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Approximating Method and Experimental Determinating Coefficients for an Electric-magnetic Drive System Model

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***Abstract*—We've developed a newer method to calculate the repulsive force between current-carrying coil and nearby metal plate. But it feels the final solution formula is complex. There are some unknown coefficients have not measured and an integral in the formula seems difficult to deal with. In the paper, we determinate these coefficients by experimental method firstly, and then propose solutions for some typical metal plate reflectors combining with Matlab procedure. Lastly, we verify the correctness of the factors and cases of classic metal plate reflectors in practice. Therefore we have solved the AC current-carrying coil of electric-magnetic drive problem completely in the article.**

***Keywords*—AC current-carrying coil; determinating coefficient; integral calculation; electric-magnetic drive**

I. INTRODUCTION

In our former researches, we deduced a kind of method only in theory to calculate the repulsive force problem for AC current-carrying coil and its nearby metal plate system. We can't figure out the final numerical result because of the coefficient is unknown and a complicated integral existing in it seems hard to deal with. So we devote to find out the final solution thoroughly by certain approximating [1] and experimental method [2] within following parts.

II. SIMPLIFICATION FORMULA FORM

The formula [3] we got in our former researches

$$F_{total} = -K_T \cdot \frac{K}{\sqrt{1+K^2}} \cdot L^2 I^2 \cdot \int \frac{d_S}{\mu d^4} \cdot \sin\left(\frac{4\pi f d}{C_{light}} + \arcsin\frac{K}{\sqrt{1+K^2}}\right) \quad (1)$$

is accurate in calculating force between a coil and induction plate, but still very complicated.

So firstly, we want to further simplify the formula in normal conditions: the distance d between a coil and induction plate is often small, and the supply frequency f is also normally not very large. These lead to a small value of $\frac{4\pi f d}{C_{light}}$. So we can simplify calculate F_{total} as follows:

$$F_{total} = -K_T \cdot \frac{K^2}{1+K^2} \cdot L^2 I^2 \cdot \int \frac{d_S}{\mu d^4}, \quad (2)$$

where $K = k_{fB\rho} \frac{fLI}{\rho d_m} = k_{\lambda B\rho} \frac{LI}{\lambda \rho d_m}$, d_m is the distance from a coil to a induction plate under the coil.

For more closer approach to theoretical value, we set $d_m = \sqrt{d_{\min} \cdot d_{\max}}$ where $d_{\min}$ is the minimum distance, $d_{\max}$ is the maximum distance between a coil and induction plate.

III. EXPERIENCE FACTORS AND INTEGRAL CALCULATIONS

A. Experience Factors

Nextly, we've measured its experience factors as follows:

$$\begin{cases} K_T = 4.28 \times 10^{-9} \\ k_{fB\rho} = 1.88 \times 10^{-11} \\ k_{\lambda B\rho} = 5.63 \times 10^{-3} \end{cases} . \quad (3)$$

Before putting into use in practice, we should to solve the $\int \frac{d_S}{\mu d^4}$ in the formula.

B. Integral Calculations in Different Cases

There are several classic cases when we calculate the integral listed as follows.

1) Case 1: Infinite plate induction surface

Initial condition: the minimum distance between transmitting coil and infinite plate induction is d ; the dimension of the plate is infinite.

Object: Numerical or analytic solution of $\int \frac{d_S}{\mu d^4}$.

Solution:

By arbitrarily selecting an infinitesimal annulus of width dr on plate as in Figure 1, we have

$$r = d\tan\alpha ,$$

$$d\alpha = \frac{dr\cos\alpha}{\frac{d}{\cos\alpha}} = \frac{dr}{d}\cos^2\alpha \Rightarrow dr = \frac{d\cdot d\alpha}{\cos^2\alpha}.$$

