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Regional Sustainable Management Problems on Networks

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***Abstract*—This paper is dedicated to the models of regional sustainable management on networks. The main objective of the paper is to propose a synthesis of the network models of influence and control with the models of sustainable management for the solution of the problems of regional sustainable management with consideration of the structural aspects. The respective formal setups are presented and interpreted for the problem domain of regional development.**

Keywords—control problems, differential games, networks, regional development, sustainable management

I. INTRODUCTION

Mathematical models of the processes of mutual interaction in social networks based on Markov chains were proposed in [1-3] and developed, for example, in [4, 5]. The idea of this approach consists in that initial opinions of the agents are changing in time by the influence of another members of a referent social group. Specifically, in the linear model all members of the group reach stable final opinions depending on the group structure.

Together with the models of influence, the models of control in social groups with a given structure of interactions is even of more interest [6]. Here models of optimal control (one control agent) and models of conflict control (several interacting agents with different interests) may be differentiated. The mathematical formalization of the models from the second group leads to game theoretic setups on networks [7, 8] including cooperative dynamic games on networks [9].

From the other side, a theory of sustainable management is developed by one of the authors [10-12]. In [13] the theory is specified for the problems of regional development. The methods of solution of the respective models are described in [14, 15]. According to this theory, two main requirements of the sustainable development are homeostasis and system compatibility. To formalize the concept of homeostasis, the theory of viability may be used [16, 17]. In general, hierarchical dynamic games create a base for the theory [18]; hierarchical dynamic game theoretic setups on networks are also known [19-21].

The main objective of the paper is to propose a synthesis of the described methods for the solution of the problems of regional sustainable management with consideration of the structural aspects.

II. METHODS

A basic model of influence in a social group is a weighted directed graph (network) in which vertices correspond to the members of the group, and arcs describe their mutual interaction. A real value (an opinion of the group's member) as a function of time is ascribed to each vertex, and a real number (weight) is ascribed to each arc (a degree of impact of one member of the group to another one or, what is the same, a degree of confidence of one member of the group to another).

Thus, in the model $X=\{x_1,...,x_n\}$ is a set of base agents; $x_i \to u_i(t)$, $i=1,...,n$ - an opinion of the i-th agent;

$u^0=(u_1^0,...,u_n^0)$ - a vector of the initial opinions of the base agents; a_{ij} - a coefficient of impact of the i-th base agent to the opinion of the j-th base agent; $A=\|a_{ij}\|$ is a matrix of influence (it defines the set of arcs of the social network model). The dynamics of opinions is determined by the rule

$$u_j(t+1) = \sum a_{ij}u_j(t) \ , \quad j=1,...,m;$$

or in the vector form

$$u(t+1) = A^T u(t), \quad u(0)=u^0, \quad t=0,1,...,T-1. \tag{1}$$

It is shown that all members of each i-th strong subgroup (a non-degenerated strong component of the digraph) come to a common final opinion determined by the formula

$$u_j^* = \sum w_k^i u_k^{i0} , \tag{2}$$

where w_i^k is a component of the stationary vector of the Markov chain with the transitive matrix A^T, $k=1,...,n_i$ - the number of members of the i-th strong subgroup. Final opinions of other agents not belonging to the strong subgroups (companions) are calculated as

$$u_j^* = \sum b_{ji} u_i^* , \quad (3)$$

where b_{ji} is a probability of transition of the agent j in the strong subgroup i as an ergodic set of the Markov chain, i=1,..., r - a total number of the strong subgroups.

The most interesting question for applications arises when one or several decision makers are not satisfied by the received final opinions. To formalize such situation the base model is extended by an introduction of a set of the impact agents $Y=\{y_1,...,y_m\}$ which can exert influence to the base agents. Then the set of vertices of the network is X U Y. The impact agents may change the initial opinions of the base agents or the factors of their interaction: $u^0:=u^0+w^0$; A:=A+B.

