Sustainable Management in Regional Fisheries: Mechanisms of Motivation of Myopic Agents

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Abstract — **This paper is dedicated to the game theoretic investigation of a dynamical model of optimal fishing with consideration of the interests of regional control agents on two levels of hierarchy and building of an original incentive mechanism. The main distinctive property of the paper consists in the consideration of a "myopic" agent who maximizes his payoff only in the one time period (one fishing season). Besides, the impulsion instead of compulsion is used as a method of control. The problem is solved by the simulation modeling, and for the investigation of the respective non-antagonistic differential two-players game a method of qualitatively representative scenarios is used. Its principal idea is that from a very big and even infinite set of the potentially existing control scenarios it is possible to choose a very small number of scenarios that reflect qualitatively different development paths of the controlled system. These scenarios are distinguished principally, while the others do not give anything essential new. The numerical verification of the formal requirements of the method using real data on the Azov Sea is made.**

Keywords — control problems, differential games, fisheries, motivation, regional development, simulation modeling, sustainable management

I. INTRODUCTION

Fishery is an important branch of the Russian economy. By the volume of catching of fish and seafood Russia has the 9-th place in the world, its share in the global catch is equal to 2,5 % by the data on 2016. The fishing activity in Russia is regulated by the Federal Fishing Agency according to the Federal laws № 166 (20.12.2004) and № 349 (03.07.2016). Fishery remains a problematic and criminalized economic branch, especially in the regions with big fish stocks. One of the most urgent problems is a development of the incentive mechanisms providing the ecological requirements of the regional sustainable development.

The main idea of any incentive system consists in that a control agent of the higher level of hierarchy impels other agents to choose one or another given actions [1]. Incentives in the control systems are formalized by the hierarchical differential games (Stackelberg or Germeier games). The Stackelberg games (Germeier games Γ_1) are described in [2], the inverse Stackelberg games (Germeier games Γ_2) with a feedback by controls - in [3]. Gorelov and Kononenko [4] have proposed an approach to the solution of the hierarchical differential games based on a reward and punishment mechanism. The game theoretic models of fisheries in different setups including asymmetric players and different planning horizons are considered, for example, in [5-8].

The authors' approach to the problem is presented in [9-15]. In [9] the authors' theory of modeling of the dynamical hierarchical control systems is described according to which the basic modeling pattern is the hierarchical system "principal - supervisor(s) - agent(s) - object" in different modifications. The principal chooses his strategy (makes his move) first and reports it to the supervisor and the agents. The principal maximizes his objective functional on the set of strategies which ensure a given state of the controlled system with considerations of the best responses of the other agents. The supervisor chooses her strategy when the choice of the principal is already known but the choices of agents are not known yet. The agents choose their strategies when the strategies of the players on the higher levels of hierarchy are known, and tends to maximize his objective functional. Acting as a leader, the player (principal or supervisor respectively) uses such methods of hierarchical control as compulsion and impulsion for the achievement of his/her objectives. From the mathematical point of view, compulsion means the leader's impact to the set of feasible controls of the follower, and impulsion means the impact to the objective function. It is supposed that all control agents have full information about their objective functionals and the objective functionals of all other players.

In [10,11,14] the algorithms of building of the solutions in hierarchical differential games for different methods of hierarchical control and information structures are proposed. A method of investigation of the hierarchical differential games based on the hypothesis of constancy of the control

actions for some periods of time is proposed in [13]. In [15] a problem of the sustainable management in the biological rehabilitation of the Azov Sea in the case of compulsion is studied.

This paper is dedicated to the game theoretic investigation of a dynamical model of optimal fishing with consideration of the interests of control agents on two levels of hierarchy and building an original incentive mechanism. The main distinctive property of the paper in comparison with the previous ones consists in the consideration of a "myopic" agent who maximizes his payoff only in the one time period (one fishing season). Besides, the impulsion instead of compulsion as in [15] is used as a method of control. This setup reflects the modern situation when the fishermen and their brigades maximize their current income without thinking about future of the whole regional ecosystem.

The agent's problem is solved analytically in each time period. The supervisor's optimal control is found by the simulation modeling, and for the investigation of the respective non-antagonistic differential two-players game a method of qualitatively representative scenarios (QRS) is used [12]. Its principal idea is that from a very big and even infinite set of the potentially existing control scenarios is possible to choose a very small number of scenarios that reflect qualitatively different development paths of the controlled system. These scenarios are distinguished principally, while the others do not give anything essential new. The numerical checking of the formal requirements of the method is fulfilled in the paper. Some real data for the Azov Sea are used [15].

