which means no thermal accumulation in the film layer. Thus, an equation can be obtained, $q_{g-l} = q_{l-s}$. In this paper, the heat flux between gas and the liquid film is focused on. It can be formulated as

$$q_{g-l} = h_g \left(T_g - T_0 \right). \tag{1}$$

q_{g-l} -- heat flux between gas and liquid, W/m^2 . q_{l-s} -- heat flux between liquid and cold surface, W/m^2 . k_g -- thermal conductivity of the gas layer, $W/(m \cdot K)$. Nu_x -- Nusselt number. T_g -- gas temperature, $°C$. T_0 -- liquid temperature, $°C$.
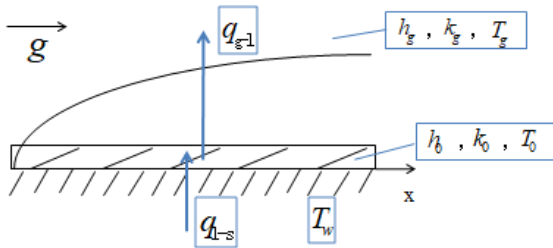

FIGURE I. SKETCH OF HEAT TRANSFER RATE IN THE INTERLAYER CONVECTION.

III. DROPWISE INTERLAYER CONVECTION

A. Mathematical Model

Whether in laminar or turbulent convection, heat transfer rate decreases along the flow direction, in which case $dh_x / dx < 0$ (shown in Figure 2.A). In order to increase the average coefficient of convective heat transfer along the flow direction, the film layer is incised (shown in Figure 2.B). The film layer is discretized so that the heat transfer coefficient of each piece of the film layer maintains in the optimal segment.

The mathematical model is established by making assumptions related to droplets.

- The increase of the average heat transfer coefficient is assumed in one dimension. The investigation is only about the heat transfer between gas and the liquid film. It has nothing to do with the height of droplets.

- The interface between gas and droplet is approximately assumed to be a plate on the premise of the discrete characteristics.

- In the model system, droplets are assumed to be uniform in size and separate from each other. The combination between droplets is negligible.

The details of discrete methods are shown in Figure 3. The film layer is separated into M pieces ($M > 1$), in which the quantity of the droplets is M_0 . The first piece of the liquid film must be a droplet. The length of each droplet is x_0 , and the length of the whole liquid film is $\bar{x} = Mx_0$. According to the assumptions, the heat transfer area in this paper is $A = \bar{x}$. Now, the transition from film condensation to dropwise condensation is obtained mathematically.

The range of the droplets quantity is discussed when the mathematical model is matched with actual situation.

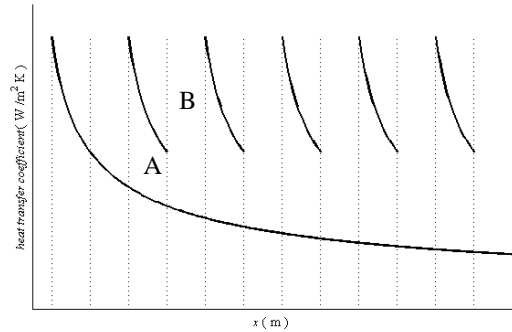- When $M_0 = M$, it is the film interlayer convection.

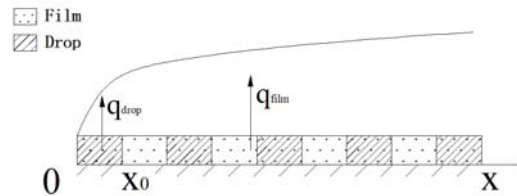FIGURE II. HEAT TRANSFER COEFFICIENT DECREASES ALONG THE FLOW DIRECTION.

FIGURE III. SKETCH OF MATHEMATICAL MODEL.

- When $M/2 < M_0 < M$, droplets will be combined with each other, not consistent with the assumption.

- When $0 < M_0 < M/2$, the contact area between gas and the solid surface is larger than that between gas and the liquid film. The convective heat transfer between gas and the solid surface cannot be negligible.

- When $M_0 \to 0$, it is original convective heat transfer across gas.

- When M_0 refers to the optimal quantity of droplets, the value of M should be discussed:

    i. If the value of M is even, $M_0 = M/2$.

    ii. If the value of M is odd, $M_0 = (M+1)/2$.

Thus, in order to simplify the analysis, the optimal value of M_0 is selected in this paper. $M_0 = M/2$ (M is even).

