

## Some Simulation Examples and Stability Analysis of Third Order Homogeneous Systems

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**Abstract:** Three kinds of third order systems was studied to check the stability of homogenous type systems. And a Lyapunov function is chose to prove the stability of homogenous nonlinear system and it was found that it is very difficult to prove the stability without homogenous theorem. Also numerical simulations were done the show the stability of three typical third order systems. And simulation result shows that the classical homogenous system is only stable in a small scope. But for linear systems, the stability is large range.

### Introduction

Currently, the homogenous theorem was widely studied for many systems[1-5]. For example, some researchers constructed a homogenous system to solve high order derivative of input signals. But the stability of homogenous nonlinear system is not easy to guarantee. Since it is not linear system, so although its formation is very similar to a Hurwitz system[6-8], we can not use matrix theorem to prove its stability. And although it is a nonlinear system, but traditional Lyapunov function method is also not easy to prove its stability[9-11]. So we tried to use simulation method to check the stability of homogeneous systems. So detail simulations were done to show the rightness of proposed conclusions.

### Problem Description

A kind of homogenous system can be written as third order system as follows:

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= -c_1 z_1^{1/3} - c_2 z_2^{3/5} - c_3 z_3^{5/7}\end{aligned}\quad (1)$$

For above system the power of third subsystem is not designed obey the rule of constructing a real homogenous system, but the last one of this paper we will take a real homogenous system as an example.

### Stability Analysis

To prove the stability of the above system, we use the Lyapunov function method and choose a Lyapunov function as

$$V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \frac{1}{2} z_3^2 \quad (2)$$

Then its derivative can be solved as

$$\dot{V} = z_1 z_2 + z_2 z_3 - c_1 z_1^{1/3} z_3 - c_2 z_2^{3/5} z_3 - c_3 z_3^{8/7} \quad (3)$$

Then it is not easy to prove the stability of the above system with classic Lyapunov function method, that is why the stability of above system is usually proved with homogeneous theorem.

## Numerical Simulation Test

Choose  $c_1 = 1, c_2 = 3, c_3 = 5$ , then the matrix  $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -c_1 & -c_2 & -c_3 \end{bmatrix}$  is stable and Hurwitz. The roots can

be solved with Matlab program as follows:

```
a=[0 1 0;
    0 0 1;
    -1 -3 -5];
eig(a)
ans =
-4.3652
-0.3174 + 0.3583i
-0.3174 - 0.3583i
```

$$\dot{z}_1 = z_2$$

Obviously, the system  $\dot{z}_2 = z_3$  is stable.

$$\dot{z}_3 = -c_1 z_1 - c_2 z_2 - c_3 z_3$$

Then we try to write a program to check the stability of the below system

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = -c_1 z_1^{1/3} - c_2 z_2^{3/5} - c_3 z_3^{5/7} + u$$

(4)

We set the input  $u$  as a constant 1, then we write a Matlab program as follows:

```
clc;clear;close all;
tf=120;dt=0.01;u=1;z1=3;z2=3;z3=3;
c1=1;c2=3;c3=15;
for i=1:tf/dt
    dz1=z2;z1=z1+dz1*dt;
    dz2=z3;z2=z2+dz2*dt;
    dz3=-c1*z1^(1/3)-c2*z2^(3/5)-c3*z3^(5/7)+u;
    dz3=-c1*z1^(1)-c2*z2^(1)-c3*z3^(1)+u;
    z3=z3+dz3*dt;
    t=i*dt;tp(i)=t;z1p(i)=z1;z2p(i)=z2;z3p(i)=z3;
end
figure(1);plot(tp,z1p,'k','LineWidth',2);xlabel('t/s');ylabel('z1')
figure(2);plot(tp,z2p,'k','LineWidth',2);xlabel('t/s');ylabel('z2')
figure(3);plot(tp,z3p,'k','LineWidth',2);xlabel('t/s');ylabel('z3')
```

and simulation result can see below figures 1 to 3.