II. METHODS

Let's investigate a hierarchical differential game theoretic model of fishing in a waterbody with consideration of the ecological requirements [15]. The supervisor is a fishing enterprise (for example, a fishing industrial holding of the regional level), the agents are fishermen and their brigades. The fish population is considered as a controlled dynamical system. Ecological requirements consist in some conditions for the state variable (fish biomass) in dynamics, when only agents have a direct impact to the fish population. Sustaining a given state of the controlled system is the main objective of the supervisor who has also additional economic interests. For simplicity we consider the case of only one agent.

The dynamics of the biomass of fish population is described by the logistic (Ferhulst-Pearl) model with consideration of the catch:

$$dP/dt = \alpha P(t)(1 - P(t)/R) - w(t)P(t), \quad P(0) = P_0 . \quad (1)$$

Here α, $R = const > 0$; R is the environmental capacity which determines the limit value of the fish biomass; the constant α characterizes the speed of reproduction in the population; t is a time coordinate; $P = P(t)$ – biomass of the fish in the instant of time t; P_0 – an initial value of the biomass; $w(t)$ – a share of catch (the agent's control).

The calculation of the biomass is made according to the equation (1) until it is positive.

Impulsion is used as a method of hierarchical control. It means that the supervisor exerts influence on the objective functional (function) of the agent, and the information structure corresponds to the Stackelberg game. As a rule, a fishing industrial holding hires fishermen only for a fishing season. Then the fishermen are fired and hired again during the next fishing season. That's why the case of a "myopic" agent (fisherman) is considered when he maximizes his payoff only on the one time period (one fishing season) during which his strategy doesn't change. When choosing his strategy the agent uses the parameters values in the beginning of the respective year (season). In the beginning of each year k=1,...,T the agent solves his optimization problem in the form:

$$(J_F)_k(s_k, w_k) = [as_k w_k - 0.5 b w_k^2] P_k \to max; \quad k = 1, ..., T. \quad (2)$$

Here $s_k = s(k)$; $s(t)$ is a share of the fisherman's reward for catching (the supervisor's control); k - a number of the year of decision making; a – the price of the unit of fish biomass; b – the fisherman's catching cost coefficient; w_k– a share of catching by the agent during the k-th year (the agent's control), k=1,...,T; P_k– the fish biomass in the beginning of the k-th year.

The supervisor (fishing enterprise) maximizes her payoff for T years with consideration of the ecological requirements. Her objective functional has the form:

$$J_L(s(\cdot), \{w_k\}) = \sum_{k=1}^{T} \left(\int_{k-1}^{k} [a(1-s(t))w_k - H(P(t) - P^*(t))^2] dt \to max. \right.$$

$$(3)$$

Here P*(t) is an ecologically optimal value of the fish biomass with consideration of the regional natural conditions; H – the coefficient of administrative penalty charged on the supervisor when a current value of the biomass deviates from the ecologically optimal one.

Explain the meaning of the second summand in the supervisor's objective functional (3). From the one side, an incomplete usage of the received quotas leads to the losses for the federal budget. From the other side, the overcatching is considered as a violation of the rules of catching of the water biological resources. This implies the penalty according to the Part 2 of the Clause 8.37 of the Russian Federation Administrative Code. The federal control in the domain of fisheries is not sufficient. Usually the control is made by the fishing enterprise itself or by the control agencies that are interested in the amount of catching that leads to the manipulation. The constraints on the controls have the form:
- supervisor

$$0 \le s(t) \le 1; 0 \le t \le T; \quad (4)$$

- agent

$$0 \le w_k \le 1; k = 1, ..., T. \quad (5)$$

Thus, the impulsion in this model consists in the choice by the fishing industrial holding of the fishermen's shares for fish catching. The condition of homeostasis for the fish population has the form $P(t) = P^*(t)$, t=1,...,T. It is assumed that this

condition provides the precise implementation of the fixed quotas for catching (an ecological optimum). If the condition is not satisfied then the quotas are overdone or underdone. In both cases the supervisor is charged a penalty with the coefficient $H \gg 1$.

Then we investigate the model (1) – (5) that is a differential game with a Stackelberg information structure [2]. The following algorithm of building the Stackelberg solution in the case of a "myopic" agent for the problem (1) – (5) is proposed.