B. Heat Transfer Rate in the Dropwise Interlayer Convection

Nusselt number $Nu_x = \dfrac{h_x x}{k} = C \operatorname{Re}_x^m \operatorname{Pr}^n$ and Reynolds number $\operatorname{Re}_x = \dfrac{\rho u_\infty x}{\mu}$ are put into (1). The convective heat

transfer coefficient between gas and liquid layer can be formulated as

$$h_g = k_g C \operatorname{Re}_x^m \operatorname{Pr}^n / x = C k_g \operatorname{Pr}^n \left(\rho_g u_\infty / \mu_g \right)^m x^{m-1}. \tag{2}$$

C -- constant in Nusselt number. m -- Reynolds number index. n -- Prandtl number index. Pr -- Prandtl number. u_∞ -- gas velovity, m/s. ρ_g -- mass density of gas, kg/m^3. μ_g -- gas viscosity, $N \cdot s/m^2$.

The average convective heat transfer coefficient along the path in the film interlayer convection can be formulated as

$$\overline{h}_{film} = \frac{\int_0^{\overline{x}} h_g dx}{\overline{x}} = \frac{C k_g \operatorname{Pr}^n \left(\dfrac{\rho_g u_\infty}{\mu_g} \right)^m \int_0^{\overline{x}} x^{m-1} dx}{\overline{x}} = \frac{k_g Nu_{\overline{x}}}{m\overline{x}}. \tag{3}$$

On the basis of the model of dropwise interlayer convection and with the same length along the path, the average convective heat transfer coefficient in the dropwise interlayer convection can be formulated as

$$\overline{h}_{drop} = \frac{M_0 \int_0^{x_0} h_g dx}{\overline{x}} = \frac{M_0 k_g Nu_{x_0}}{m\overline{x}}. \tag{4}$$

Thus, the heat flux and heat transfer rate of the film interlayer convection and the dropwise interlayer convection can be separately formulated as

$$q_{film} = \overline{h}_{film} \left(T_g - T_0 \right), Q_{film} = \overline{h}_{film} A \left(T_g - T_0 \right), \tag{5}$$

$$q_{drop} = \overline{h}_{drop} \left(T_g - T_0 \right), Q_{drop} = \overline{h}_{drop} A \left(T_g - T_0 \right). \tag{6}$$

C. Contrast between Film Interlayer Convective Heat Transfer and Dropwise Interlayer Convection Heat Transfer

The heat flux between gas and cold solid surface is larger than that between gas and liquid film. Thus, contrasts between the heat transfer rate of liquid film and that of droplets are most focused on.

The word B represents the ratio of the heat transfer rate between dropwise interlayer convection and film interlayer convection. It can be formulated as

$$B = \frac{Q_{drop}}{Q_{film}} = \frac{M_0 \times \overline{h}_{drop}}{\overline{h}_{film}} = M_0 \times \left(\frac{x_0}{\overline{x}} \right)^m = \frac{M_0}{M^m}. \tag{7}$$

This formula shows that the ratio of the heat transfer rate between dropwise interlayer convection and film interlayer convection is related to the number of liquid droplets and the separate parts. It also shows that the Reynolds number index m affects the ratio, what means the flow regime is an important influence factor.

According to the foregoing, the number of liquid droplets is expected to be $M_0 = M/2$. Then the superiority of dropwise over film condensation will be analyzed from the number of liquid droplets. The ratio can be expressed as

$$B_{M_0} = \frac{M_0}{M^m} = \frac{M_0}{\left(2M_0 \right)^m} = \frac{M_0^{1-m}}{2^m}. \tag{8}$$

In order to prove that droplet formation is superior to film formation in terms of maintaining high condensation and heat transfer rates, it should be satisfied that the ratio is greater than 1 ($B_{M_0} > 1$). After mathematical analysis, it can be expressed as $\ln M_0 > \dfrac{m \ln 2}{1-m} \Rightarrow \begin{cases} \ln 2, & m = 0.5 \leftarrow La \min ar \\ 4\ln 2, & m = 0.8 \leftarrow Turbulent \end{cases}$. Then the number of droplets must meet the conditions $\begin{cases} M_0 > 2, m = 0.5 \leftarrow La \min ar \\ M_0 > 16, m = 0.8 \leftarrow Turbulent \end{cases}$. At the same time, the number of separate parts must meet the conditions $\begin{cases} M > 4, m = 0.5 \leftarrow La \min ar \\ M > 32, m = 0.8 \leftarrow Turbulent \end{cases}$. In the experiment of dropwise condensation, once the droplets appear, the number of the liquid droplets is greater than this lower limit. Thus, it can be concluded that the heat transfer rate of dropwise condensation is larger than that of film condensation.

Then it will be analyzed that whether the transfer rates that are more than an order of magnitude larger than those associated with film condensation can be achieved in dropwise condensation.

