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Analysis of digital differentiator's differential effect

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Abstract：In order to analyze the performance of the differential device, the cosine signals' spectrum is caculated to compare. Differentiate has two methods: mathematical differentiate and digital differentiator, so two methods can be used to estimate the differential effect of digital differentiator. Digital differentiator has the following parameters: sampling frequency, cutoff frequency, sampling points, resolution, etc. When these parameters change, differential results will inevitably change with those parameters. The experimental results show , only when each parameter is selected correctly, the ideal effect of differentiate can be achieved.

Introduction

In recent years, the differential operator in many fields of engineering application and science and technology caused wide attention and in-depth study, one of the more important is the fractional order operations, including the fractional order derivative and fractional integrals, fractional Fourier transform and various kinds of fractional order transformation, they are useful tools which can analysis and processing nonlinear, non-causal ,non-minimum phase, non-gaussian and non-stationary problems. And the fractional order operation is based on differential operation, differential operation in terms of signal singularity detection and extraction has special effects. This article is based on the theory of digital differentiator, detailed study of the selection of various parameters for the differentiator performance impact.

Principle of differentiator

If the continuous function $f(t)$ of Fourier transform is $F(t)$,and the $f(t)$ differential is $\frac{df(t)}{dt}$, then：

$F[\frac{df(t)}{dt}] = j2p\ fF[j2p\ f]$，则：

$$\frac{df(t)}{dt} = F^{-1}[F[\frac{df(t)}{dt}]] = F^{-1}[j2p\ fF[j2p\ f]]$$

$$=F^{-1}[j2p\ f] * f(t)$$

Therefore，$F^{-1}[j2p\ f]$ is the impulse response，is also simulated differentiator, and remember to $h(t)$：

$$h(t)=\frac{1}{2p}\int_{-\infty}^{+\infty} j2p\ fe^{j2p\ ft} d2p\ f$$

Take cut-off frequency for f_t, meaning that the ideal frequency response curve,

$$H_d(f) = \begin{cases} 1 & f \le f_t \\ 0 & f > f_t \end{cases}$$

Change $h(t)$ integral upper and lower limits

, get:

$$h(t)=\frac{1}{2p}\int_{-f_t}^{+f_t} j2p\, fe^{j2p\, ft}d2p\, f\ ,$$

After the integral available:

$$h(t)=\frac{p\, f_t t[2\cos(2p\, f_t t)-\sin(2p\, f_t t)]}{p t^2},$$

To above formula discrete digital sampling , replace t

with nT_s,

The above formula is changed:

$$h(n)=\frac{p\, f_t nT_s[2\cos(2p\, f_t nT_s)-\sin(2p\, f_t nT_s)]}{p(nT_s)^2}$$

In order to smooth response function h(n), reduce the GIBBS phenomenon and corrugated,

h(n) multiplied by the hanning window function and moves to the right, can get the following formula:

$$h(n)=\frac{q[2\cos(q)-\sin(q)]}{2p((n-M_0)T_s)^2}(1-\cos\frac{pn}{M_0})$$

M_0 represent half the width of the window,

moves to the right is to make the scope of time domain response h (n) in the n > 0, meet the causality。

Sampling frequency, cut-off frequency and sampling points, the resolution

Cut-off frequency f_t refers to the need analysis of the highest frequency, is also the highest signal frequency after filtering。According to the sampling theorem, f_t and the relationship between the sampling frequency Fs is commonly: F_s=2.56 f_t; While selection of the highest frequency analysis is determined by the rotation speed of equipment and expectations to determine the fault nature。

The relationship between sampling points N and the spectrum line number M is as follows:

N=2.56M, Which spectrum line number M and frequency resolution *ΔF and* cut-off frequency f_t has the following relationship: ΔF= f_t/M

