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Geometry of transport networks in the region

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***Abstract* — The article deals with the problem of constructing transport networks, the geometric structure of which contributes to the reduction of freight transportation costs for any pair of "point of departure" / "destination". To this end, the loss (T) loss factor of the transport network T is introduced. This factor is introduced as a quantity indicating how many percent the shortest path along the transport network is longer than the straight path. The article gives examples of transport networks of different geometric structure, as well as the results of computer calculations, the loss factors for these examples.**

Keywords—transport network, triangulation, geometric structure, loss coefficient

I. Introduction

The transport network is a set of transport routes (lines, roads) of a certain territory or region, connecting transport nodes and settlements. Transport networks are characterized by density, length, throughput (the maximum possible number of cargoes passing through this section for a certain time). They include transport highways - the main transport routes that are of great importance in the system of production and territorial links. When solving various economic problems, appropriate mathematical models of the transport network are constructed. For example, when solving the problem of minimizing the capital costs of building a network, a geometric model consisting of points connected by a system of rectilinear segments appears. In this case, the solution of the problem is reduced to the search for a geometric network for which the total length of the segments is minimally possible. This and similar problems are called the Steiner problem. The results of solving Steiner's problems are called minimal networks. One of the key properties of a minimal network is that it is a geometric tree - a flat, connected graph without cycles. The existence of a cycle, as is not difficult to see, contradicts the minimality property. The geometry and topology of minimal networks are actively studied by both domestic and foreign mathematicians. To get acquainted with their work, we refer the reader to the survey work [1].

However, the classical statement of the Steiner problem on the construction of a minimal network does not take into account the many different factors that arise when solving real economic problems. Therefore, a number of analogues of Steiner's problems arose and algorithms for finding their approximate solutions were developed. So, for example, in one of the last papers devoted to the topic under discussion [2], Steiner's problem with constraints in the form of the maximum total length of communication sections from any terminal vertex to the collection point-the tree root-is considered. We should also mention [3], in which three types of geometric models of transport networks are considered: Euclidean, orthogonal and polar, which are determined by the corresponding functions of the metric. In this article, the models are discussed in the framework of the classical formulation of the Steiner problem.

Despite the high interest in problems such as Steiner's problem, it is nevertheless easy to see that real transport networks, for example cities, are not constructed at all like geometric trees. In [4] the following typification of transport networks by their topological and geometric structure is proposed: radial scheme, radial-circular scheme, rectangular scheme, rectangular-diagonal scheme, triangular scheme, free scheme. As possible geometric characteristics of the networks, the following are indicated there:

a) network density;

b) the degree of non-directness of messages;

c) the degree of loading of the central transport hub by transit correspondents;

d) network capacity (availability of duplicate directions);

e) configuration of intersections of trunk lines.

Unfortunately, the author ignored the consideration of the magnitude of point b) - the degree of non-straightness of the messages: the paper does not show how this degree can be determined and calculated. In this paper, we propose some variants for determining the degree of non-rectilinearity of the transport network and present some results of the investigation of this quantity. The main goal of our research is to obtain such geometric configurations of transport networks for which the value of non-rectilinearity introduced below would be as small as possible.

II. Materials and Methods (Model)

We will consider the transport network model in the form of a flat graph *T*. The vertices of such a graph are the destination points, and the edges are rectilinear segments connecting these points. For each pair of vertices *p*,*q* we denote by $|pq|$ is the usual Euclidean distance between these vertices, and $|pq|_T$ – is the length of the shortest path between the vertices *p*,*q* when moving along the edges of the graph *T*. The ratio of these quantities $\lambda(p,q) = |pq|_T / |pq|$ is called the coefficient of extension of the graph *T*. It is clear that the value $\lambda(p,q) \geq 1$. The loss factor of the transport network with graph *T* is defined as follows

$$\lambda(T)=\max_{p,q} \lambda(p,q),$$

$$loss(T)=(\lambda(T)-1)100\%.$$

It should be noted that this value characterizes the natural and unavoidable losses of such resources as time, fuel during transportation on the network *T*. Also, we note that in addition to the maximum loss value, one can consider their average value

$$\lambda_m(T)= \sum_{p \neq q}(\lambda(p,q))/N(N-1),$$

$$loss_m(T)=(\lambda_m(T)-1)100\%,$$

where *N* – number of vertices of the graph. First of all, we note that the introduced loss values can reach arbitrarily large values (see Fig. 1).