The ratio should meet the condition $B_{M_0} > 10$. It will be analyzed mathematically in the same way $\ln M_0 > \dfrac{\ln \left(2^m \times 10 \right)}{1-m} \Rightarrow \begin{cases} 2\ln \left(2^{0.5} \times 10 \right), & m = 0.5 \leftarrow La \min ar \\ 5\ln \left(2^{0.8} \times 10 \right), & m = 0.8 \leftarrow Turbulent \end{cases}$. Then the number of droplets must meet the conditions $\begin{cases} M_0 > 200, m = 0.5 \leftarrow La \min ar \\ M_0 > 1.6 \times 10^6, m = 0.8 \leftarrow Turbulent \end{cases}$. At the same time, the number of separate parts must meet the conditions $\begin{cases} M > 400, m = 0.5 \leftarrow La \min ar \\ M > 3.2 \times 10^6, m = 0.8 \leftarrow Turbulent \end{cases}$. When the flow regime of gas in dropwise interlayer convection is laminar, adjust the number and diameter of the liquid droplets to meet the conditions, then the conclusion may be achieved that the heat transfer rate of dropwise interlayer convection is more than an

order of magnitude larger than that of film interlayer convection.

It can also be concluded that relative to turbulent flow, laminar flow is the better situation to achieve the superiority of dropwise interlayer convection.

IV. INFLUENCE OF THE NUMBER OF DROPLETS ON THE SUPERIORITY OF DROPWISE INTERLAYER CONVECTIVE HEAT TRANSFER

An example of condensation is presumed. The outer surface of a semi-infinite vertical square tube, which is $1m$ wide, is exposed to saturated steam at atmospheric pressure and is maintained at $50°C$ by the flow of cool water through the tube. Steam flows through the width direction with the rate $1m/s$ and the flow regime is laminar. The heat transfer rates of one side of the tube will be analyzed.

According to the appendix Table A.6 in the book *Fundamentals of Heat and Mass Transfer* [1], saturated vapor ($p = 1.0133bar$): $T_g = 100°C$, $\rho_g = 0.596kg/m^3$, $k_g = 24.8 \times 10^{-3} W/(m \cdot K)$, $\mu_g = 12.02 \times 10^{-6} N \cdot s/m^2$, $Pr_g = 0.984$. Saturated liquid: $T_0 = 75°C$. In the interlayer convection, the other parameters of steam: $C = 0.332$, $m = 1/2$, $n = 1/3$, $u_\infty = 1m/s$.

When film condensation occurs between steam and the cold tube, parameters should be put into (5). Then the heat flux between steam and flow condensation is

$$q_{film} = \overline{h}_{film}\left(T_g - T_0\right)$$
$$= \frac{Ck_g \, Pr_g{}^n \left(\dfrac{\rho_g u_\infty}{\mu_g}\right)^m \overline{x}^m \left(T_g - T_0\right)}{m\overline{x}} = 91.179W/m^2. \quad (9)$$

On the same surface, when dropwise condensation occurs between steam and the cold tube, parameters should be put into (6). Then the heat flux between steam and droplet condensation is

$$q_{drop} = \overline{h}_{drop}\left(T_g - T_0\right) = q_{film} \cdot \frac{M_0}{M^m}. \quad (10)$$

Change the number of droplets M_0 , and the change graph of dropwise interlayer convective heat transfer rate with the number of droplets can be drawn in Figure 4.

It can be seen from the figure that when a few liquid droplets appear, the heat transfer rate of dropwise interlayer convection is already larger than that of film interlayer convection. The heat transfer rate increases with the increase of the number of droplets. When the droplet number reaches a certain value, the heat transfer rate that is more than an order of

magnitude larger than that of film interlayer convection can be achieved in dropwise interlayer convection.
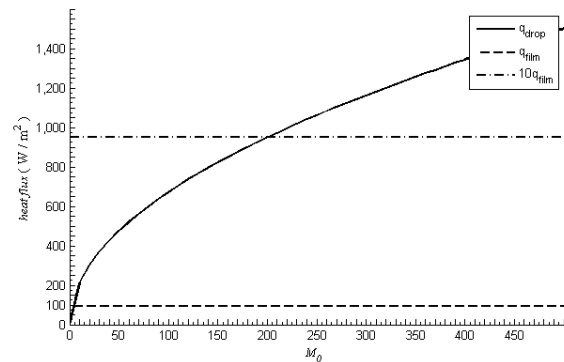

FIGURE IV. CHANGE GRAPH OF DROPWISE INTERLAYER CONVECTIVE HEAT TRANSFER RATE WITH DROPLETS NUMBER.

V. CONCLUSIONS

In this paper, the concept of interlayer convection is introduced. Based on the concept, the mathematical model about the transition from film interlayer convection to dropwise interlayer convection is established. Theoretical analysis is given and the conclusions are achieved that the heat transfer rate of dropwise condensation is larger than that of film condensation. In addition, relative to turbulent flow, laminar flow is the better situation to achieve the superiority of dropwise interlayer convection. Adjust the number and diameter of the liquid droplets to meet certain conditions, the conclusion that the heat transfer rate of dropwise interlayer convection is more than an order of magnitude larger than that of film interlayer convection can also be achieved.

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