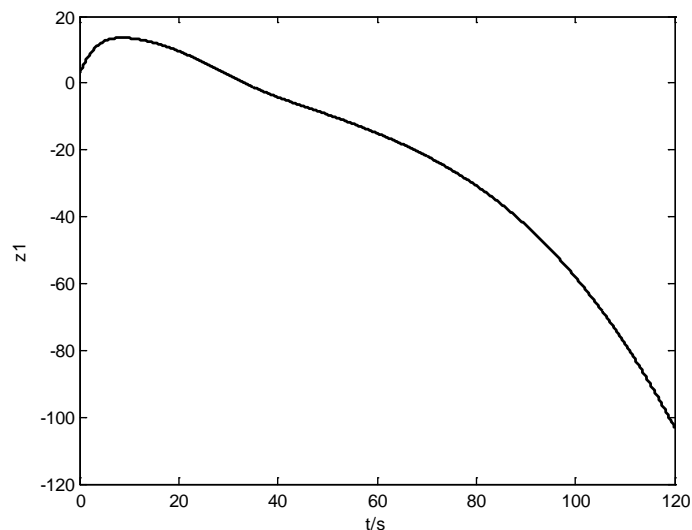


Fig.1 Curve of state  $z_1$

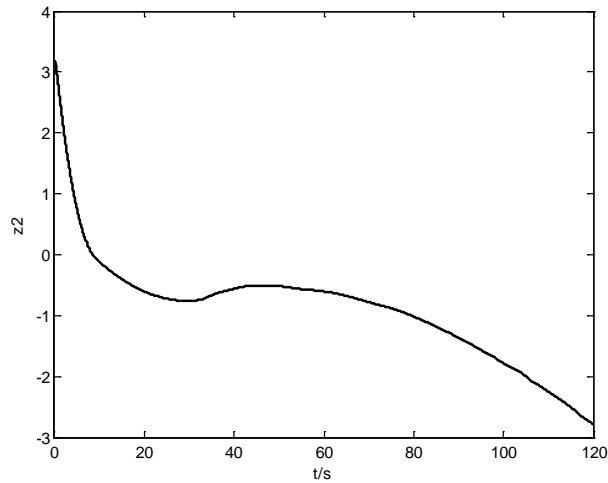


Fig.2 Curve of state  $z_2$

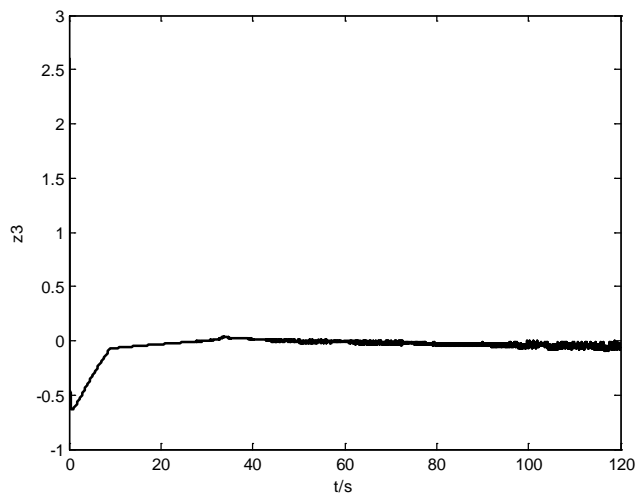


Fig.3 Curve of state  $z_3$

Obviously, the above system is unstable. And for the below system

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= -c_1 z_1 - c_2 z_2 - c_3 z_3 + u\end{aligned}\quad (5)$$

Then we try to write a program to check the stability of the below system. We also do numerical simulations by change the above model with Matlab language as

`dz3=-c1*z1^(1)-c2*z2^(1)-c3*z3^(1)+u;`

and below figure 4 to 6 show the simulation results.

