

# Study and Application of $1\frac{1}{2}$ Spectrum and Its Cepstrum In Fault Diagnosis

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**Abstract:** In order to improve the correct rate of fault diagnosis, through combining cepstrum and  $1\frac{1}{2}$  dimension spectrum,  $1\frac{1}{2}$  dimension cepstrum was defined. Theoretically,  $1\frac{1}{2}$  dimension spectrum, can suppress the Gauss noise completely, cepstrum can reduce false peaks in spectra, but the noise has a significant influence on the cepstrum analysis results.  $1\frac{1}{2}$  cepstrum, can combine with the advantages of  $1\frac{1}{2}$  dimension spectrum and cepstrum. Through the application of  $1\frac{1}{2}$  dimension spectrum, cepstrum and  $1\frac{1}{2}$  cepstrum in the overflow valve is fault diagnosis, the results show that the  $1\frac{1}{2}$  cepstrum can obtain a better effect.

## Introduction

Higher order spectrum technology is a signal processing technology in recent years, is right to Non-gaussian and nonlinear, noncausal signal processing and gaussian colored noise and blind signal processing with very useful important analysis tool. Higher order spectrum can make up the second order statistics (power spectrum) do not contain the defects of phase information. When the signal containing additive colored gaussian noise, in theory, higher order cumulant (higher order spectrum) can restrain the influence of noise, improve the analysis and identification precision [1-4]. Fractal generally consists of three elements, namely, shape, dimension and opportunities. Generally come from some very rules in the characteristics of the signal to find out its structural characteristics, the fractal dimension of finding fault with its normal state relations and used as the basis for fault quantitative identification. Common fractal dimension has the capacity dimension, correlation dimension and the generalized dimension, etc. In this paper, based on this, define  $1\frac{1}{2}$  cepstrum, and then pour  $1\frac{1}{2}$  cepstrum by calculation the correlation dimension of overflow valve fault diagnosis, and the diagnosis effect compared with  $1\frac{1}{2}$  dimensional spectrum.

## Cepstrum and $1\frac{1}{2}$ cepstrum

The inverse Fourier transform power spectrum of numerical called cepstrum. Sets the power spectrum of signal  $x(n)$  to be, it is defined as:

$$S_x(\omega) = \sum_{m=-\infty}^{m=\infty} R_x(m) e^{-j\omega m} \quad (1)$$

$$R_x(m) = E\{x(n)x(n+m)\},$$

for signal autocorrelation function of  $x(n)$ . Signal  $\{x(n)\}$  third-order cumulants for:

$$c_{3x}(\tau_1, \tau_2) = E\{x(n)x(n+\tau_1)x(n+\tau_2)\} \quad (2)$$

Again, to make  $\tau_1 = \tau_2 = m$ , taking the Fourier transform to get dimensional spectrum:

$$B(\omega) = \sum_{m=-\infty}^{\infty} c_{3x}(m, m) e^{-j\omega m} \quad (3)$$

The inverse Fourier transform power spectrum of logarithmic called cepstrum:

$$C_x(n) = F^{-1}(\log S(\omega)) \quad (4)$$

Cepstrum can turn the complex signal simplification, easy to identify concerns signal components. Set: a system of input for  $x(t)$ , the pulse response function of the system for  $h(t)$ , then the system output is:

$$y(t) = x(t) * h(t)$$

By the Fourier transform

$$y(\omega) = x(\omega) \cdot H(\omega)$$

The power spectrum is:

$$S_y(\omega) = S_x(\omega) |H(\omega)|^2$$

On both sides of the exponential:

$$\log S_y(\omega) = \log S_x(\omega) + 2 \log |H(\omega)|$$

After both sides take inverse Fourier transform again:

$$F^{-1}\{\log S_y(\omega)\} = F^{-1}\{\log S_x(\omega)\} + F^{-1}\{2 \log |H(\omega)|\}$$

Namely get cepstrum:

$$C_y(n) = C_x(n) + C_h(n)$$

Shows, the power spectrum of the output signal  $S_y(f)$  as a result of the transmission channel interference and influence, it is difficult to directly identify, in cepstrum,  $C_x(\tau)$  and  $C_h(\tau)$  is linear and relationship, and is located in a different position, so easy to identify.