How many of sampling points related to require how much frequency resolution。For example: the machine speed 3000r/min=50Hz, if you want to analysis the failure frequency is 8 times frequency, demand the frequency resolution of spectrum diagram *ΔF*=1Hz, the sampling frequency and sampling points is set to:

cut-off frequency f_t=8·50Hz=400Hz;

samplingfrequency Fs=2.56· f_t=2.56·400Hz=1024Hz;

sampling points N=2.56·（f_t/ΔF）=2.56·（400Hz/1Hz）=1024 Hz

spectrum line number M=N/2.56=1024/2.56=400

Sampling points is not the bigger the better, and vice versa. For rotating machine must meet the whole cycle sampling, to eliminate the frequency of malformation, simply raising resolution also cannot eliminate the frequency of malformation。Do not produce frequency mixing of the lowest sampling frequency F_s asked 2 times the cut-off frequency f_t, the reason by 2.56 times, mainly considering relative to the Nyquist frequency, can maintain a certain margin, but also related to computer binary representation. Its main purpose is to avoid confusion signal, to ensure that the high frequency signal is not distorted into a low frequency signal. The choice of sampling length *T* must first ensure that could reflect the whole picture signals, the transient signal should include the whole transient process; A cycle of periodic signal, theoretically to collect a periodic signal is ok. Second to consider the frequency resolution, sampling length *T* under the condition of the cut-off frequency f_t to determine and frequency resolution $\triangle f$ is inversely proportional, namely, the longer *T*, $\triangle f$ is smaller, the higher the frequency resolution.

Differential signal and its power spectrum

Set x (n) for a power signal, and its power spectrum are defined as follows:

$$S(w)=\frac{1}{2T}\left|X_T(jw)\right|^2 \qquad (1)$$

After discretization, it is defined as:

$$S(w)=\frac{1}{N}\left|\sum_{n=0}^{N-1}x(n)\mathrm{e}^{-jwn}\right|^2 \qquad (2)$$

For the cosine signal, the Fourier transform is:

$$\cos(w_0t)\leftrightarrow p[d(w+w_0)+d(w-w_0)] \qquad (3)$$

After the cosine function differential, the Fourier transform is:

$$d\cos(w_0t)/dt\leftrightarrow jw_0p[d(w+w_0)+d(w-w_0)]$$

It can be seen that each differential, the amplitude of cosine function of Fourier transform to expand w_0 times, and its power spectrum amplitude is expanding $w_0^{\,2}$ Times. If it differential n times, its power spectrum amplitude to expand $w_0^{\,2n}$ times.

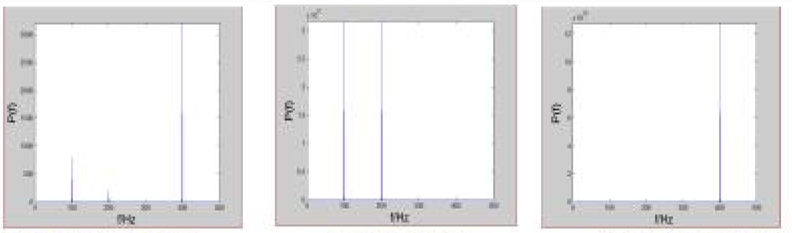
The experimental results

In this paper, first of all, on the basis of the theory of the differentiator，reference（5），to establish a differential function *Dif(x,t,* f_t*,M,* M_0*,N)*, Among the parameters are represented in turn: the corrected data, sampling period, the length of the Fourier transform, half window width and the number of signals. In order to effectively determine differentiator differential effect, specially set the

following signal:
$y = 2cos(2p f_1 t)$
$+ cos(2p f_2 t)+ 2cos(2p f_3 t)$ (5)
Among f_1=100HZ, f_2=200HZ, f_3=400HZ,
Respectively by mathematical method and the differentiator methods to the above signal take a differential and quadratic differential, and then calculate the power spectrum of them respectively, by comparing to analyze the differential effect of differentiator. Mathematical method is adopting the method of derivative to get the signal y of a differential and quadratic differential。According to the type (1) to (4), type (5) of the power spectrum before differential should respectively in 100 HZ, 200 HZ and 400 HZ has three peaks, and the peak value of 100 hz and 400 HZ should be equal. Figure 1 for the sampling frequency is 800 HZ, sampling time about 1 second, to the original signal, and using mathematical methods find out a differential and quadratic differential signal to calculate the power spectrum .Can be seen in the figure, at the time of sampling frequency is 800 HZ and the power spectrum is not correct. Because according to the method of derivation, its power spectrum amplitude for f1, f2, f3 signal, in no differential state, its power spectrum amplitude is:
（2^2，1^2，2^2）；
A differential state:（$(2x2p\ x100)^2$, $(2p\ x200)^2$,$(2x2p\ x400)^2$）；
Quadratic differential state:（$(2x2p\ x100)^4$, $(2p\ x200)^4$,$(2x2p\ x400)^4$）. Therefore, at the time of sampling frequency is 800 hz, three kinds of state power spectrum amplitude ratio is wrong. When the sampling frequency is 1000 hz, three kinds of state power spectral amplitude ratio is correct, As shown in figure 2, no differential, the amplitude of f1 and f3 basic quite; And when a differential and quadratic differential, the amplitude of signal f1 and f2 basic equal, is in line with the theoretical analysis. The signal when increase the sampling frequency to 10000 hz, the situation basically remain unchanged.As shown in figure 3, this is because the signal of the highest frequency is 400 HZ, After to the signaly take method of digital differentiator for differential, as shown in figure 4. And when a differential and quadratic differential, the amplitude of signal f1 and f2 basic equal, is in line with the theoretical analysis. The signal when increase the sampling frequency to 10000 hz, the situation basically remain unchanged.As shown in figure 3, this is because the signal of the highest frequency is 400 HZ, according to the Nyquist theorem, sampling frequency at least two times as much can restore the original signal correctly, but just double the effect is not ideal. After to the signal y take method of digital differentiator for differential, as shown in figure 4.Parameters using (x, 0.0001, 00, 400327, 68100, 0100), the result compared with figure 2, is roughly the same.