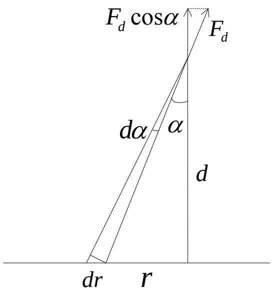

FIGURE I. INFINITE PLATE INDUCTION SURFACE

We get the area of the infinitesimal annular surface

$$\begin{aligned} \boldsymbol{drs} &= 2\pi \boldsymbol{r}\cdot \boldsymbol{dr} \\ &= 2\pi \boldsymbol{d}\tan\alpha \frac{\boldsymbol{d}\cdot \boldsymbol{d}\alpha}{\cos^2\alpha} \\ &= 2\pi \boldsymbol{d}^2 \frac{\tan\alpha}{\cos^2\alpha}\cdot \boldsymbol{d}\alpha . \end{aligned}$$

and then

$$\begin{aligned} \int \frac{\boldsymbol{d_S}}{\mu \boldsymbol{d}^4} &\equiv \frac{1}{\mu}\int_0^{\alpha\max} \frac{\boldsymbol{drs}\cdot\cos\alpha}{(\frac{\boldsymbol{d}}{\cos\alpha})^4}\cdot\cos\alpha \\ &= \frac{1}{\mu}\int_0^{\alpha\max} 2\pi \boldsymbol{d}^2 \frac{\tan\alpha}{\cos^2\alpha}\cdot\frac{\cos\alpha^2}{(\frac{\boldsymbol{d}}{\cos\alpha})^4}\cdot \boldsymbol{d}\alpha \\ &= \frac{2\pi}{\mu \boldsymbol{d}^2}\int_0^{\alpha\max} \tan\alpha\cos^4\alpha\cdot \boldsymbol{d}\alpha \\ &= \frac{2\pi}{\mu \boldsymbol{d}^2}\int_0^{\alpha\max} \sin\alpha\cos^3\alpha\cdot \boldsymbol{d}\alpha \\ &= -\frac{2\pi}{\mu \boldsymbol{d}^2}\int_0^{\alpha\max} \cos^3\alpha\cdot \boldsymbol{d}\cos\alpha = \frac{2\pi}{\mu \boldsymbol{d}^2}\cdot\frac{1}{4}\cos^4\alpha\,|_{\alpha\max}^{0} \\ &= \frac{\pi}{2\mu \boldsymbol{d}^2}(1-\cos^4\alpha\max) . \end{aligned} \quad (4)$$

For the whole surface of a infinitely large plate, $\alpha\max = \pi/2$, so we have

$$\int \frac{d_S}{\mu d^4} \equiv \frac{\pi}{2\mu d^2} . \quad (5)$$

2) Case 2: Spherical induction surface

Initial condition: the minimum distance between transmitting coil and infinite plate induction is h; the radius of the sphere is R.

Object: Numerical or analytic solution of $\int \frac{d_S}{\mu d^4}$.

Solution:

We take an infinitesimal annulus GG_2 with the center axis AO on spherical induction surface (refer with: Figure 2). Combing the figure, we get these relations:

$$GE = R\cdot\sin\beta , \quad (6)$$

$$\frac{OG}{AG} = \frac{\sin\alpha}{\sin\beta} \Rightarrow AG = R\cdot\frac{\sin\beta}{\sin\alpha} , \quad (7)$$

$$\begin{aligned} \angle G_1GG_2 &= \angle DGF \\ &= \angle EGF + \angle EGD \\ &= \alpha + \beta \end{aligned} , \quad (8)$$

$$\begin{aligned} GG_2 &= GG_1\cos\angle G_1GG_2 \\ &= R\cdot d\beta\cdot\cos(\alpha+\beta) \end{aligned} , \quad (9)$$

$$\begin{aligned} \tan\alpha &= \frac{GE}{AE} = \frac{OG\cdot\sin\beta}{h+R-R\cdot\cos\beta} \\ \Rightarrow \alpha &= \arctan\frac{R\cdot\sin\beta}{h+R-R\cdot\cos\beta} \end{aligned} , \quad (10)$$

