

The Application of Vector Lyapunov Functions in Iterative Learning Control

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Abstract—This paper presents iterative learning control of multiple state vector system, whose initial states are equal. By introducing the vector Lyapunov function, it involves interesting criteria to guarantee the robust convergence of the tracking error in the sense of the λ -norm. Finally, the validity of the proposed method is verified by an example.

Keywords—iterative learning control; convergence; vector lyapunov function; λ -norm

I INTRODUCTION

Since iterative learning control, which belongs to the intelligent control methodology, is proposed by Arimoto et al. in 1984(See[1]), this feed-forward control approach for fully utilizing the previous control information and improving the transient performance of studied systems that is suitable for repetitive movements has been a major research area and a hot issue in recent years. Its goal is to get full range of tracking tasks on finite interval (See [2-15]).

It is well known that the more complex the considered dynamical system is, the more difficult it is to find a Lyapunov function. This arouse us to employ several Lyapunov function, which is called to be a vector Lyapunov function, for each component provides information about a part of the dynamics. Hence, the corresponding theory, namely, the method of vector Lyapunov function, offers a very flexible process (see [16-22]).

There are few research results on iterative learning control for systems with time-varying coefficient as well as vector Lyapunov function. In this paper, using vector Lyapunov function and λ -norm, we obtain some sufficient conditions to guarantee the output states of the considered time-varying coefficient system to converge to the desired trajectories. It is shown that the proposed method can achieve the ILC convergence property significantly.

Before ending this section, it is worth pointing out the main contribution of this paper is the design of iterative controllers from vector Lyapunov function, which is different from the designed controllers by positive definite Lyapunov function, in past literature.

II PRELIMINARIES

Throughout this paper, the 2-norm for the n -dimensional vector $x = (x_1, x_2, \dots, x_n)^T$ is defined as

$\|x\| = (\sum_{i=1}^n x_i^2)^{1/2}$, while the λ -norm for a function is defined as $\|\cdot\|_\lambda = \sup_{t \in [0, T]} \{e^{-\lambda t} \|\cdot\|\}$, where the superscript T represents

the transpose and $\lambda > 0$. $|A| = (|a_{ij}|)_{n \times n}$, where $A = (a_{ij})_{n \times n} \in R^{n \times n}$ is a matrix.

Lemma 1[10,23]

Consider

$$\sup_{t \in [0, T]} \{e^{-\lambda t} \int_0^t \|x(\tau)\| d\tau\} \leq \frac{1}{\lambda} \|x(t)\|_\lambda.$$

III MAIN RESULTS

Consider the following multiple state vector system

$$\begin{aligned} \dot{x}_k &= F(t, x_k, y_k) + u_{x,k}(t), \\ z_{x,k}(t) &= Cx_k + Du_{x,k}(t), \\ u_{x,k+1}(t) &= u_{x,k}(t) + Me_{x,k}(t), \\ \dot{y}_k &= G(t, x_k, y_k) + u_{y,k}(t), \\ z_{y,k}(t) &= Cy_k + Du_{y,k}(t), \\ u_{y,k+1}(t) &= u_{y,k}(t) + Me_{y,k}(t), \end{aligned} \quad (1)$$

where $x_k, y_k \in R^n$ are the state vectors, $u_{x,k}, u_{y,k} \in R^n$ are input vectors of x_k, y_k , and $z_{x,k}, z_{y,k} \in R^n$ are output vectors of x_k, y_k , respectively. k is the number of iterations, $k \in \{1, 2, 3, \dots\}$ and $t \in [0, T]$, T is a constant.

Let

$e_{x,k}(t) = z_{x,d}(t) - z_{x,k}(t)$, $e_{y,k}(t) = z_{y,d}(t) - z_{y,k}(t)$, where $z_{x,d}(t)$, $z_{y,d}(t)$ are reference outputs of x_k, y_k , respectively. So we have

$$\begin{aligned} e_{x,k+1}(t) &= z_{x,d}(t) - z_{x,k+1}(t) = z_{x,d}(t) - z_{x,k}(t) + z_{x,k}(t) - z_{x,k+1}(t) \\ &= e_{x,k}(t) + z_{x,k}(t) - z_{x,k+1}(t), \end{aligned}$$

$$e_{y,k+1}(t) = e_{y,k}(t) + z_{y,k}(t) - z_{y,k+1}(t). \quad (2)$$

We define the operator $V: R^n \times R^n \rightarrow R^n$ such that

$$(1) \quad V(0,0) = 0;$$

$$(2) \quad V(x+z, y+w) = V(x, y) + V(z, w) \quad \text{for any vectors } x, z, y, w \in R^n,$$

(3) there is a constant $\gamma > 0$ such that $V(Cx, Cy) = C^\gamma V(x, y)$ for any vectors $x, y \in R^n$ and matrix $C \in R^{n \times n}$;

(4) the derivative $\frac{\partial V}{\partial x}$ and $\frac{\partial V}{\partial y}$ of $V(x, y)$ exist, where $x, y \in R^n$.