  1. For determination of the agent's controls T parametrical nonlinear optimization problems (2), (5) are solved with given supervisor's controls $s_k = s(k)$, k=1,...,T. The solution of each problem k depends on P_k – the value of fish biomass in the beginning of the k-th year (k=1,...,T) which is determined from (1).

  2. The received values $\{w_k^* = w_k(s(k))\}$, k=1,...,T, are substituted in (1), (3). The solution of the respective control problem (1), (3) - (4) is denoted by $s^* = s^*(t)$.

  3. The Stackelberg equilibrium has the form $(s^*(t), w_k^*(s^*(k)))$, k=1,...,T.

The solution of the agent's problem (2), (5) for each k=1,...,T is found analytically by the Lagrange multipliers method. Namely, we have

$$d(J_F)_k/dw_k = a(1-s_k) - bw_k = 0, k=1,...,T.$$

Notice that $d^2(J_F)_k/dw_k^2 < 0$, k=1,...,T.

The derivative of the agent's objective function is equal to zero if $w_k^0 = a(1-s_k)/b$, k=1,...,T. Therefore,

$$w_k^*=0, s_k=1; w_k^*=a(1-s_k)/b, s_k > 1-b/a;$$
$$w_k^*=1, \text{ otherwise, } k=1,...,T. \tag{6}$$

Then the supervisor's objective functional takes the form

$$J_L(s(\cdot),\{w_k^*(s_k)\}) = \sum_{k=1}^{T}\sum_{k=1}^{N}\int_{k-1}^{k} as(t)w_k^* - H(P(t)-P^*(t))^2]dt \rightarrow \max,$$

$$\tag{7}$$

where w_k^* (k=1,...,T) are determined by the formula (6).

Assuming that both the supervisor and the agent can change their strategies only once a year, we may rewrite the functional (7) as an objective function of T variables:

$$J_L(\{s_k\},\{w_k^*(s_k)\}) = \sum_{k=1}^{T}(as_kw_k^* - H\sum_{k=1}^{N}\int_{k-1}^{k}(P(t)-P^*(t))^2dt) \rightarrow \max. \tag{8}$$

Here $s_k = s(k)$, k=0,1,...,T. The constraints (4) take the form

$$0 \leq s_k \leq 1, k=1,...,T. \tag{9}$$

In this case the supervisor's strategy is a grid function $\{s_k\}$, k=1,...,T, which determines his controls for each time period

(year, fishing season). The supervisor's problem (1), (8), (9) is solved by the QRS method. Explain it in more details.

An explicit solution of the supervisor's problem (1), (8), (9) is complicated by the presence in (6) of three variants of the agent's best response in dependence of the supervisor's control. Thus the agent's optimal reaction as a function of time is continuous but not differentiable, and Pontryagin's maximum principle is not applicable directly. That's why the supervisor's problem is solved by the simulation modeling based on the scenarios method. In the simulation modeling the complete enumeration of the set of feasible controls of the supervisor is impossible. Therefore, the question is how to choose an observable finite set of scenarios for the numerical calculations. The principal idea of the QRS method is that from a very big and even infinite set of the potentially existing control scenarios is possible to choose a very small number of scenarios that reflect qualitatively different development paths of the controlled system. These scenarios are distinguished principally, while the others do not give anything essential new.

Let's specify this idea for the model (1), (8), (9). The agent's best responses are determined by the formulas (6), so the supervisor's QRS are discussed.

Definition. A set QRS = $\{s^{(1)},...,s^{(m)}\}$ is called the set of qualitatively representative strategies (QRS-set) of the supervisor in the game (1), (6), (8), (9) with precision Δ if the two conditions are satisfied:
(a) for any two elements of this set $s^{(i)}, s^{(j)} \in$ QRS holds

$$\left|\sum_{l=1}^{T}J_L(s^{(i)},\{w_l^*(s^{(i)})\}) - \sum_{l=1}^{T}J_L(s^{(j)},\{w_l^*(s^{(j)})\})\right| > \Delta;$$

(b) for any feasible strategy of the supervisor which does not belong to this set $s^{(m)} \notin$ QRS, an element exists $s^{(j)} \in$ QRS:

$$\left|\sum_{l=1}^{T}J_L(s^{(m)},\{w_l^*(s^{(m)})\}) - \sum_{l=1}^{T}J_L(s^{(j)},\{w_l^*(s^{(j)})\})\right| \leq \Delta.$$

Here $\Delta > 0$ is a constant. Thus, QRS imply the essential difference in the supervisor's payoffs while the difference inside the QRS-set is not essential in this sense.

