Stability Analysis of Second Order and Third Order Kind of Tracking Differentiators Based on Lyapunov Method

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Abstract. two kind of complex tracking differentiators are studied and its stability are proved by constructing Lyapunov function method. And the necessary condition for the input signal is that its derivative should be bounded. And according to the proof process, the stability and design of this kind of tracking differential has no better effect than traditional first order filter method to solve the derivative of a signal.

Introduction

It is very common to use program and computer to solve the derivative of a input signal such as guidance of a target[1-4], we will need the speed information of target to predict the next position of target. So how to use computer and numerical simulation to solve the derivative of a input signal is a very useful question. Many researchers proposed many methods[5-7] to solve this problem, such as traditional first order filter and second order filter method. A kind of tracking differentiator method was proposed and widely studied in the past ten years[8-10]. In this paper, we researched the stability of this kind of tracking differentiator, and we found that, the real effect of this tracking differentiator will be almost the same as traditional first order filter method although no simulation were done to compare its real effect of solving derivative of input signals.

Problem Description

A kind of second order tracking differentiator can be described as follows:

$$\begin{aligned}\dot{z}_0 &= v_0 \\ v_0 &= -\lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} \text{sign}(z_0 - f(t)) + z_1 \\ \dot{z}_1 &= -\lambda_1 L \text{sign}(z_1 - v_0)\end{aligned}\quad (1)$$

The Main Problem Is To Prove Its Stability.

Stability Analysis

Define a new variable as

$$\varepsilon_1 = z_0 - f(t) \quad (2)$$

Then

$$\begin{aligned}\dot{\varepsilon}_1 &= \dot{z}_0 - \dot{f}(t) = v_0 - \dot{f}(t) \\ &= -\lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} \text{sign}(z_0 - f(t)) + z_1 - \dot{f}(t) \\ &= -\lambda_0 L^{1/(n+1)} |\varepsilon_1|^{n/(n+1)} \text{sign}(\varepsilon_1) + z_1 - \dot{f}(t)\end{aligned}\quad (3)$$

And

$$\begin{aligned}\dot{z}_1 &= -\lambda_1 L \text{sign}(\lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} \text{sign}(z_0 - f(t))) \\ &= -\lambda_1 L \lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} (z_0 - f(t))\end{aligned}\quad (4)$$

Then the whole system can be rewritten as

$$\dot{\varepsilon}_1 = -\lambda_0 L^{1/(n+1)} |\varepsilon_1|^{n/(n+1)} \text{sign}(\varepsilon_1) + z_1 - \dot{f}(t) \quad (5)$$

$$\dot{z}_1 = -\lambda_1 L \lambda_0 L^{1/(n+1)} |\varepsilon_1|^{n/(n+1)} \varepsilon_1 \quad (6)$$

Choose a Lyapunov function as

$$V = \frac{1}{2} z_1^2 + \frac{1}{2} \varepsilon_1^2 \quad (7)$$

Then its derivative can be solved as

$$\begin{aligned} \dot{V} &= \frac{1}{2} \dot{z}_1^2 + \frac{k}{2} \varepsilon_1^2 \\ &= -\lambda_0 L^{1/(n+1)} |\varepsilon_1|^{n/(n+1)} |\varepsilon_1| + \varepsilon_1 z_1 - \varepsilon_1 \dot{f}(t) \\ &\quad - k \lambda_1 L \lambda_0 L^{1/(n+1)} |\varepsilon_1|^{n/(n+1)} z_1 \varepsilon_1 \end{aligned} \quad (8)$$

If $\dot{f}(t)$ is bounded, then there exist a k such that

$$\dot{V} \leq -\lambda_0 L^{1/(n+1)} |\varepsilon_1|^{n/(n+1)} |\varepsilon_1| + |\varepsilon_1| \quad (9)$$

Then if L is big enough, the system is stable, and ε_1 is converged to zero.