For the sake of convenient, we denote that the set, whose elements are operators $V: R^n \times R^n \rightarrow R^n$ and satisfy the above four conditions, is \mathbb{S} .

For any operator $V \in \mathbb{S}$, the following conclusion can be gotten

$$\begin{aligned} V(e_{x,k+1}(t), e_{y,k+1}(t)) &= V(e_{x,k}(t) + z_{x,k}(t) - z_{x,k+1}(t), e_{y,k}(t) + z_{y,k}(t) - z_{y,k+1}(t)) \\ &= V(e_{x,k}(t), e_{y,k}(t)) + V(z_{x,k}(t) - z_{x,k+1}(t), z_{y,k}(t) - z_{y,k+1}(t)) \\ &= V(e_{x,k}(t), e_{y,k}(t)) + V(C(x_k(t) - x_{k+1}(t)), C(y_k(t) - y_{k+1}(t))) \\ &\quad + D(u_{x,k}(t) - u_{x,k+1}(t), C(y_k(t) - y_{k+1}(t)) + D(u_{y,k}(t) - u_{y,k+1}(t))) \\ &= V(e_{x,k}(t), e_{y,k}(t)) + C^\gamma V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t)) \\ &\quad + D^\gamma V(u_{x,k}(t) - u_{x,k+1}(t), u_{y,k}(t) - u_{y,k+1}(t)) \\ &= V(e_{x,k}(t), e_{y,k}(t)) + C^\gamma V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t)) \\ &\quad - (DM)^\gamma V(e_{x,k}(t), e_{y,k}(t)) \\ &= (I - (DM)^\gamma) V(e_{x,k}(t), e_{y,k}(t)) + C^\gamma V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t)), \end{aligned} \quad (3)$$

where I is identical matrix. In fact:

$$\begin{aligned} z_{x,k}(t) - z_{x,k+1}(t) &= C(x_k(t) - x_{k+1}(t)) + D(u_{x,k}(t) - u_{x,k+1}(t)), \\ z_{y,k}(t) - z_{y,k+1}(t) &= C(y_k(t) - y_{k+1}(t)) + D(u_{y,k}(t) - u_{y,k+1}(t)), \end{aligned}$$

$$u_{x,k}(t) - u_{x,k+1}(t) = -M e_{x,k}(t), u_{y,k}(t) - u_{y,k+1}(t) = -M e_{y,k}(t).$$

Then from (3) we can obtain that

$$\|V(e_{x,k+1}(t), e_{y,k+1}(t))\| \leq \|I - (DM)^\gamma\| \cdot \|V(e_{x,k}(t), e_{y,k}(t))\| + \|C^\gamma\| \cdot \|V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t))\|. \quad (4)$$

For $\|V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t))\|$ in (4), we have

$$\begin{aligned} \frac{d e^\tau V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t))}{dt} &= r e^\tau V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t)) \\ &\quad + e^\tau \left(\frac{\partial V}{\partial(x_k(t) - x_{k+1}(t))} \cdot \frac{d(x_k(t) - x_{k+1}(t))}{dt} \right) \\ &\quad + e^\tau \left(\frac{\partial V}{\partial(y_k(t) - y_{k+1}(t))} \cdot \frac{d(y_k(t) - y_{k+1}(t))}{dt} \right) \end{aligned}$$

$$\begin{aligned} &= r e^\tau V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t)) \\ &\quad + e^\tau \left(\frac{\partial V}{\partial(x_k(t) - x_{k+1}(t))} \cdot (F(t, x_k, y_k) - F(t, x_{k+1}, y_{k+1})) \right) \\ &\quad + e^\tau \left(\frac{\partial V}{\partial(y_k(t) - y_{k+1}(t))} \cdot (G(t, x_k, y_k) - G(t, x_{k+1}, y_{k+1})) \right) \end{aligned}$$

$$\begin{aligned} &+ e^\tau \left(\frac{\partial V}{\partial(x_k(t) - x_{k+1}(t))} \cdot (-M e_{x,k}(t)) \right) \\ &+ e^\tau \left(\frac{\partial V}{\partial(y_k(t) - y_{k+1}(t))} \cdot (-M e_{y,k}(t)) \right) \end{aligned}$$

$$\begin{aligned} &= r e^\tau V_k(t) + e^\tau (V_{x,k}(t) \cdot f_k(t) + V_{y,k}(t) \cdot g_k(t)) \\ &\quad - e^\tau (V_{x,k}(t) \cdot M e_{x,k} + V_{y,k}(t) \cdot M e_{y,k}). \end{aligned}$$