This leads to the optimal control problems or dynamic games on networks. Denote by $v(t)=(v_1(t),...,v_n(t))$ a vector of control impacts on all base agents in the instant of time t, satisfying the constraints

$$v(t) \in V . \quad (4)$$

If the situation is considered from the point of view of the only impact agent then an optimal control problem arises:

$$J(v(\cdot))=\int e^{-\rho t} g(v(t),u(t))dt \rightarrow \max, \ t=0,T \quad (5)$$

with constraints (1) and (4), where g is a current objective function of the impact agent, T - period of consideration (finite or infinite), ρ - a discount factor. The criterion of optimality of the impact agent can also be in the discrete form

$$I(v(\cdot))=\sum e^{-\rho t} g(v(t),u(t)) \rightarrow \max, \ t=0,T. \quad (6)$$

In a more general case when there are several impact agents l=1,...,m, each of them has his own optimality criterion of the type (5) or (6) and chooses his strategy $v_l(t) \in V_l$ of impact to a set of base agents. It generates a differential game in normal form

$$J_l(v(\cdot))=\int e^{-\rho t} g_l(v(t),u(t))dt \rightarrow \max, \ t=0,T; \quad (7)$$

$$v_l(t) \in V_l, \ l=1,...,m \quad (8)$$

with state dynamics (1), on the base of which a cooperative game may be constructed [9]. Some specific setups of such control problems are presented in [6].

If a hierarchy is given on the set of impact agents then Stackelberg games or inverse Stackelberg games arise that are of great importance [19-21].

III. Results and Discussion

An interpretation of the models of influence and control on networks for the case of regional management is presented in Table 1.

TABLE 1. INTERPRETATION OF THE MODEL

Model element	Mathematical meaning	Regional management
Base agent	Vertex of the network	Municipal district or enterprise
Impact agent	Vertex of the network	Federal government, regional administration, big investors
Base agent's opinion	A real number ascribed to each vertex of the network representing a base agent (function of time)	An essential factor of the base agent's economic and social development
Confidence (impact)	Presence of the arc from one vertex to another	Economic and cultural interactions between the base agents
Coefficient of confidence (impact) of one base agent to another	A real number ascribed to each arc of the network	A quantitative characteristic of the confidence (impact)
Final opinion	The limit value of an opinion	A stable final (limit) value of the opinion after a long period of time
Strong subgroup	A non-degenerated strong component of the network	Determines its own common final opinion and dependant opinions of the other agents (companions)
Companions	A subset of vertices representing the degenerated strong components	Final opinions are determined completely by the strong subgroups
Impact on initial opinions	An additive component to the vector of initial opinions	Administrative and economic mechanisms of the regional management
Impact on coefficients of confidence	An additive or other component to the matrix of influence	Administrative and economic mechanisms of the regional management
Control objective	A domain in the state space of the network	A range of the desirable values of the opinions

One of the key requirements of sustainable development is the homeostasis of the controlled system [10]. In terms of the considered model this condition can be written in the form

$$u_i^* \in U_i^* , \ i=1,...,n. \quad (9)$$

This condition is used as an additional constraint in the optimal control problem (1),(4)-(5) or in the differential game (1), (7)-(8).

However, an even more important requirement of the sustainable development is the system compatibility. Really, let's consider a differential game (1), (7)-(9). In fact, each player l=1,...,m has his own preferences of the type (9), namely

$$u_i^{l*} \in U_i^{l*} , \ i=1,...,n, \ l=1,...,m. \quad (10)$$

Moreover, in a game theoretic setup each player has also his own objective functional (7), so that a solution of the game with consideration of the conditions of homeostasis can be only a trade-off which satisfies all players to an extent.

There are two ways of the solution of this problem. First, the players may join voluntarily and then act cooperatively. In fact, in this case their grand coalition becomes an only player, and the differential game is reduced to the optimal control

problem. Practically, such a situation is improbable due to an essential difference between the interests of agents.

Second, a special agent (Center) may be chosen for the coordination of the actions of all agents. The Center has an administrative and/or economic power and can exert influence to other agents; she is responsible for the homeostasis (9).

In turn, the Center has her own interests reflecting by an objective functional

$$J_0 = \int e^{-\rho t} g_0(w(t),v(t),u(t))dt \rightarrow \max,\ t=0,T, \qquad (11)$$

where $w(t)=(w_1(t),...,w_n(t))$ is a vector of controls of the Center and

$$w(t) \in W.$$

To develop a classification of information structures in the hierarchical differential games with many players can be used three attributes characterizing the Center's strategy [12]:

1) absence/presence of a feedback of the Center's strategy on the state of a controlled dynamic system. This attribute has two basic values: open-loop strategies which depend only on the instant of time t, and closed-loop strategies which depend on the game position (t,x(t)) [18];
2) absence/presence of a feedback of the Center's strategy with the agents' strategies. In the first case we deal with a Stackelberg game, and games of the second type are inverse Stackelberg games [22];
3) methods of hierarchical control. Here we differentiate compulsion, when the Center influences the agents' sets of feasible strategies, and impulsion, when she influences on the agents' objective functionals [10].

