Approximating Method and Experimental Determinating Coefficients for an Electric-magnetic Drive System Model

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Abstract—We’ve developed a newer method to calculate the repulsive force between current-carrying coil and nearby metal plate. But it feels the final solution formula is complex. There are some unknown coefficients have not measured and an integral in the formula seems difficult to deal with. In the paper, we determinate these coefficients by experimental method firstly, and then propose solutions for some typical metal plate reflectors combining with Matlab procedure. Lastly, we verify the correctness of the factors and cases of classic metal plate reflectors in practice. Therefore we have solved the AC current-carrying coil of electric-magnetic drive problem completely in the article.

Keywords—AC current-carrying coil; determinating coefficient; integral calculation; electric-magnetic drive

I. INTRODUCTION

In our former researches, we deduced a kind of method only in theory to calculate the repulsive force problem for AC current-carrying coil and its nearby metal plate system. We can’t figure out the final numerical result because of the coefficient is unknown and a complicated integral existing in it seems hard to deal with. So we devote to find out the final solution thoroughly by certain approximating [1] and experimental method [2] within following parts.

II. SIMPLIFICATION FORMULA FORM

The formula [3] we got in our former researches

\[ F_{\text{total}} = -K_T \frac{K}{\sqrt{1 + K^2}} I_1 I_2^2 \int \frac{d_\perp}{\mu d^2} \sin \left( \frac{4\pi d_\perp f d}{C_{\text{light}}} + \arcsin \frac{K}{\sqrt{1 + K^2}} \right) \]

is accurate in calculating force between a coil and induction plate, but still very complicated.

So firstly, we want to further simplify the formula in normal conditions: the distance \( d \) between a coil and induction plate is often small, and the supply frequency \( f \) is also normally not very large. These lead to a small value of \( \frac{4\pi f d}{C_{\text{light}}} \). So we can simplify calculate \( F_{\text{total}} \) as follows:

\[ F_{\text{total}} = -K_T \frac{K^2}{1 + K^2} I_1^2 I_2^2 \int d_\perp \frac{d_\perp}{\mu d^2}, \]

(2)

where \( K = k_{\mu r} \frac{fL}{\mu d_m} = k_{\mu r} \frac{L}{\mu d_m} \), \( d_m \) is the distance from a coil to a induction plate under the coil.

For more closer approach to theoretical value, we set \( d_m = \sqrt{d_{\text{min}} d_{\text{max}}} \), where \( d_{\text{min}} \) is the minimum distance, \( d_{\text{max}} \) is the maximum distance between a coil and induction plate.

III. EXPERIENCE FACTORS AND INTEGRAL CALCULATIONS

A. Experience Factors

Nextly, we’ve measured its experience factors as follows:

\[ K_T = 4.28 \times 10^{-9} \]
\[ k_{\mu r} = 1.88 \times 10^{-11} \]
\[ k_{\mu r} = 5.63 \times 10^{-12} \].

(3)

Before putting into use in practice, we should to solve the \( \int \frac{d_\perp}{\mu d^2} \) in the formula.

B. Integral Calculations in Different Cases

There are several classic cases when we calculate the integral listed as follows.

1) Case 1: Infinite plate induction surface

Initial condition: the minimum distance between transmitting coil and infinite plate induction is \( d \); the dimension of the plate is infinite.

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Object: Numerical or analytic solution of $\int \frac{d_s}{\mu l^d}$. 

Solution:

By arbitrarily selecting an infinitesimal annulus of width $dr$ on plate as in Figure 1, we have

$$r = d \tan \alpha,$$

$$d\alpha = \frac{dr \cos \alpha}{\cos \alpha} = \frac{dr}{\cos \alpha} \Rightarrow dr = \frac{d \cdot d\alpha}{\cos \alpha}.$$

![FIGURE 1. INFINITE PLATE INDUCTION SURFACE](image)

We get the area of the infinitesimal annular surface

$$d\alpha = 2\pi \cdot dr = 2\pi \tan \alpha \cdot d\alpha = 2\pi \tan \alpha \cdot \frac{d \cdot d\alpha}{\cos \alpha}.$$

and then

$$\int d\frac{d_s}{\mu l^d} = \int \frac{1}{\mu l^d} \int_{\theta_{\min}}^{\theta_{\max}} \int_{\alpha_{\min}}^{\alpha_{\max}} d\theta \cdot \cos \alpha \cdot \frac{d}{\cos \alpha} \cdot d\alpha = \frac{\pi}{2\mu l^d} \left(1 - \cos^4 \alpha_{\max}\right).$$

For the whole surface of a infinitely large plate, $\alpha_{\max} = \pi/2$, so we have

$$\int d\frac{d_s}{\mu l^d} = \frac{\pi}{2\mu l^d}. \quad (5)$$

2) Case 2: Spherical induction surface

Initial condition: the minimum distance between transmitting coil and infinite plate induction is $h$; the radius of the sphere is $R$.