Third Order Tracking Differentiator

There a third order tracking differential can be described as follows

$$\begin{aligned} \dot{z}_0 &= v_0 \\ v_0 &= -\lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} \text{sign}(z_0 - f(t)) + z_1 \\ \dot{z}_1 &= v_1 \\ v_1 &= -\lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \text{sign}(z_1 - v_0) + z_2 \\ \dot{z}_2 &= -\lambda_2 L \text{sign}(z_2 - v_1) \end{aligned} \quad (10)$$

Define a new variable as

$$\varepsilon_0 = z_0 - f(t) \quad (11)$$

then

$$\dot{\varepsilon}_0 = \dot{z}_0 - \dot{f}(t) = -\lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0 + z_1 - \dot{f}(t) \quad (12)$$

And

$$\begin{aligned} \dot{z}_1 &= -\lambda_1 L^{1/(n+1)} |z_1 - v_0|^{-1/n} (z_1 - v_0) + z_2 \\ &= -\lambda_1 L^{1/(n+1)} \left| \lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} \text{sign}(z_0 - f(t)) \right|^{-1/n} \\ &\quad (\lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} \text{sign}(z_0 - f(t))) + z_2 \\ &= -\lambda_1 L^{1/(n+1)} \lambda_0 L^{1/(n+1)} \left| |\varepsilon_0|^{n/(n+1)} \text{sign}(\varepsilon_0) \right|^{-1/n} (\lambda_0 L^{1/(n+1)} |\varepsilon_0|^{n/(n+1)} \text{sign}(\varepsilon_0)) + z_2 \\ &= -\lambda_1 L^{1/(n+1)} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0 + z_2 \end{aligned} \quad (13)$$

Then it can rewritten as

$$\dot{z}_1 = -\lambda_1 L^{1/(n+1)} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0 + z_2 \quad (14)$$

And the last system can be rewritten as

$$\begin{aligned}
\dot{z}_2 &= -\lambda_2 L \text{sign}(z_2 - v_1) \\
&= -\lambda_2 L \text{sign}(\lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \text{sign}(z_1 - v_0)) \\
&= -\lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \text{sign}(\text{sign}(z_1 - v_0)) \\
&= -\lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} (z_1 - v_0) \\
&= -\lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} (\lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{n/(n+1)} \text{sign}(z_0 - f(t))) \\
&= -\lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \lambda_0 L^{1/(n+1)} |z_0 - f(t)|^{-1/(n+1)} \varepsilon_0
\end{aligned} \tag{15}$$

It can also be rewritten as

$$\begin{aligned}
\dot{z}_2 &= -\lambda_2 L \text{sign}(z_2 - v_1) \\
&= -\lambda_2 L \text{sign}(\lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \text{sign}(z_1 - v_0)) \\
&= -\lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \text{sign}(\text{sign}(z_1 - v_0)) \\
&= -\lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} z_1 + \lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} v_0 \\
&= -\lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} z_1 + \lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0 \\
&\quad - \lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} z_1
\end{aligned} \tag{16}$$

Then the whole system can be rewritten as

$$\dot{\varepsilon}_0 = -\lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0 + z_1 - \dot{f}(t) \tag{17}$$

$$\dot{z}_1 = -\lambda_1 L^{1/(n+1)} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0 + z_2 \tag{18}$$

$$\dot{z}_2 = -2\lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{(n-1)/n} z_1 + \lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0 \tag{19}$$

If we choose a Lyapunov function as

$$V = \frac{1}{2} \varepsilon_0^2 + \frac{k_1}{2} z_1^2 + \frac{k_2}{2} z_2^2 \tag{20}$$

Then there exist k_1 and k_2 its derivative can be solved as

$$\begin{aligned}
\dot{V} &\leq -\lambda_1 L^{1/(n+1)} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0^2 - \dot{f}(t) \varepsilon_0 \\
&\quad + \lambda_2 L \lambda_1 L^{1/(n+1)} |z_1 - v_0|^{n-1/n} \lambda_0 L^{1/(n+1)} |\varepsilon_0|^{-1/(n+1)} \varepsilon_0 z_2
\end{aligned} \tag{21}$$

Then if $\dot{f}(t)$ is bounded, the stability of the above tracking differential is proved by Lyapunov stability theorem.

Conclusions

The stability of second order tracking differentiator and third order differentiator are proved by constructing a Lyapunov function method. And we found that the second order tracking differentiator is easy to prove its stability, also the input signal should be bounded and its derivative also should be bounded, then the stability of above tracking differentiator can be guaranteed.

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