In more details, compulsion means that in (8)

$$V_l = V_l(w),$$

and impulsion means that in (7)

$$J_l = J_l(w),\ l=1,...,m.$$

In turn, the agents can choose at least one of the three modes of behavior:

(a) isolation, when the agents act independently and come to a Nash equilibrium;
(b) cooperation, when they pool resources and combine efforts to maximize the summarized objective functional (team solution);
(c) collaboration, when the agents voluntarily maximize the Center's objective functional. The idea of the system compatibility is that individually optimal (Nash or dominant) strategies of the agents in their totality maximize the Center's objective functional (11). That means that the interests of the Center and all agents are compatible and the agents are also interested to provide the requirement of homeostasis (9) in spite of their own preferences (10).

The described conditions of sustainable management are presented in the Table 2 [11].

TABLE 2. CONDITIONS OF SUSTAINABLE MANAGEMENT

Conditions	Description	Mathematical formalization
Homeostasis (two levels)	Weak form: all state variables of an environmental-economic system must take their values in a given range	The neutral Lyapunov stability (or Lagrange stability) of the steady state describing the ideal requirements to a regional environmental-economic system
	Strong form: additionally, the state variables converge to their ideal values	The asymptotic Lyapunov stability of the steady state describing the ideal requirements to a regional environmental-economic system
Coordination of interests (three levels)	Strategic stability: existence of a trade-off considering to an extent the interests of all associated agents	Nash equilibrium in a dynamic game theoretic model describing the conflict interaction of the associated agents
	Dynamic stability: no agent has incentives to defect from the trade-off solution for an arbitrary long time	Time consistency or subgame perfectness of the solution
	System compatibility: the trade-off outcome maximizes the system payoff	Pareto optimality of the time consistent or subgame perfect Nash equilibrium

In the case of the presented model, the condition (9) unites the weak and strong forms of homeostasis because it is already given as a limit condition.

A quantitative measure of the system compatibility is given by the respective index

$$SCI = J_0^{max} - J_0^* , \qquad (12)$$

where J_0^{max} is the globally maximal value of the Center's objective functional (11), and J_0^* is equal to its value in the worst of Nash equilibria in the game of agents in normal form (1), (7)-(8). So, the regional system is completely compatible if SCI=0.

In regional development, on the higher control level a regional administration (RA) is situated. It is treated as the Center; in some cases the federal government also can play the role of Center. The middle control level is presented by municipal districts (MD). A more complicated variant is also possible when urban municipal formations and their districts are considered separately. The lower control level is formed by local active agents (LA): enterprises, organizations, firms, individual entrepreneurs.

In the majority of cases LA are economic agents having certain financial, human, and other resources and tending to maximize their income (profit). However, non-commercial organizations and local control agencies can also be considered as LA having non-economic objectives. Economic LA can have social and ecological objectives too.

In some cases, MD also can be considered as LA. MD solve the problems of social-economic development of the respective territories with consideration of the environmental requirements using certain budget resources.

The objectives and possibilities of RA are structurally similar to the ones of MD but differ from them by the volume and sources of financing. RA can establish its own laws, and have additional means of influence to MD and LA [13].

Most typical political technologies of regional elites include:

- a concentration of financial resources;
- a power pressure to the opponents;
- an establishment of control and strategic partnership with the most powerful regional economic agents;
- using of information flows from the federal center for the protection of the elites' interests;
- negotiation process with the political leaders;
- a direct or indirect control on the information flows and media, an active construction of a positive image of the regional administration by media;
- a formation of the regional mythology;
- a harder selection of the new candidates to the regional elites [23].

IV. Conclusion

Thus, the described models permit to solve the following problems of the regional sustainable management.

1.Analysis: a segmentation of the set of regional active agents such as municipal districts or enterprises, a differentiation of the strong subgroups that determine the common final opinions inside the subgroups and the final opinions of all companions as linear convolutions of the final opinions of the strong subgroups.

2. Forecasting: calculation of the final opinions in their natural dynamics without any external influence.

3. Optimal control: choosing of the optimal impacts to the base agents from the point of view of one impact agent. Specifically, the requirements of homeostasis may be considered in this setup as the additional constraints.