$$\begin{aligned} \text{where } V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t)) &= V_k(t), \\ \frac{\partial V_k}{\partial(x_k(t) - x_{k+1}(t))} &= V_{x,k}, \frac{\partial V_k}{\partial(y_k(t) - y_{k+1}(t))} = V_{y,k}, \\ F(t, x_k, y_k) - F(t, x_{k+1}, y_{k+1}) &= f_k(t), \\ G(t, x_k, y_k) - G(t, x_{k+1}, y_{k+1}) &= g_k(t). \end{aligned}$$

we integrate $e^\tau V_k(t)$ with respect to t and obtain

$$\begin{aligned} e^\tau V_k(t) &= e^\tau V_k(s) + \int_s^t [r e^{\rho} V_k(\rho) + e^{\rho} (V_{x,k}(\rho) \cdot f_k(\rho) + V_{y,k}(\rho) \cdot g_k(\rho))] d\rho \\ &\quad - \int_s^t e^{\rho} (V_{x,k}(\rho) \cdot M e_{x,k} + V_{y,k}(\rho) \cdot M e_{y,k}) d\rho. \end{aligned}$$

$$e^{rs} \|V_k(t)\| = e^{rs} \|V_k(s)\| + \int_s^t e^{r\rho} \|rV_k(\rho) + (V_{x,k}(\rho) \cdot f_k(\rho) + V_{y,k}(\rho) \cdot g_k(\rho))\| d\rho \\ + \int_s^t e^{r\rho} \|V_{x,k}(\rho) \cdot Me_{x,k} + V_{y,k}(\rho) \cdot Me_{y,k}\| d\rho.$$

$$re^{rt} \|V_k(t)\| + e^{rt} \|V_k(t)\|_t' = e^{rt} \|rV_k(t) + (V_{x,k}(t) \cdot f_k(t) + V_{y,k}(t) \cdot g_k(t))\| \\ + e^{rt} \|V_{x,k}(t) \cdot Me_{x,k} + V_{y,k}(t) \cdot Me_{y,k}\|.$$

If there exists a constant ϖ such that

$$\|V_{x,k}(t) \cdot Me_{x,k} + V_{y,k}(t) \cdot Me_{y,k}\| \leq \varpi \|V(e_{x,k}(t), e_{y,k}(t))\|, \quad (5)$$

the following conclusion

$$\|V_k(t)\|_t' \leq \|rV_k(t) + (V_{x,k}(t) \cdot f_k(t) + V_{y,k}(t) \cdot g_k(t))\| - r\|V_k(t)\| \\ + \varpi \|V(e_{x,k}(t), e_{y,k}(t))\| \quad (6)$$

is true.

It is easy to prove that $\|rV_k(t) + (V_{x,k}(t) \cdot f_k(t) + V_{y,k}(t) \cdot g_k(t))\| - r\|V_k(t)\|$ is monotonically

decreasing on r , thus the limit $\lim_{r \rightarrow +\infty} (\|rV_k(t) + (V_{x,k}(t) \cdot f_k(t) + V_{y,k}(t) \cdot g_k(t))\| - r\|V_k(t)\|)$ exists. From 2-norm of the n -dimensional vector, we obtain

$$\lim_{r \rightarrow +\infty} (\|rV_k(t) + (V_{x,k}(t) \cdot f_k(t) + V_{y,k}(t) \cdot g_k(t))\| - r\|V_k(t)\|) \\ = \frac{(V_{x,k}(t) \cdot f_k(t) + V_{y,k}(t) \cdot g_k(t))^T V_k(t)}{\|V_k(t)\|} = h_k(t).$$

$$\|V_k(t)\|_t' \leq h_k(t) + \varpi \|V(e_{x,k}(t), e_{y,k}(t))\|. \quad (7)$$

Based on the above calculation, the following theorem is obtained.

Theorem 1 If the system (1) and the operator $V \in \mathbb{N}$ satisfy the condition (5), then the conclusions (4) and (7) are true.

Corollary 1 when $x_k(t) \in R^n, y_k(t) \in R^m$, and $n > m$, let $y_k(t) \rightarrow \begin{pmatrix} y_k(t) \\ 0_{n-m} \end{pmatrix} = \tilde{y}_k(t)$,

$$G(t, x_k(t), y_k(t)) \rightarrow \begin{pmatrix} G(t, x_k(t), y_k(t)) \\ 0_{n-m} \end{pmatrix} = \tilde{G}(t, x_k(t), \tilde{y}_k(t)), \quad \text{then}$$

one obtains the similar result with above Theorem.