$$\beta\max = \arccos\frac{R}{R+h} . \quad (11)$$

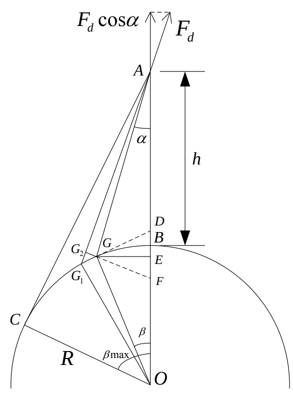

FIGURE II. SPHERICAL INDUCTION SURFACE

So, we can get

$$\int \frac{d_S}{\mu d^4} \equiv \int_0^{\beta \max} \frac{[R \cdot d\beta \cdot \cos(\alpha+\beta)] \cdot (2\pi R \cdot \sin\beta)}{\mu (R \cdot \frac{\sin\beta}{\sin\alpha})^4} \cdot \cos\alpha$$

$$= \frac{2\pi}{\mu R^2} \int_0^{\beta \max} \frac{\cos(\alpha+\beta) \cdot \sin^4\alpha \cdot \cos\alpha}{\sin^3\beta} d\beta \quad . \qquad (12)$$

When we know the height h, spherical radius R, and permeability μ, above integral can be solved numerically within Matlab software as follows.

```
% solving the integral of spherical induction surface.
h=120*1e3;% the minimum distance between the coil and the spherical surface is 120km
R=6371*1e3;% spherical radius is 6371km
mu0=4*pi*1e-7;% permeability of vacuum/atmosphere
betamax=acos( R/(R+h) );% the maximum border angle about sphere center
syms beta;% integration variable
alpha=atan( R*sin(beta) / (h+R-R*cos(beta)) );% the maximum border angle of the coil
FinalValue = 2*pi/(mu0*R^2) * int( (cos(alpha+beta) * (sin(alpha))^4 * cos(alpha)) / (sin(beta))^3 ,beta,0,betamax );% integral expression
FV = eval(FinalValue) % getting the solution of the integral
```

3) Case 3: N regular polygon induction surface with sides of a

Initial condition: the minimum distance between transmitting coil and n regular polygon plate induction is h; the side dimension of the regular polygon is a.

Object: Numerical or analytic solution of $\int \frac{d_S}{\mu d^4}$.

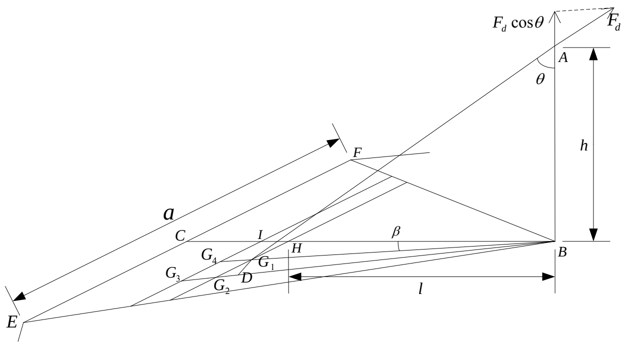

FIGURE III. SPHERICAL N REGULAR POLYGON INDUCTION SURFACE WITH SIDES OF a

Solution:

We take the range of $G_1G_2G_3G_4$ as an infinitesimal area S on the induction surface. Combing Figure 3, we get these relations as follows:

$$BG_1 = \frac{BH}{\cos\beta} = \frac{l}{\cos\beta}, \qquad (13)$$

$$G_1D \perp G_2D \Rightarrow \angle G_2G_1D = \beta, \qquad (14)$$

$$G_1D \approx BG_1 \cdot d\beta \, . \qquad (15)$$

Combing (13), (14), and (15) products

$$G_1G_2 = \frac{G_1D}{\cos\angle G_2G_1D} = \frac{BG_1 \cdot d\beta}{\cos\beta} = \frac{\frac{l}{\cos\beta} \cdot d\beta}{\cos\beta} = \frac{l}{\cos^2\beta} d\beta ,$$