Due to  $\frac{1}{2}$  dimensional spectrum and power spectrum similar in form, only the power spectrum of the autocorrelation function is replaced with third-order cumulants diagonal, so this article will

$\frac{1}{2}$  dimensional spectrum  $B(\omega)$  in type (1), instead of power spectrum  $S_x(\omega)$  namely get

$\frac{1}{2}$  dimensional spectrum, pour in the calculation of actual  $\frac{1}{2}$  cepstrum using the type:

$$C_B(n) = F^{-1} |\log B(\omega)| \quad (5)$$

The discrete signal may be obtained from (1) :

$$\begin{aligned}
 S_x(\omega) &= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x^*(n)x(n+m)e^{-j\omega n} \\
 &= \left( \sum_{n=-N}^N x^*(n)e^{j\omega n} \right) \left( \sum_{m=-\infty}^{\infty} x(n+m)e^{-j\omega(n+m)} \right) \quad (6) \\
 &= \left( \sum_{n=-N}^N x(n)e^{j\omega n} \right)^2 = |X(j\omega)|^2
 \end{aligned}$$

Its corresponding cepstrum is:

$C_x(n) = F^{-1}(\log S_x(\omega)) = F^{-1}(\log |X(j\omega)|^2)$  (7) For  $1\frac{1}{2}$  dimensional spectrum, also will be the following transformation type (3) :

$$\begin{aligned}
 B(\omega) &= \sum_{n=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} x^*(n)x^2(n+m)e^{-j\omega n} \\
 &= \left( \sum_{n=-N}^N x^*(n)e^{j\omega n} \right) \left( \sum_{m=-\infty}^{\infty} x^2(n+m)e^{-j\omega(n+m)} \right) \quad (8) \\
 &= X(j\omega)^* Y(j\omega)
 \end{aligned}$$

Among them,

$$Y(j\omega) = \sum_{n=-\infty}^{\infty} x^2(n)e^{-j\omega n}$$

The corresponding  $1\frac{1}{2}$  cepstrum to:

$$C_B(n) = F^{-1}(\log B(\omega)) = F^{-1}(\log(X^*(j\omega)Y(j\omega))) \quad (9)$$

By type (6) it can be seen that power spectrum is a positive number sequences, does not include the phase information. By type (7), it is concluded that power cepstrum is a symmetric sequence, as a result, it can't distinguish between minimum phase and maximum phase sequence. Type (8) show that the  $1\frac{1}{2}$  dimensional spectrum contains the amplitude and phase information, its nature of

cepstrum also contains the amplitude and phase information. (6), points out that literature  $1\frac{1}{2}$

dimensional spectrum can signal coupling information. To sum up,  $1\frac{1}{2}$  cepstrum at the same time combines the advantages of cepstrum and  $1\frac{1}{2}$  dimensional spectrum.