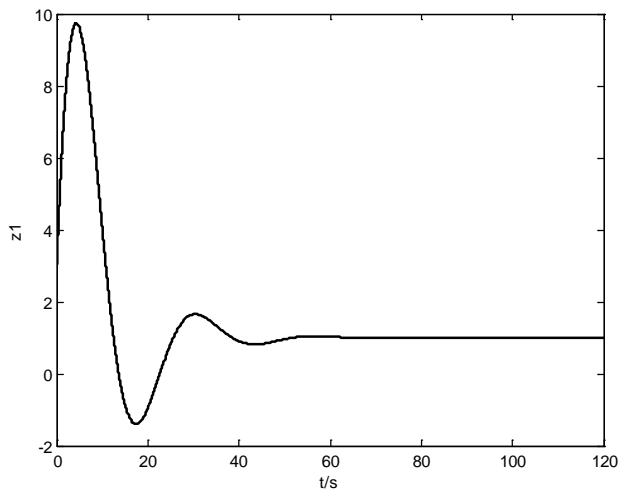


Fig.4 Curve of state  $z_1$

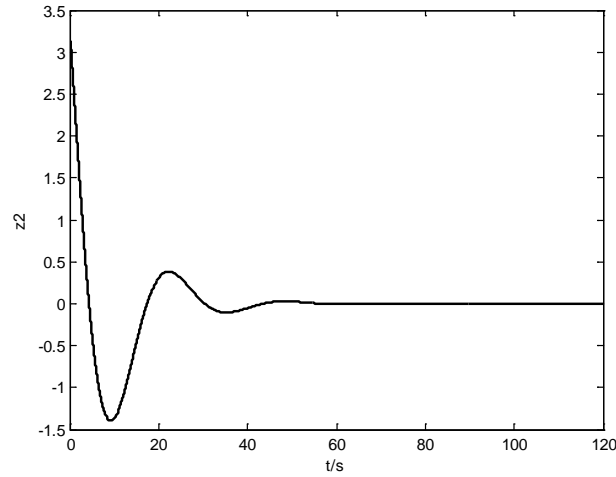


Fig.5 Curve of state  $z_2$

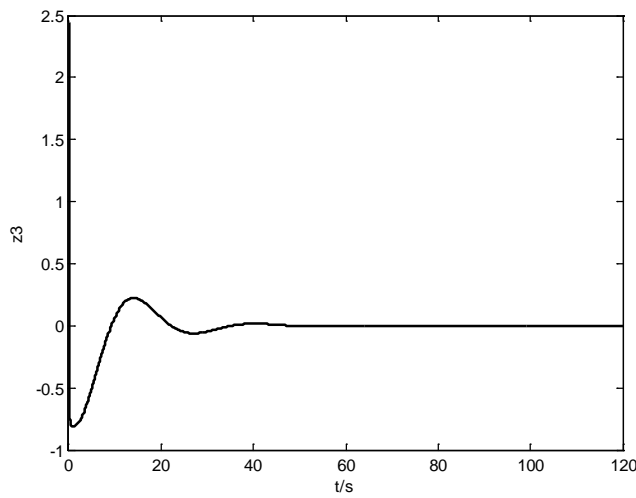


Fig.6 Curve of state  $z_3$

And the above simulation result shows that the system is stable, and the final value of state  $z_1$  can track the input constant 1.

And if we set  $l_3 = 1, l_2 = 3/4, l_1 = \frac{l_2 l_3}{2l_3 - l_2} = 3/5$ , and the model is revised as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= -c_1 z_1^{l_1} - c_2 z_2^{l_2} - c_3 z_3^{l_3} + u \end{aligned} \quad (6)$$

And we do simulation to check the stability, its simulation results can see below figure 6 to 9.