$$S \approx G_1G_2 \cdot HI = \frac{l}{\cos^2\beta} d\beta \cdot dl ,$$

$$\theta = \arctan\frac{BG_1}{h} = \arctan\frac{\frac{l}{\cos\beta}}{h} = \arctan\frac{l}{h \cdot \cos\beta} ,$$

and then

$$d = AG_1 = \frac{h}{\cos\theta} = \sqrt{h^2 + (\frac{l}{\cos\beta})^2} ,$$

$$\beta \max = \angle CBE = \frac{1}{2} \cdot \frac{2\pi}{n} = \frac{\pi}{n} ,$$

$$l \max = CB = \frac{\frac{a}{2}}{\tan\angle CBE} = \frac{a}{2\tan\frac{\pi}{n}} .$$

With above equations, the integral can be further deal with as follows:

$$\int \frac{d_S}{\mu d^4} \equiv 2n \int_0^{l \max} \int_0^{\beta \max} \frac{S \cdot \cos\theta}{\mu d^4} \cdot \cos\theta$$

$$= 2n \int_0^{l \max} \int_0^{\beta \max} \frac{\frac{l \cdot \cos^2\theta}{\cos^2\beta} \cdot d\beta \cdot dl}{\mu [h^2 + (\frac{l}{\cos\beta})^2]^2}$$

$$= 2n\int_0^{l\max}\int_0^{\beta\max}\frac{l\cdot\dfrac{h^2}{h^2+(\frac{l}{\cos\beta})^2}}{\mu[h^2+(\frac{l}{\cos\beta})^2]^2\cdot\cos^2\beta}\cdot d\beta\cdot dl$$

$$= 2n\int_0^{l\max}\int_0^{\beta\max}\frac{l\cdot h^2}{\mu[h^2+(\frac{l}{\cos\beta})^2]^3\cdot\cos^2\beta}\cdot d\beta\cdot dl$$

$$= \frac{2nh^2}{\mu}\int_0^{l\max}\int_0^{\beta\max}\frac{l}{[h^2+(\frac{l}{\cos\beta})^2]^3\cdot\cos^2\beta}\cdot d\beta\cdot dl \quad (16)$$

When we know the n regular polygon sides' dimension a, height h, and permeability μ, the integral can be solved numerically within Matlab software as follows.

```
% solving the integral of n regular polygon induction surface.
h=9.95e-3;% the minimum distance between the coil and the polygon surface
n=4;% the case of a square
a=0.2;% square edge length 200mm
mu0=4*pi*1e-7;% permeability of vacuum/atmosphere
betamax= pi/n ; % the maximum border angle
lmax= a / ( 2*tan(pi/n) ) ; % the maximum border length
F = @(beta,l) l ./ ( ( h.^2 + (l./cos(beta)).^2 ).^3 .* (cos(beta)).^2 ); % integral expression
Q = dblquad(F,0,betamax,0,lmax); % solving the integral
eff=2*n*h^2/mu0; % the common coefficient
FV = eff*Q % final numerical solution of aim integral
Integral=pi/(2*mu0*h^2)% contrasting the solution with the infinitely large plate
```

4) Case 4: Rectangle induction surface with sides of $a\times b$

Initial condition: the minimum distance between transmitting coil and rectangle induction is h; the side dimensions of the rectangle are $a\times b$.

Object: Numerical or analytic solution of $\int\frac{d_S}{\mu d^4}$.

The solution method is as similar as Case 3, and the final solution we got is shown as follows

$$\int\frac{d_S}{\mu d^4}\equiv\frac{4h^2}{\mu}\Big(\int_0^{a/2}\int_0^{\arctan(b/a)}\frac{l}{[h^2+(\frac{l}{\cos\beta})^2]^3\cdot\cos^2\beta}\cdot d\beta\cdot dl + \int_0^{b/2}\int_0^{\arctan(a/b)}\frac{l}{[h^2+(\frac{l}{\cos\beta})^2]^3\cdot\cos^2\beta}\cdot d\beta\cdot dl\Big) \quad . \quad (17)$$