4. Conflict control: choosing of a trade-off solution with consideration of the interests of several interacting impact agents. This setup is the most general and may be treated as a sustainable management problem. In this case, one of the impact agents plays the role of Center who is responsible for the homeostasis and can exert influence to other impact agents to ensure this objective. Namely, compulsion means the Center's influence to the sets of strategies of the agents, and impulsion means her influence to their objective functionals. These control methods have clear interpretations as administrative and economic mechanisms of the regional management. The Center has her own interests, and the requirement of system compatibility is introduced to characterize the trade-off. A quantitative measure of the system compatibility is given by the respective index (12).

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References

[1] French J.R. A formal theory of social power // The Psychological Review. 1956. №63. P. 181-194.

[2] Harary F. A Criterion for Unanimity in French's Theory of Social Power // Studies in Social Power. - Michigan: Institute of Sociological Research, 1959. - P.168-182.

[3] De Groot M.H. Reaching a Consensus // Journal of American Statistical Association. 1974. №69. P.118-121.

[4] Friedkin N.E., Johnson E.C. Social Influence and Opinions // Journal of Mathematical Sociology. 1990. №15. P.193-205.

[5] Jackson M. Social and Economic Networks. - Princeton University Press, 2008.

[6] Barabanov I.N., Korgin N.A., Novikov D.A., Chkhartishvili A.G. Dynamic models of informational control in social networks // Automation and Remote Control, 2010, 71(11), 2417-2426.

[7] Roughgarden T. Selfish Routing and the Price of Anarchy. - MIT Press, 2005.

[8] Novikov D.A. Games and networks // Automation and Remote Control, 2014, 75(6), 1145-1154.

[9] Gao H., Pankratova Ya. Cooperation in Dynamic Network Games // Contributions to Game Theory and Management. Vol.X. Ed. by L.Petrosyan, N.Zenkevich. - SPb., 2017. - P.42-67.

[10] Ougolnitsky G. Sustainable Management. – N.Y.: Nova Science Publishers, 2011.

[11] Ougolnitsky G.A. Sustainable Management as a Key to Sustainable Development // Sustainable Development: Processes, Challenges and Prospects. Ed. D. Reyes. - N.Y.: Nova Science Publishers, 2015. - P.87-128.

[12] Ougolnitsky G.A. Differential Games in Environmental Management // Environmental Management: Past, Present and Future / Ed. by E. Wright. - N.Y.: Nova Science Publishers, 2017. P. 1-52.

[13] Ougolnitsky G.A. A System Approach to the Regional Sustainable Management // Advances in Systems Science and Applications, 2017, 17(2), 52-62.

[14] Ougolnitsky G.A., Usov A.B. Solution algorithms for differential models of hierarchical control systems // Automation and Remote Control, 2016, 77(5), 872-880.

[15] Ougolnitsky G.A., Usov A.B. Computer Simulations as a Solution Method for Differential Games // Computer Simulations: Advances in Research and Applications. Eds. M.D. Pfeffer and E. Bachmaier. - N.Y.: Nova Science Publishers, 2018.

[16] Aubin J.-P. Viability Theory. - Springer-Verlag, Birkhauser, 1991.

[17] Bonneuil N., Boucekkine R. Viable Nash Equilibria in the Problem of Common Pollution // Pure and Applied Functional Analysis, 2017, 2(3), 427-440.

[18] Basar T., Olsder G.Y. Dynamic Noncooperative Game Theory. - SIAM: Philadelphia, 1999.

[19] Basar T., Srikant R. A Stackelberg Network Game with a Large Number of Followers // J. Optimization Theory and Applications, 2002, 115(3), 479-490.

[20] Johari R., Tsitsiklis J. Efficiency Loss in a Network Resource Allocation Game // Mathematics of Operations Research, 2004, 29(3), 407-435.

[21] Shen X., Basar T. Incentive-Based Pricing for Network Games with Complete and Incomplete Information // Advances in Dynamic Game Theory: Numerical Methods, Algorithms, and Applications to Ecology and Economics. Eds. S.Jorgensen, M.Quincampoix, T.L.Vincent. - Birkhauser, 2007. - P.431-458.

[22] Gorelov M.A., Kononenko A.F. Dynamic models of conflicts. III. Hierarchical games / Automation and Remote Control, 2015, 76(2), 264-277.

[23] Lapina N., Chirikova A. Strategies of Regional Elites: Economics, Power Models, Political Choice. - Moscow: INION RAS, 2000 [in Russian].