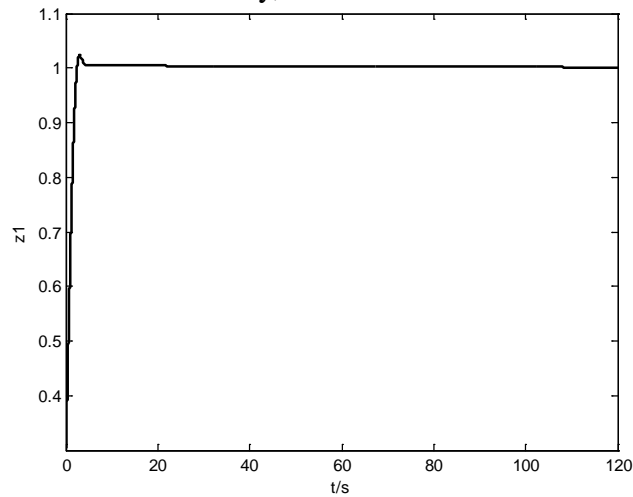


Fig.7 Curve of state  $z_1$

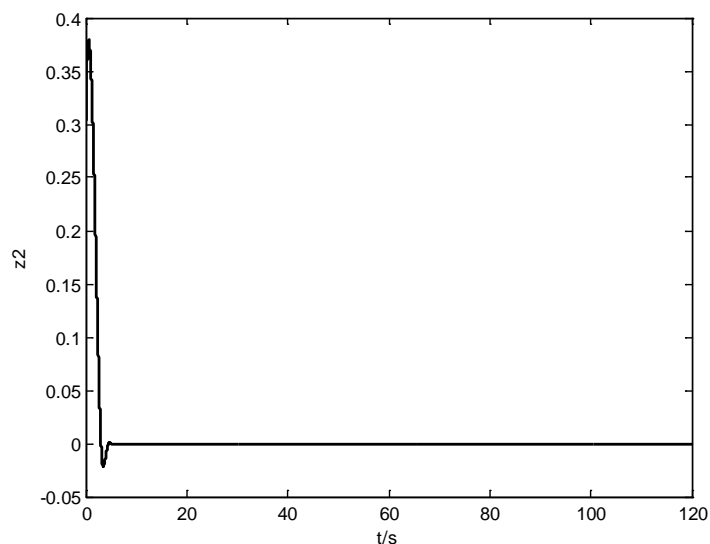


Fig.8 Curve of state  $z_2$

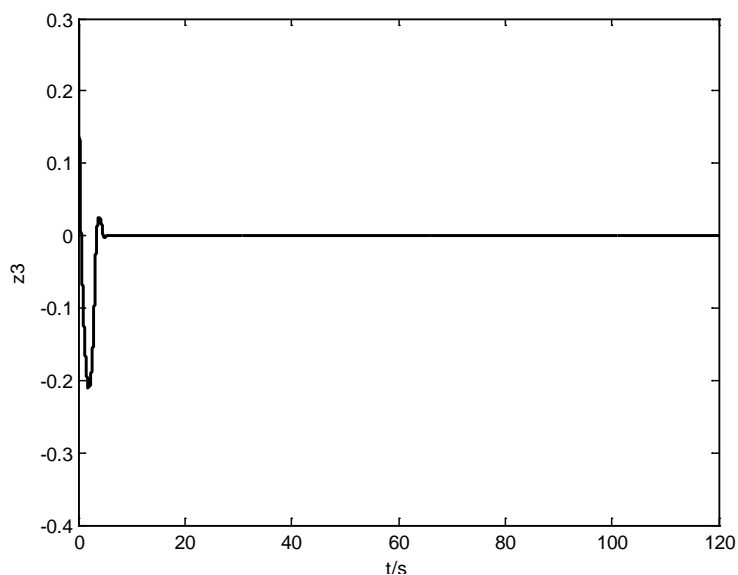


Fig.9 Curve of state  $z_3$

But if we increase the initial value of above system, then the simulation result is changed, which can see figure 10 to 12. Then it is means that the system is still unstable. So the so-called homogeneous method can only guarantee the small scope stability of a system. Also the robustness is not good.