The supervisor's QRS-set QRS=S_1^{QRS} X ... X S_T^{QRS} for the problem (1), (6), (8), (9) is built as follows. The set S_k^{QRS} is the supervisor's QRS-set at the instant k=1,...,T (in the k-th year). If in each time instant the set S_k^{QRS} contains Z elements then the QRS-set, in turn, contains

$$m = |QRS| = \prod_{k=1}^{T}|S_k^{QRS}| = Z^T \text{ elements.}$$

The proposed incentive mechanism for the agent (fisherman) from the supervisor (fishing industrial holding) is reduced to the building of the Stackelberg equilibrium for the problem (1), (6), (8), (9). The algorithm of building of the

Stackelberg equilibrium on the supervisor's QRS-set for this problem consists in the following actions.

1. All input functions and parameters of the model (1), (6), (8), (9) are given. Set k=1.

2. The next (k-th) supervisor's strategy is fixed:

$$s^{(k)} = (s_1^{(k)},...,s_T^{(k)}), \; s_i^{(k)} \in S_i^{QRS}, \; i=1,...,T.$$

3. According to (6) and the values of components of the vector $s^{(k)}$, the values w_i^*, i=1,...,T are determined and substituted in (1), (8). The greater value of the objective function (8) and the respective set of control variables are saved.

4. If not all feasible supervisor's strategies are screened then go to the next strategy (k:=k+1) and return to the step 2.

5. When all supervisor's QRS are screened (totally 3^T strategies) then the strategy attaining the greatest value to the functional (1) is determined, i.e.

$$s^* = \arg \max_{s^{(k)} \in QRS} J_L(s^{(k)},\{w_i^*(s^{(k)})\}). \quad i=1$$

6. The Stackelberg equilibrium for the model (1), (6), (8), (9) on the supervisor's QRS-set has the form $(s^*,\{w_i(s^*)\})$, i=1,...T.

III. RESULTS AND DISCUSSION

The results of numerical calculations according to this algorithm for several typical sets of the input data are presented below. First, the initial supervisor's QRS-set was formed. For this purpose, an initial set of the supervisor's strategies of computer simulations were checked according to the definition of the QRS-set. It is verified whether an initial set of scenarios is the QRS-set or not? If not then the initial set is extended or reduced in dependence on which requirement in the definition is not satisfied.

In the process of formation of the QRS-set in each instant of time the set of three strategies $S_k^{(0)}=\{0,1/2,1\}$, k=1,...,T, is taken as the initial set of the supervisor's strategies. Then the initial set of the supervisor's strategies

$$S^{(0)} = S_1^{(0)} X ... X S_T^{(0)} \quad \text{contains}$$

$$|S^{(0)}| = \prod_{k=1}^{T} |S_k^{(0)}| = 3^T \text{ elements.}$$

For each simulation scenario the equation (1) was solved numerically, for example, by the finite differences method with an implicit difference scheme of the first-order approximation in time, or by the Runge-Kutta method.

Suppose that T = 4. In this case the players may change their strategies four times during the period of modeling. As a result, the initial set of the supervisor's strategies contains 81 strategies. For each of these 81 strategies the first condition

from the definition of the QRS-set was verified, and then the second one for the not chosen strategies.

Example1. The numerical calculations aimed to the verification of the conditions of the supervisor's QRS-set in the case T = 4 years = 1462 days and $|S_k^{(0)}|$=3, k=1,...,T, were conducted for the following input data set: a=50000 c.u.; $P^0(t)$=0.09mg/cm^3; H=600000c.u.cm^2; ρ=0.01; R=0.7; P_0=0.05mg/cm^3; α=0.1. Here c.u. stand for conditional units of income or cost; cm – centimeters. Then the optimal strategies of the agents are s_k^*=0.5; w_k^*=1, k=1,2,3,4, and their payoffs in the Stackelberg equilibrium are determined by the formulas

$$J_F^* = \sum_{k=1}^{T} (J_F^*)_k = 1788 \text{ c.u.; } J_L = 35098 \text{ c.u.}$$

In the process of verification of the first condition in the definition of the QRS-set it is received that the QRS-set contains only 21 strategies when Δ=9446 c.u. All QRS s(j) = $(s_1^{(j)},s_2^{(j)},s_3^{(j)},s_4^{(j)})$, j=1,...21, for the input data from Example 1 are presented in the Table I.