Object: Numerical or analytic solution of $\int d\frac{d_s}{\mu l^d}$. 

Solution:

We take an infinitesimal annulus $GG_1$ with the center axis $AO$ on spherical induction surface (refer with: Figure 2). Combing the figure, we get these relations:

$$GE = R \cdot \sin \beta, \quad (6)$$

$$\frac{OG}{AG} = \sin \beta \Rightarrow AG = R \cdot \frac{\sin \beta}{\sin \alpha}, \quad (7)$$

$$\angle GGG_1 = \angle DGF = \angle EGF + \angle EGD = \alpha + \beta, \quad (8)$$

$$GG_2 = GG_1 \cos \angle GGG_2 = R \cdot d\beta \cdot \cos(\alpha + \beta), \quad (9)$$

$$\tan \alpha = \frac{GE}{AE} = \frac{OG \cdot \sin \beta}{h + R - R \cdot \cos \beta} \Rightarrow \alpha = \arctan \left(\frac{R \cdot \sin \beta}{h + R - R \cdot \cos \beta}\right), \quad (10)$$

$$\beta_{\max} = \arccos \left(\frac{R}{R + h}\right). \quad (11)$$

![FIGURE II. SPHERICAL INDUCTION SURFACE](image)
So, we can get
\[
\int \frac{d\phi}{\mu R^2} = \int_0^{\phi_{\text{max}}} \frac{R \cdot d\phi \cdot \cos(\alpha + \beta) - (2\pi R \cdot \sin \beta) \cdot \cos \alpha}{\mu (R \cdot \sin \beta) \cdot \sin \alpha} \cdot \cos \alpha = \frac{2\pi}{\mu R^2} \int_0^{\phi_{\text{max}}} \cos(\alpha + \beta) \cdot \sin^4 \alpha \cdot \cos \alpha \cdot d\beta \sin \beta.
\]  
(12)

When we know the height \( h \), spherical radius \( R \), and permeability \( \mu \), above integral can be solved numerically within Matlab software as follows.

% solving the integral of spherical induction surface.
% the minimum distance between the coil and the spherical surface is 120km
% sphereal radius is 6371km
% permeability of vacuum/atmosphere
% \( \beta_{\text{max}} = \arccos(R/R+h); \% \) the maximum border angle about sphere center
% \% integration variable
% alpha=atan( R*sin(beta) / (h+R-R*cos(beta)) ); \% the maximum border angle of the coil
% FinalValue = 2*pi/(mu0*R^2) * int( (cos(alpha+beta) * (sin(alpha))^4 * cos(alpha)) / (sin(beta))^3 ,beta,0,betamax ); \% integral expression
% FV = eval(FinalValue) % getting the solution of the integral

3) Case 3: \( N \) regular polygon induction surface with sides of \( a \)

Initial condition: the minimum distance between transmitting coil and \( n \) regular polygon plate induction is \( h \); the side dimension of the regular polygon is \( a \).

Object: Numerical or analytic solution of \( G_{G_1}, \cos \theta \).

Solution:

We take the range of \( G_{G_1}G_2G_3G_4 \) as an infinitesimal area \( S \) on the induction surface. Combing Figure 3, we get these relations as follows:

\[ G_{G_1} = \frac{BH}{\cos \beta} = \frac{l}{\cos \beta} \cos \theta. \]  
(13)

\[ G_{D} \perp G_{D} \Rightarrow \angle G_{G_1}G_{D} = \beta, \]  
(14)

\[ G_{D} \approx G_{B_1} \cdot d\beta. \]  
(15)

Combing (13), (14), and (15) products

\[ G_{G_1} = \frac{G_{D}}{\cos \angle G_{G_1}G_{D}} = \frac{G_{B_1} \cdot d\beta \cos \beta}{\cos \beta} = \frac{l \cdot d\beta}{\cos \beta}, \]
(\text{16})

\[ S \approx G_{G_2} \cdot \text{HI} = \frac{l}{\cos \beta} \cdot d\beta \cdot dl, \]
(\text{17})

\[ \theta = \arctan \frac{BG}{h} = \arctan \frac{l}{\cos \beta} \approx \arctan \frac{l}{h \cdot \cos \beta}, \]
and then

\[ d = AG = \frac{h}{\cos \theta} = \sqrt{h^2 + (\frac{l}{\cos \beta})^2}, \]
(\text{18})

\[ \beta_{\text{max}} = \angle CBE = \frac{2\pi}{n} \approx \frac{\pi}{n}, \]
(\text{19})

\[ l_{\text{max}} = CB = \frac{a}{\tan \angle CBE} = \frac{a}{2 \tan \frac{\pi}{n}}. \]