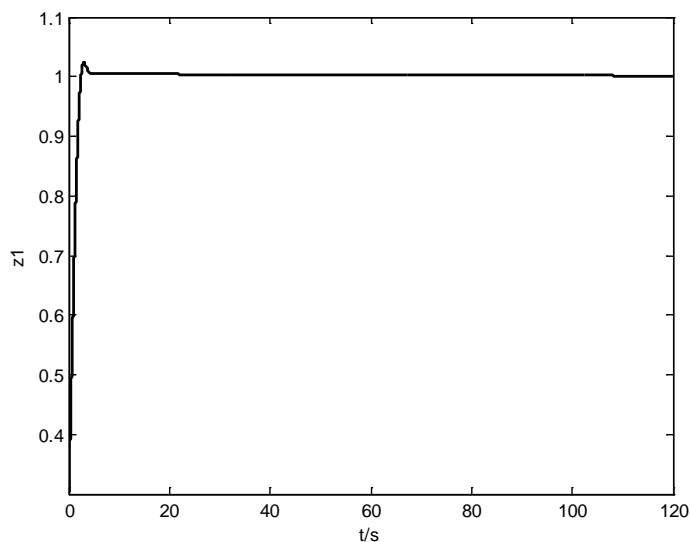


Fig.10 Curve of state  $z_1$

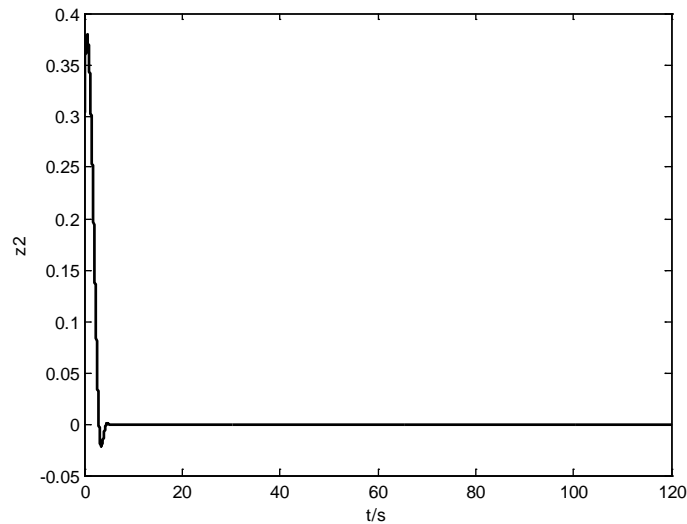


Fig.11 Curve of state  $z_2$

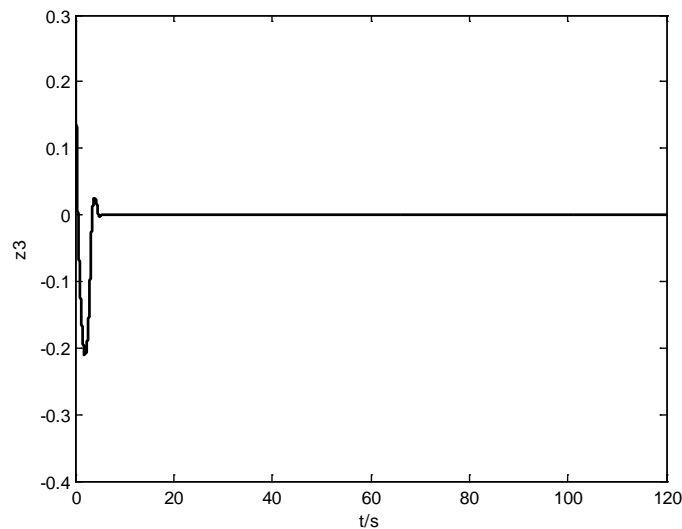


Fig.12 Curve of state  $z_3$

## Conclusions

Three kinds homogenous type third order system are compared by numerical simulation method. And it shows that the traditional linear Hurwitz system can show large range stability for any initial conditions. But the homogenous type nonlinear system can only stable for some small initial values. So the homogenous nonlinear system is stable for a small scope.

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