For the verification of the second condition from the definition of the QRS-set in each instant of time (k=1,2,3,4) the strategies which do not belong to the QRS-set $s_k \in \{0.2, 0.3,0.4,0.6,0.7,0.8,0.9\}$ and their combinations were used. For each of the verified strategies a QRS exists such that the difference in the supervisor's payoffs (3) for these two strategies doesn't exceed Δ. Thus, the set of 21 elements presented in the Table I can be used as the QRS-set in the model (1)-(5) when Δ=9446 c.u. This reduction of the set of potential scenarios of the supervisor permits to find her optimal strategies and the respective players' payoffs by the simulation modeling.

Example 2. When the price of unit of the fish biomass decreases (*a*=10 000 c.u.) or the penalty coefficient increases (H=1000000 c.u.cm^2/mg^2) then the players' optimal strategies do not change in comparison with the Example 1, and their payoffs expectably fall and are determined by the values J_F^*=328 c.u.; J_L=5898 c.u. and J_F^*=328 c.u.; J_L=7066 c.u. respectively. The QRS-set is also the same.

Example 3. In the case of the input data from the Example 1 and increasing of the catching cost the catch share chosen by the fisherman diminishes (when *b*=10000 c.u. then w_k^*=0.5, and when *b*=10000 c.u. then w_k^*=0.1, k=1,2,3,4). The supervisor's optimal strategies do not change. The payoffs of both agents fall abruptly till J_F^*=91 c.u.; J_L=2248 c.u. and J_F^*=18 c.u.; J_L=-671 c.u. respectively. In this case the supervisor's losses are possible. The QRS-set contains 25 strategies when Δ=9446 c.u.

For any other input data set the QRS-set will be different but in any case it will be a subset of the initial set of her strategies QRS belongs to $S^{(0)}=S_1^{(0)} X ... X S_T^{(0)}$), where $S_k(0)=\{0,1/2,1, k=1,...,T\}$. The number Δ characterizes the precision of choosing of a strategy in the QRS-set. Its change (decreasing or increasing) implies the change of cardinality of the QRS-set (increasing or decreasing respectively).

TABLE I.   RESULTS OF THE NUMERICAL CALCULATIONS

Supervisor's strategies				$J_L^{(j)}$	$min \; |J_L^{(k)} - J_L^{(j)}|$ $1 \leq k \leq 21, \; k \neq j$
$s_1^{(j)}$	$s_2^{(j)}$	$s_3^{(j)}$	$s_4^{(j)}$		
0	0	0	0	-1 401	9124
0.5	0.5	0.5	0.5	35 098	9125
0.5	0.5	0	0	16 873	9150
0.5	0	0	0	7 723	9124
0	0.5	0.5	0.5	25 973	9100
1	1	0.5	0.5	-299 241	9150
1	1	1	1	-317 812	9446
1	0.5	0.5	0.5	-290 091	9125
1	0	0	0.5	-308 366	9125
0	1	0	0	-236 328	9124
0.5	1	0	0	-227 204	9124
0.5	1	0.5	0.5	-208 978	9101
0.5	1	0.5	1	-218 079	9101
0	0	1	0	-154 967	9125
0.5	0.5	1	0.5	-127 592	9150
0.5	0	1	0.5	-136742	9100
0.5	0	1	0	-145 842	9100
0.5	0.5	0.5	1	-46 427	9150
0	0	0	1	-73 827	9125
0.5	0	0.5	1	-55 577	9125
0	0	0.5	1	-64 702	9125

IV. CONCLUSION

The proposed incentive mechanism is based on the "myopia" of agents and is reduced to the building of the Stackelberg equilibrium.

The supervisor's optimal strategies are found by the simulation modeling based on the QRS method. The QRS-set if formed as follows. An initial set of the supervisor's strategies is given for which the first condition from the definition of the QRS-set is verified. As a result, the initial set is extended or reduced. After that the second condition from the definition of the QRS-set is verified. The cardinality of the QRS-set depends on the chosen precision (number Δ). If Δ decreases then the cardinality increases. The building of the QRS-set permits to find the Stackelberg equilibrium by means of the simulation modeling based on the scenarios method.

The values of model parameters should reflect natural and economic conditions of a specific region.

Acknowledgments

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