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A new pedestrian-footbridge interaction model

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Abstract. In this paper "the plane bifilar pendulum model" is proposed to understand excessive lateral vibration of a suspension footbridge under crowd excitation. We use a plane bifilar pendulum to describe a suspension bridge by considering its structural features, which consists of two strings and a central rigid body representing the cables and deck of the footbridge respectively. In addition, the vertical and lateral forces exerted by crowd on the deck both are considered to be harmonic with constant amplitudes. According to Lagrange method, we found that the dynamic behavior of the suspension footbridge under crowd-induced excitation can be described by a Hill equation. The solution and its stability of the plane pendulum model are theoretically analyzed based on the perturbation method, the correctness of which is verified by numerical simulations. By applying the analytical results to the London Millennium Bridge (a famous suspension bridge), we can easily explain the occurrence of excessive lateral vibration with 0.48 and 0.96 Hz and the "lock-in" phenomenon. Our research suggests that structural features of a suspension footbridge should not be ignored in the investigation of the pedestrian-footbridge interaction.

Introduction

More and more slender footbridges with low natural frequencies (less than 3 Hz) have become popular in the past few years with the improvement of mechanical characteristics of materials and technological advancements. Experimental measurements showed that a pedestrian can induce both vertical and horizontal dynamic time-varying forces on the surface of a structure. And the frequency of vertical force is equal to the pacing frequency about in the range of $1.4 - 2.4$ Hz, while that of the horizontal force is half the pacing frequency. Then the frequency is between 0.7 and 1.2 Hz [1]. Therefore, many modern footbridges are very sensitive to human actions in the lateral direction.

The London Millennium Bridge is the best-known footbridge closed after opening due to excessive lateral vibration. It was found that the first frequency of the lateral modes of the bridge is 0.48 Hz, which is so far below normal range of lateral forcing frequencies induced by pedestrians. It is difficult to explain the occurrence of so low lateral frequency by applying direct or internal resonance theory. Dynamic interaction mechanisms to explain excessive lateral vibration of footbridges have been focused in recent years. By assuming that the force exerted by pedestrians on a moving bridge is harmonic and its amplitude relates to the lateral displacement of the bridge, Piccardo and Tubino [2] described the motion of the Millennium Bridge using a Mathieu equation. In their model, unstable oscillations of the bridge occur if the frequency of the lateral frequency of the footbridge approaches half the lateral step frequency. Then parametric excitation mechanism was proposed by them to understand the occurrence of lateral mode frequency of 0.48 Hz in the Millennium Bridge. It need to be noticed that Dallard et al. [3] assumed from the tests that the lateral force induced by pedestrians is proportional to the lateral velocity of the Millennium Bridge, which is different from that of Piccardo and Tubino.

As can be seen from literature about pedestrian-footbridge interaction, two points are worth highlighting concerning the current study of lateral vibration of footbridges. One is that the emphasis almost has been put on the lateral force model of pedestrians; the other is that the structural features of footbridges were seldom taken into account. The footbridge in almost

all dynamic models was regarded as a damped single-degree-of-freedom system no matter what its structure type. Piccard and Tubino by proposing a new lateral force model of pedestrians to explain the occurrence of lateral mode frequency of 0.48 Hz in the Millennium Bridge. This inspired us to find clues to the mystery about the Millennium Bridge by considering its structural feature in dynamic model. McRobie et al. [4] investigated experimentally on the lock-in phenomenon in suspension bridges by using a section model consisted of two diagonal bars representing the cables and the central rigid body the deck according to wind engineering practice. Considering the features of a suspension bridge, it may be appropriate to be modeled by using a section cut from the bridge if the attention is paid to its lateral vibration. By simplifying the vertical and lateral forces exerted by pedestrians to be harmonic, shown as in Fig.1, we find that the dynamics model describing pedestrian-footbridge interaction is governed by a Hill's equation. We call this model "the plane bifilar pendulum model". By applying this plane model we can easily explain the occurrence of the first and second lateral frequencies (0.48 and 0.96 Hz) and the "lock-in" phenomenon in the Millennium Bridge. Our research indicates that the structural type of a footbridge maybe also import in the study of pedestrian-bridge interaction, which usually was neglected in previous works.

The rest of the paper is organized as follows: In section 2 the governing equation of the plane pendulum model described in Fig.1 is established. In sections 3, the stability charts of the governing equation are derived. Conclusions are given in section 4.