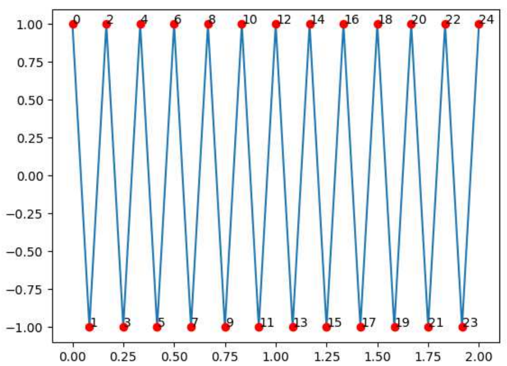

Fig. 1. To construct a network with arbitrarily large *loss(T)*.

Of great interest, especially in the English-language scientific literature (see [5] - [11] and sources cited there) raises the question of estimating the value of $\lambda(T)$ (and hence the *loss(T)*) for geometric networks that split the planar region into triangles (in terms of [4] - for triangular schemes). In computing geometry, such networks are called triangulations. A special class of triangulations is considered, the so-called Delaunay triangulations [12], [13]. A number of interesting esults were obtained in this direction. In particular, from the results of the article Ge Xia [10] it follows that for every triangular network *T*, for which the Delone condition is satisfied, the value $\lambda(T) \leq 1,998$. Theoretically, this estimate may be interesting, but from a practical point of view, the above upper estimate is too crude, since in this case the loss coefficient is estimated by the inequality $loss(T) \leq 99,8\%$. While losses on a rectangular grid (see Figure 2) do not exceed $41,4\%$. This follows from the estimate $\lambda(T) \leq \sqrt{2} \approx 1,414...$.

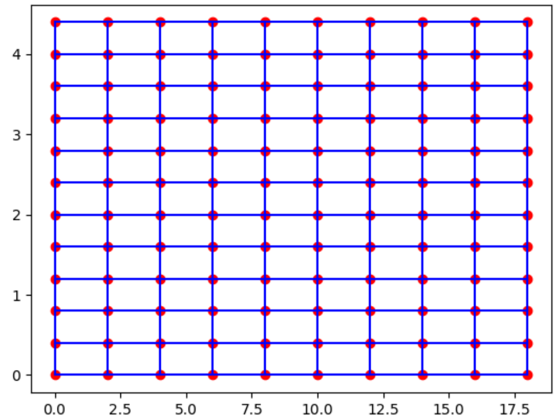

Fig. 2. For such a network $loss(T) \leq 41,4\%$.

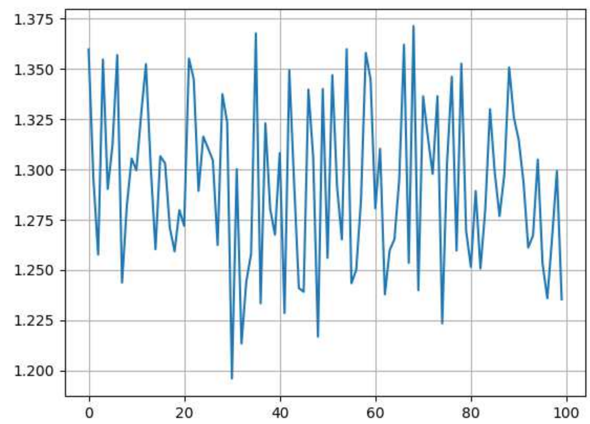

Fig. 3. Graph of values $\lambda(T)$

We undertook a number of computer experiments on the statistical study of the *loss(T)* value for randomly generated Delaunay triangulations on a plane. Below is a graph of *loss(T)* values for 100 Delaunay triangulations by a random set of 24 points and the corresponding empirical distribution function.

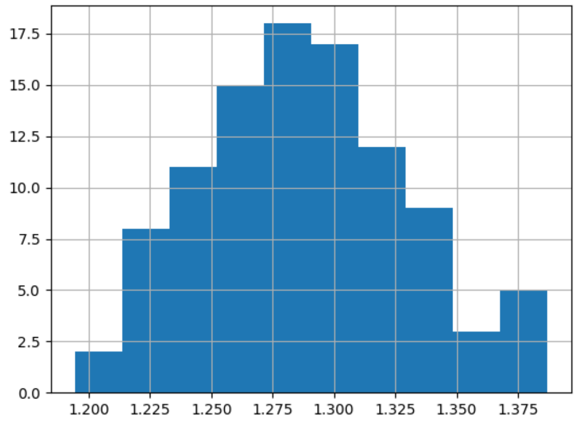

Fig. 4. Empirical density of distribution of values $\lambda(T)$

Similarly, the distribution of average losses can be calculated. In the experiment, 100 sets of points are generated, for which the corresponding coefficients for the Delaunay triangulation are calculated. A plot of the values is shown in Fig. 3, the corresponding distribution density is shown in Fig. 4.