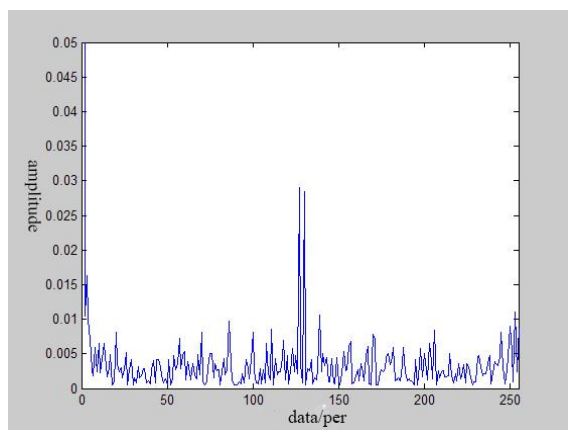


Figure 1 normal state  $1\frac{1}{2}$  cepstrum spectra

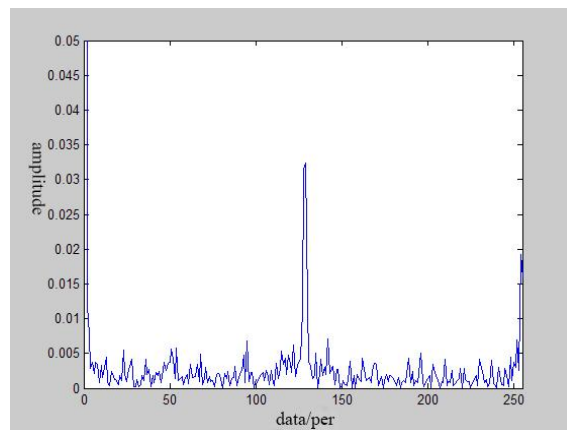


Figure 2 fault state  $1\frac{1}{2}$  cepstrum spectra

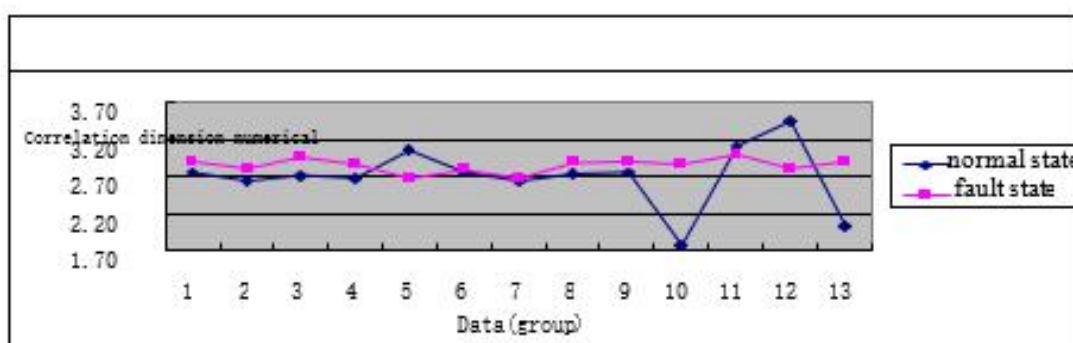


Figure3 normal state and fault state  $1\frac{1}{2}$  dimension cepstrum correlation dimensions line char

## Spectra

In this experiment, the date of 26 groups in the normal state and fault state were first calculated the  $1\frac{1}{2}$  cepstrum by equation (5). In the normal state and fault state, the  $1\frac{1}{2}$  cepstrum of each group of the oil pressure equivalent to 2MPa are shown in FIG.1 to FIG. 2. It can be seen from figure 1 and figure 1, the  $1\frac{1}{2}$  cepstrum in the normal state is more dense than the fault one in the distribution of the spectrum. From the  $1\frac{1}{2}$  dimension spectrum which in two states, the distribution of the spectrum peak is not obvious. In general, the  $1\frac{1}{2}$  cepstrum is a little better to distinguish than the  $1\frac{1}{2}$  dimension spectrum in the two states.

## Experimental Results

In order to effectively distinguish the fault, this text quantified  $1\frac{1}{2}$  cepstrum and  $1\frac{1}{2}$  dimension spectrum using fractal methods. Since  $1\frac{1}{2}$  cepstrum obtained the time sequence signal of reduction, the correlation dimension of two states  $1\frac{1}{2}$  cepstrum was calculated by using the method of literature [6]. In order to observe the results effectively, we drew a point line chart, as shown in figure 3. And this figure represents the number of data sets in the horizontal coordinates, the unit is the group, and the ordinate representation calculates the correlation dimension, no dimensionality. It can be seen from the figure 3, signal correlation dimension in normal and fault states has more apparent differences on the whole, further analysis, the value of correlation dimensions of number 6 data points in fault state as boundary values, i.e., according to the correlation dimension is greater than or equal to 2.7994, the judgment is fault state, with less than 2.7994, judgment as normal state, for the fault state, misjudgment data number is 2, and for the normal state, misjudgment data number is 3, the overall accuracy is  $(26-5)/26 = 21/26$ , more than 80%, the result is satisfactory. In order to make the comparison, the diagnosis effect of  $1\frac{1}{2}$  dimension spectrum was also analyzed. Because of the smooth nature of  $1\frac{1}{2}$  dimension spectrum, the capacity dimension of  $1\frac{1}{2}$  dimension spectrum in two state was calculated using the method of literature [7]. The results show that the normal state and fault state are indistinguishable. To further comparison, this experiment also calculated the power spectrum and the cepstrum of the following formula (1) and (2), namely of cepstrum was calculated by two kinds of state of the correlation dimension, the power spectrum to calculate the capacity of the two states, the calculation results show that both cepstrum and power spectrum, the two states of fractal dimension are indistinguishable.

## Analysis and Conclusion of

### Experimental Results

This experiment drew  $1\frac{1}{2}$  cepstrum by combining cepstrum and  $1\frac{1}{2}$  dimension spectrum, and used  $1\frac{1}{2}$  cepstrum for fault diagnosis, obtained the good effect. The same experimental data, through the method of literature (3) and (4), the accuracy of fault diagnosis are below 70%, the results show that by combine the advantages of cepstrum and  $1\frac{1}{2}$  dimension spectrum, in some of the fault diagnosis can be achieve better effect.

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