The plane bifilar pendulum model

Fig.1 illustrates our dynamic model for lateral vibrations of a suspension bridge under crowed-induced excitation. In order to simplify the problem, we assume that vertical and lateral forces induced by pedestrians act on the center of mass of the deck (point C in Fig.1), which are denoted by $F_V(t)$ and $F_L(t)$ respectively. The length of the same two cables and the width of the deck are are L_1 and $2L_2$ respectively. Furthermore, the tensile and bending deformations of cables and deck are not taken into consideration for simplicity. The mass of deck is assumed to be M and the mass of cables is very small compared to M that can be neglected. Let a coordinate system center at point O. The angles between two cables and y axis both are assumed to be q_0 when the bridge is at rest; the rotation angles of two cables leaving from equilibrium positions are assumed to be q_1 and q_2 respectively when the bridge sways. The governing equation of the model can be given by using Lagrange method:

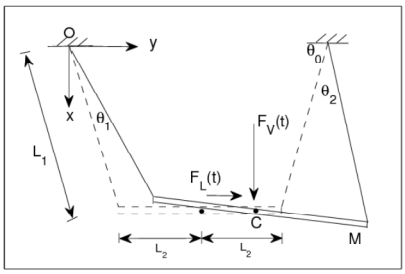


Figure 1. The plane bifilar pendulum model

$$\ddot{q}_1 + 2a_1\dot{q}_1 + \left(a_2 + a_3F_L(t) + a_4F_V(t)\right)q_1 = a_5F_L(t), \tag{1}$$

where

$$a_1 = \frac{1}{2}\frac{C_L L_2^2 \sin^2 q_0 + C_{q_C}\cos^2 q_0}{mL_2^2 \sin^2 q_0 + J_C \cos^2 q_0},\quad a_2 = \frac{mg\left(L_2 + L_1\cos^3 q_0\right)\left(L_2 + L_1 - L_1\cos q_0\right)^2}{mL_1L_2\sin^3 q_0\left(L_2 + L_1 - L_1\cos q_0\right)^2 + J_C L_1 L_2 \sin q_0 \cos^2 q_0},$$

$a_3 = \frac{L_2 \cos q_0 (L_2 + L_1 \cos q_0)}{L_1 (mL_2^2 \sin^2 q_0 + J_C \cos^2 q_0)}$, $a_4 = \frac{L_2 (L_2 + L_1 \cos^3 q_0)}{L_1 \sin q_0 (mL_2^2 \sin^2 q_0 + J_C \cos^2 q_0)}$, $a_5 = -\frac{L_2^2 \sin q_0}{L_1 (mL_2^2 \sin^2 q_0 + J_C \cos^2 q_0)}$.

It is worth pointing out that the natural circle frequency of the plane bifilar pendulum model, denoted by w and derived by letting $a_{1,3,4,5} = 0$ in Eq.(5), is given by $w = \sqrt{a_2}$. The above result is simplified as $w = \sqrt{g/L_1}$ if we take $q_0 = p/2$, which coincides with the natural circle frequency of a plane single pendulum. For further analyzing the dynamic of the plane bifilar pendulum, we consider the vertical and lateral forces produced by pedestrians varying harmonically are expressed by [5]

$$F_i(t) = l\, a_i g m_p(x) \cos(\Omega_i t), i = V, L,$$

where l is the percentage of synchronized pedestrians, $a_i, i = V, L,$ the so-called "dynamic loading factors" and depend on the considered load harmonic and on the load directions. g is the gravity acceleration, $\Omega_i, i = V, L$,the dominant walking frequencies. $m_p(x)$ is the distribution of the pedestrian mass walking with frequencies $\Omega_i, i = V, L$, along the bridge, and $m_p(x) = N_p m_{ps}/L$, in which N_p is the number of pedestrians on the footbridge, m_{ps} the mass of a single pedestrian and L the footbridge span length. Substituting the expressions of $F_V(t)$ and $F_L(t)$ into Eq.(1), and rescaling time according to $t = \Omega_L t$, Eq.(1) can be rewritten as

$$\ddot{q}_1 + 2x\dot{q}_1 + (u - e\cos t - h\cos(mt))q_1 = ke\cos t, \tag{2}$$

where " $\cdot$ " represents derivatives with respect to t for convenience, and

$$x = \frac{a_1}{\Omega_L}, u = \frac{a_2}{\Omega_L^2} = \frac{w^2}{\Omega_L^2}, e = \frac{a_3 l\, a_L g m_p(x)}{\Omega_L^2}, h = \frac{a_4 l\, a_V g m_p(x)}{\Omega_L^2}, m = \frac{\Omega_V}{\Omega_L}, k = -\frac{L_2 \sin^2 q_0}{L_2 + L_1 \cos^3 q_0}.$$

By introducing the transformation $q_1(t) = e^{-xt} y(t)$, Eq.(2) is converted into the following expression, without a first derivative,

$$\ddot{y} + (d - y(t))y = ke\cos t, \tag{3}$$

where $d = u - x^2$, $y(t) = e\cos t + h\cos(mt)$.