With above equations, the integral can be further deal with as follows:

\[ \int \frac{d\phi}{\mu \beta^2} = \frac{2\pi}{\mu} \int_0^{\phi_{\text{max}}} \frac{S \cdot \cos \theta \cdot \cos \phi}{\mu \beta^4} \cdot d\beta \cdot dl \]
(\text{20})

\[ = \frac{2\pi}{\mu} \int_0^{\phi_{\text{max}}} \frac{l \cdot \cos \phi \cdot \theta}{\mu \beta^4} \cdot d\beta \cdot dl \]
(\text{21})

\[ = \frac{2\pi}{\mu} \int_0^{\phi_{\text{max}}} \frac{l \cdot \cos \phi \cdot \theta}{\mu \beta^4} \cdot d\beta \cdot dl \]
(\text{22})
When we know the regular polygon sides’ dimension $a$, height $h$, and permeability $\mu$, the integral can be solved numerically within Matlab software as follows.

\[
\int_{\beta=0}^{\beta_{\text{max}}} \int_{l=0}^{l_{\text{max}}} \frac{l}{h^2 + \left(\frac{l}{\cos \beta}\right)^2} \cdot \cos \beta \, dl \, d\beta = 2h^2 \mu \int_{\beta=0}^{\beta_{\text{max}}} \int_{l=0}^{l_{\text{max}}} \frac{1}{h^2 + \left(\frac{l}{\cos \beta}\right)^2} \cdot \cos \beta \, dl \, d\beta .
\]

(16)

When we know the $n$ regular polygon sides’ dimension $a$, height $h$, and permeability $\mu$, the integral can be solved numerically within Matlab software as follows.

% solving the integral of $n$ regular polygon induction surface.

\[
h = 9.95 \times 10^{-3} \text{ m}; \text{ the minimum distance between the coil and the polygon surface}
\]

$\mu = 4\pi \times 10^{-7} \text{ H/m}; \text{ the permeability of vacuum/atmosphere}$

$\beta_{\text{max}} = \frac{\pi}{n}; \text{ the maximum border angle}$

$l_{\text{max}} = \frac{a}{2\tan\left(\frac{\pi}{n}\right)}; \text{ the maximum border length}$

\[
F = \frac{1}{\mu} \int_{\beta=0}^{\beta_{\text{max}}} \int_{l=0}^{l_{\text{max}}} \frac{l}{h^2 + \left(\frac{l}{\cos \beta}\right)^2} \cdot \cos \beta \, dl \, d\beta
\]

\[
Q = \text{dblquad}(F,0,\beta_{\text{max}},0,l_{\text{max}}); \text{ solving the integral}
\]

\[
eff = 2n h^2 / \mu; \text{ the common coefficient}
\]

\[
FV = eff \cdot Q \text{ final numerical solution of aim integral}
\]

\[
\int_{0}^{h_{\text{min}}} \text{d}z = \int_{0}^{h_{\text{max}}} \text{d}z \int_{\beta_{\text{min}}}^{\beta_{\text{max}}} \int_{l_{\text{min}}}^{l_{\text{max}}} \frac{l}{h^2 + \left(\frac{l}{\cos \beta}\right)^2} \cdot \cos \beta \, dl \, d\beta .
\]

(17)

When the height $h$ is very small, for more approaching to the ideal value, we set $h = \sqrt{h_{\text{min}} \cdot h_{\text{max}}}$, where $h_{\text{min}}$ is the minimum height of the coil, and $h_{\text{max}}$ is the maximum height.

C. Experimental Proof

Lastly, we’ve verified its validity in practical testing (refer with: Table 1). Figure 4 shows our tested model. When we supply the coil with a adjustable AC power, we can get loss weight data as in Table 1. From the coil parameter and measuring data, we can check out the the experimental result is well according to above formula calculation result (refer with: Equation 17). When we change some different coils and metal plates [4], it is also verified correct [5] in all of the other cases [6].

TABLE I. SUPPLY MAXIMUM FREQUENCY 400HZ, MAXIMUM AC CURRENT 0.668A, CIRCUIT RESISTANCE 144Ω

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>400</th>
<th>200</th>
<th>100</th>
<th>60</th>
<th>45</th>
</tr>
</thead>
<tbody>
<tr>
<td>On-load AC voltage (V)</td>
<td>290.5</td>
<td>176.5</td>
<td>135.4</td>
<td>115</td>
<td>116.5</td>
</tr>
<tr>
<td>Output power (W)</td>
<td>46</td>
<td>68</td>
<td>74.9</td>
<td>70.2</td>
<td>73.7</td>
</tr>
<tr>
<td>Mass indication (g)</td>
<td>25</td>
<td>50</td>
<td>70</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td>Loss Mass (g)</td>
<td>95</td>
<td>70</td>
<td>50</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Loss weight (N)</td>
<td>0.931</td>
<td>0.686</td>
<td>0.49</td>
<td>0.294</td>
<td>0.196</td>
</tr>
</tbody>
</table>

D. Summary

By far, we’ve solved the AC current-carrying coil problem completely. Though it is solved and measured coefficients in low-frequency case, that’s also considerate satisfied to high-frequency coil [7] in theory. It provides us a newer thinking to
interpret the electromagnetic wave thrust problem and can help us to understand the hot area of electric-magnetic drive system more deeply.

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