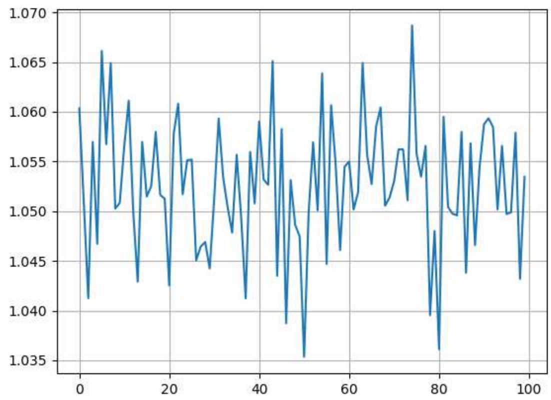

Fig. 5. Graph of values $\lambda_m(T)$

The study of these graphs suggests that there may exist triangulations for which the coefficient $\lambda(T)$ is close to *1,2*, and the value of the coefficient $\lambda_m(T)$ is close to *1,04*. Next, we show the results of research on this issue and present some geometric structures of transport networks, calculating the loss coefficients for them (see Fig.5 and Fig.6).

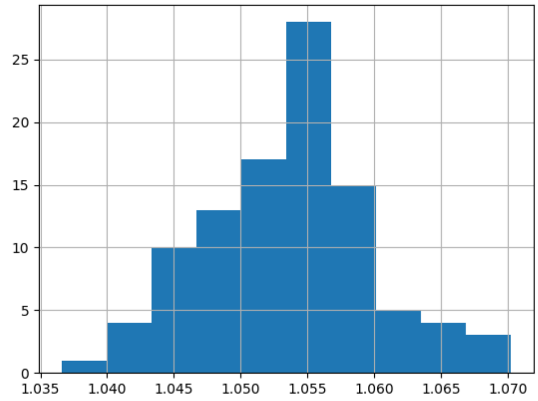

Fig. 6. Empirical density of distribution of values $\lambda_m(T)$

III. RESULTS AND DISCUSSION

If we confine ourselves to considering uniform triangular grids in which all triangles are similar and have angles α,β,γ, α+β+γ=π (see Fig. 7), then for geometric reasons it is not difficult to obtain such an estimate

$$\lambda(T) \le \mathbf{max}\{\cos^{-1}\alpha/2, \cos^{-1}\gamma/2, \cos^{-1}\beta/2\}.$$

Not complex calculations show that the best value from this estimate is obtained for an equilateral triangle α=β=γ=π/3. In this case

$$\lambda(T) \le 2/\sqrt{3} \approx 1{,}155 \cdot$$

and, accordingly, the loss will be $loss(T) \approx 15{,}5\%$. At the same time, computer calculations show that the average losses amount is of the order of $loss_m(T) \approx 10\%$.

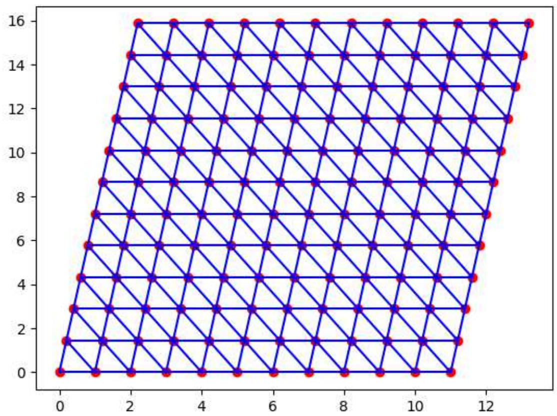

Fig. 7. For such a network *loss(T) ≤ 15,5%*.

It makes sense to consider not only triangular grids, which are less common in planning the transport networks of cities and regions. In fact, the most common geometric structures are represented by quadrangular networks. By the way, the term "city blocks" is, to some extent, reflects this property of real transport networks. First of all, let's look at networks that generalize networks from rectangles (see Fig. 8).