Inspection of Eqs.(2) and (3) shows that parameter e and h are naturally small since the distributed mass of pedestrians on the bridge is generally small compared with that of the bridge; parameter m, the ratio between vertical and lateral walking frequencies, usually is considered equal to 2 according to experimental measurements. If $m = 2$, the governing equation (3) describing the motion of the plane bifilar pendulum model is a Hill equation with 2 harmonic modes. In next section, we will analyze the condition of occurrence of lateral large amplitude in the planar pendulum model.

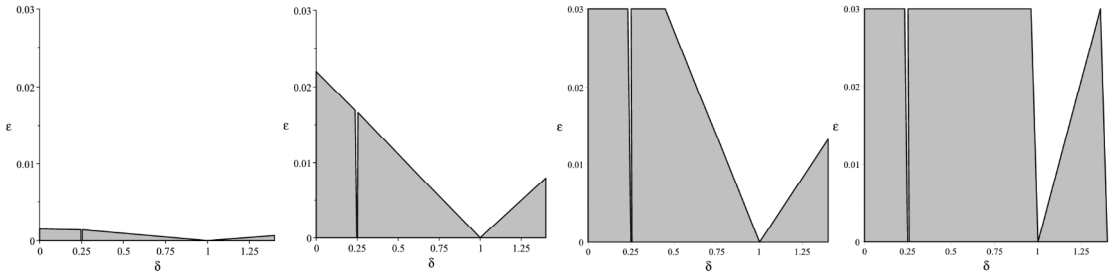
Stability analysis for the plane bifilar pendulum model

It is said that solutions to Eq.(3) are stable if all solutions are bounded, and unstable otherwise. In this section, we will find the transition curves of Eq.(3) with $m = 2$ in the $d - e$ parameter plane. From Floquet theory, any solution along the transition surfaces of Eq.(3) has minimum period $T = 2p/\Omega_L$ or $T = 4p/\Omega_L$ if $m = 2$. The aim in this section is to show why unstable solutions of Eq.(3) are most likely to occur under the condition d near $1/4$ or 1 when $m = 2$. Assume that d can be written in the following form

$$d = d_0 + ed_{11} + hd_{12} + e^2 d_{21} + h^2 d_{22} + ehd_{23} + \cdots,$$

where d_{ij} are constants to be determined. By using the perturbation method, the stability charts of Eq.(3) in the $d - e$ parameter plane are presented in Fig.2, in which the solution is stable in the shaded regions. As shown in Fig.2(a), at this point the area of the stable regions in the $d - e$ parameter plane is very small, which means that it is easy to cause a greater sway of the bridge for almost any value of

d by adding the number of pedestrians. Considering the definition of d, which represents the ratio between the lateral natural frequency of the bridge and the lateral walking frequency, it implies that a growing lateral vibration of the bridge is almost invertible as long as there are more people walking on it. As the number of pedestrians on the bridge is increased, the lateral vibration amplitude begins to increase. As shown in Fig.2 (b), (c) and (d), the lateral amplitude getting larger results in stable regions increasing rapidly. If d is far from resonance points 1/4 and 1, unstable vibrations stabilize quickly with the lateral amplitude increasing. Therefore, a large amplitude vibration of the bridge eventually appears only when $d \approx 1/4$ or $d \approx 1$ is reached.



(a) (b) (c) (d)

Figure 2. Transition curves of Eq.(3) with $m = 2$ in the $d - e$ parameter plane for $k = -0.5$ and (a) ± 0.001 (b) $a = \pm 0.01$ (c) ± 0.02 (d) $a = \pm 0.07$. The solution is stable in the shaded regions.

Conclusions

In this paper, we proposed "the plane bifilar pendulum model" to investigate the dynamics of a suspension bridge under crowd excitation. We model the suspension bridge by the plane bifilar pendulum, which consists of two strings and a rigid body separately representing the cables and the deck, shown as in Fig.1. By applying the analytical results in this paper to the London Millennium Bridge, we can easily explain the occurrence of excessive lateral vibration with 0.48 and 0.96 Hz and the "lock-in" phenomenon. The dramatic difference between the plane bifilar pendulum model and others' models is that the structural feathers of a suspension bridge are taken into account. This may reveal that structural feathers of the suspension bridge can never be ignored to study the excessive lateral vibration.

Acknowledgements

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