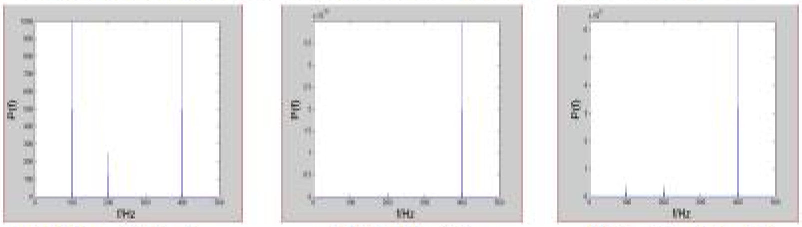
Analysis and Conclusion

In this article, through analyzing the principle of differentiator, and then establish a differentiator function, and based on the cosine function of differential signal to analyze the performance of the differentiator. By changing various parameters of the differentiator, and then comparing with through the mathematical method of differential signal power spectrum, points out the change of each parameter can lead to what the results, as for the cause of this change, due to space limitation, the author of this article will analyze in another article. The results of analysis in this paper for the differentiator and the choice of parameter values have certain guiding significance



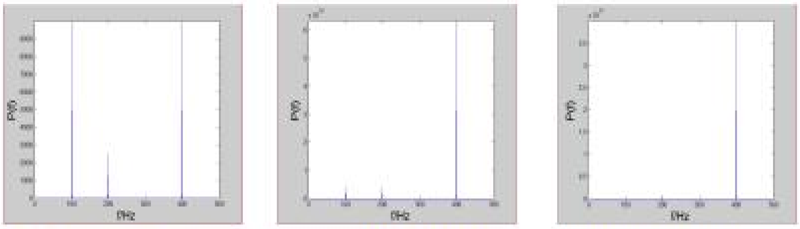
(a) No differential signal (b) A differential (c) Quadratic differential

Figure 1, mathematical differential method of power spectrum （Sampling frequency is 800HZ）



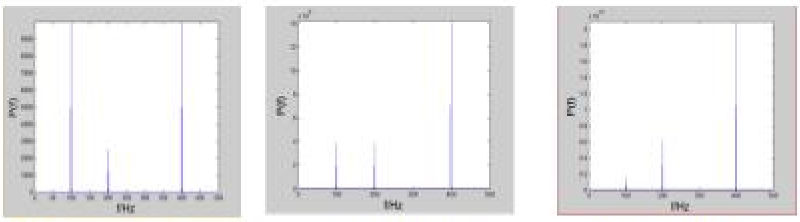
(a) No differential signal (b) A differential (c) Quadratic differential

Figure 2, mathematical differential method of power spectrum （Sampling frequency is 1000HZ）



(a) No differential signal (b) A differential (c) Quadratic differentia

Figure 3, mathematical differential method of power spectrum （Sampling frequency is 10000HZ）



(a) No differential signal (b) A differential (c) Quadratic differentia

Figure 4, mathematical differentiator method of power spectrum, parameters for （x, 0.0001, 400, 32768, 1000, 10000）

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