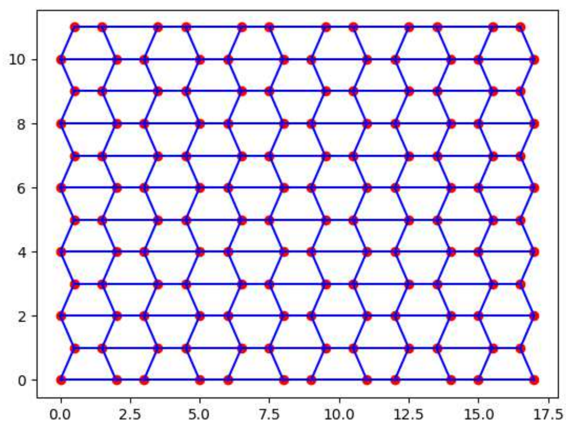

Fig. 8. Quadrangular network based on trapeziums

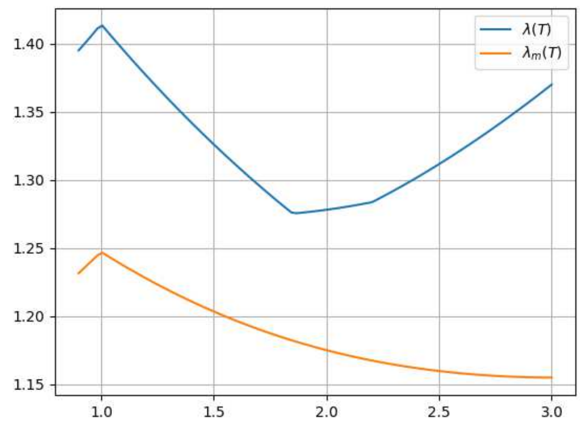

Fig. 9. Dependence $\lambda(T)$ and $\lambda_m(T)$ on b/a

Let us consider the dependence of the $loss(T)$ and $loss_m(T)$ values on the ratio b/a of the larger trapezoidal base-the cell of the quadrangular grid to its smaller base with a fixed value of the trapezium height. The graph shows that the maximum losses can be reduced to 27%, and the average - to 15.3%. Here it should be specially noted that in this sense the square network is not optimal, since, as seen from the graph for such a network, the $loss(T) \approx 41\%$, and $loss_m(T) \approx 25\%$ (see Fig. 9).

The second type of quadrangular networks is constructed according to the scheme indicated in Fig. 10.

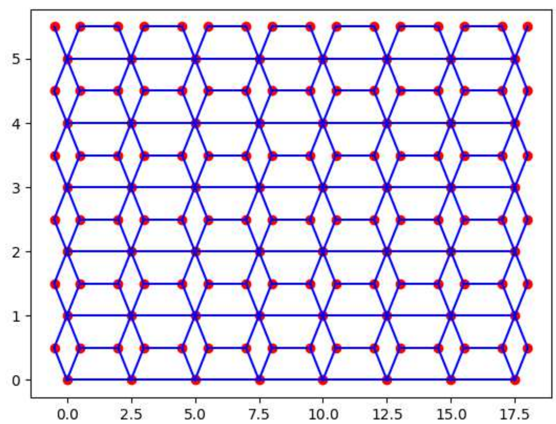

Fig. 10. Quadrangular network of trapeziums and rhombuses

As in the previous examples, let us consider the dependence of the $loss(T)$ and $loss_m(T)$ values on the ratio b/a for a fixed value of the ratio *c/a=0,5* Here *b* is the greater base of the trapezoid, *a* is the length of its smaller base, *2c* is the length of the horizontal diagonal of the rhombus. As a result of calculations, we obtain such graphs (see Fig. 11).

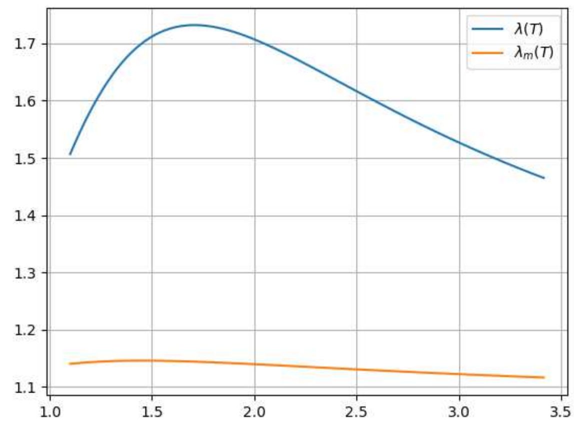

Fig. 11. Dependence $\lambda(T)$ and $\lambda_m(T)$ on *b/a*

It can be seen from the graph that at a ratio of *b/a* close to *3,4* the average loss is of the order of $loss_m(T)\approx 12\%$. This is clearly better than for a square grid.

In the future it is supposed to conduct research in at least two ways. First, to search for geometric structures of transport networks that do not have explicit symmetries. Secondly, it is supposed to take into account congestion of transport networks due to their geometry – in other words, it is supposed to take into account for each edge of the network graph the number of minimal paths passing through this edge.

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