Numerical calculation programme code is as follows.

```
% solving the rectangle induction surface
h=9.95e-3;% the minimum distance between the coil and the rectangle surface (m).
a=0.2;% rectangle side length
b=0.2;% rectangle side length
mu0=4*pi*1e-7;% permeability of vacuum/atmosphere
F1 = @(beta,l) l ./ ( ( h.^2 + (l./cos(beta)).^2 ).^3 .* (cos(beta)).^2 ); % integral expression
Q1 = dblquad(F1,0,atan(b/a),0,a/2); % solving the integral
F2 = @(beta,l) l ./ ( ( h.^2 + (l./cos(beta)).^2 ).^3 .* (cos(beta)).^2 ); % integral expression
Q2 = dblquad(F2,0,atan(a/b),0,b/2); % solving the integral
eff=4*h^2/mu0; % the common coefficient
FV = eff*(Q1+Q2) % final numerical solution of aim integral
```

When the height h is very small, for more approaching to the ideal value, we set $h=\sqrt{h_{\min}\cdot h_{\max}}$, where $h_{\min}$ is the minimum height of the coil, and $h_{\max}$ is the maximum height.

C. Experimental Proof

Lastly, we've verified its validity in practical testing (refer with: Table 1). Figure 4 shows our tested model. When we supply the coil with a adjustable AC power, we can get loss weight data as in Table 1. From the coil parameter and measuring data, we can check out the the experimental result is well according to above formula calculation result (refer with: Equation 17). When we change some different coils and metal plates [4], it is also verified correct [5] in all of the other cases [6].

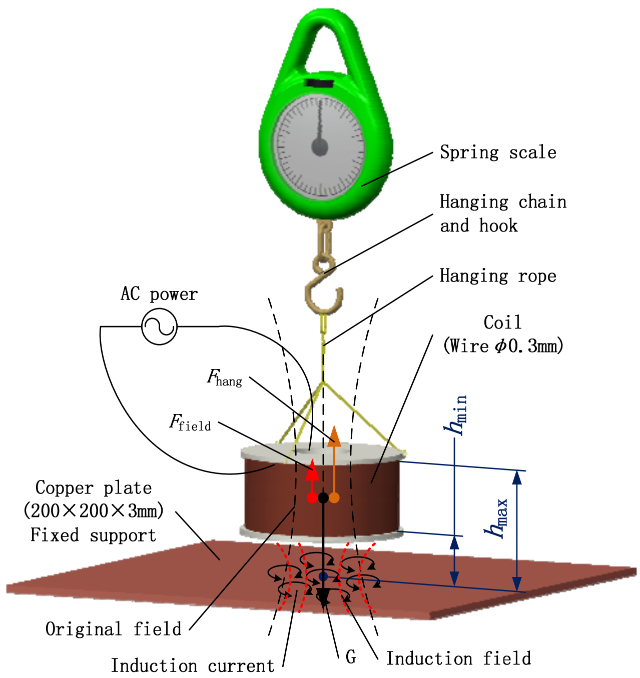

FIGURE IV. TESTED MODEL AND ITS FIELD DISTRIBUTIONS

TABLE I. SUPPLY MAXIMUM FREQUENCY 400HZ, MAXIMUM AC CURRENT 0.668A, CIRCUIT RESISTANCE 144Ω

Frequency (Hz)	400	200	100	60	45
On-load AC voltage (V)	299.5	176.5	135.4	115	116.5
Output power (W)	46	68	74.9	70.2	73.7
Mass indication (g)	25	50	70	90	100
Loss Mass (g)	95	70	50	30	20
Loss weight (N)	0.931	0.686	0.49	0.294	0.196

D. Summary

By far, we've solved the AC current-carrying coil problem completely. Though it is solved and measured coefficients in low-frequency case, that's also considerate satisfied to high-frequency coil [7] in theory. It provides us a newer thinking to interpret the electromagnetic wave thrust problem and can help us to understand the hot area of electric-magnetic drive system more deeply.

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