Remark When there are three parts x_k, y_k, z_k about the multiple state vector system (1), we can construct $V(e_{x,k}(t), e_{y,k}(t), e_{z,k}(t)) \in \mathbb{N}$ and imitate the above proof to get the similar conclusion with Theorem 1.

Corollary 2 For system (1), taking $V(e_{x,k}(t), e_{y,k}(t)) = \alpha e_{x,k}(t) + \beta e_{y,k}(t)$, $V(x_k(t) - x_{k+1}(t), y_k(t) - y_{k+1}(t))$

$= \alpha(x_k(t) - x_{k+1}(t)) + \beta(y_k(t) - y_{k+1}(t))$, the following results are drawn:

$$\|\alpha e_{x,k+1}(t) + \beta e_{y,k+1}(t)\| \leq \|I - DM\| \|\alpha e_{x,k}(t) + \beta e_{y,k}(t)\| + \|C\| \|\alpha(x_k - x_{k+1}) + \beta(y_k - y_{k+1})\|, \quad (8)$$

$$\|p_k(t)\|_t' \leq \frac{p_k^T(t) q_k(t)}{\|p_k(t)\|} + \|M\| \|\alpha e_{x,k}(t) + \beta e_{y,k}(t)\|, \quad (9)$$

where

$$p_k(t) = \alpha(x_k - x_{k+1}) + \beta(y_k - y_{k+1}),$$

$$q_k(t) = \alpha[F(t, x_k, y_k) - F(t, x_{k+1}, y_{k+1})]$$

$$+ \beta[G(t, x_k, y_k) - G(t, x_{k+1}, y_{k+1})].$$

In fact,

$$\alpha e_{x,k+1}(t) + \beta e_{y,k+1}(t) = \alpha e_{x,k}(t) + \beta e_{y,k}(t) + \alpha(z_{x,k} - z_{x,k+1}) \\ + \beta(z_{y,k} - z_{y,k+1}) \\ = (\alpha e_{x,k}(t) + \beta e_{y,k}(t)) + \alpha[C(x_k - x_{k+1}) + D(u_{x,k} - u_{x,k+1})] \\ + \beta[C(y_k - y_{k+1}) + D(u_{y,k} - u_{y,k+1})] \\ = (I - DM)(\alpha e_{x,k}(t) + \beta e_{y,k}(t)) \\ + C[\alpha(x_k - x_{k+1}) + \beta(y_k - y_{k+1})].$$

Imitating the inference of (7), we have

$$\frac{de^{rt} p_k(t)}{dt} = re^{rt} p_k(t) + e^{rt} q_k(t) - e^{rt} M[\alpha e_{x,k}(t) + \beta e_{y,k}(t)],$$

$$e^{rt} p_k(t) = e^{rs} p_k(s) + \int_s^t e^{ru} [rp_k(u) + q_k(u) - M(\alpha e_{x,k}(u) + \beta e_{y,k}(u))] du,$$

$$e^{rt} \|p_k(t)\| \leq e^{rs} \|p_k(s)\| + \int_s^t e^{ru} \|rp_k(u) + q_k(u)\| du + \int_s^t \|M\| \|\alpha e_{x,k}(u) + \beta e_{y,k}(u)\| du,$$

$$e^{rt} \|p_k(t)\|_t' \leq e^{rt} [\|rp_k(t) + q_k(t)\| + \|M\| \|\alpha e_{x,k}(t) + \beta e_{y,k}(t)\|] - re^{rt} \|p_k(t)\|.$$

Going on to the next item, we will infer the important

conclusion of this paper.

Conditions (A), (B) are satisfied if the following conditions hold:

(A) The operator $V_{k,i}(t) \in \mathbb{S}, i=1,2$. There exist functions $p_{ji}(t), i, j=1,2$, and such that

$$\frac{(V_{x,k,i}(t) \cdot f_k(t) + V_{y,k,i}(t) \cdot g_k(t))^T V_{k,i}(t)}{\|V_{k,i}(t)\| \cdot \|V_{k,j}(t)\|} \leq p_{ji}(t)$$

$$\|V_{x,k,i}(t) \cdot Me_{x,k} + V_{y,k,i}(t) \cdot Me_{y,k}\| \leq \varpi_{i1} \|V_1(e_{x,k}(t), e_{y,k}(t))\| + \varpi_{i2} \|V_2(e_{x,k}(t), e_{y,k}(t))\|,$$

where ϖ_{ij} are constants.

Theorem 2 Suppose the operator F, G in system (1) satisfy $F(t, 0, 0) = G(t, 0, 0) = 0$. There are operators $V_{k,i} \in \mathbb{S}, V_i(e_{x,k}(t), e_{y,k}(t)), i=1,2$, such that conditions (A), (B), and

$$V_i(e_{x,k}(t), e_{y,k}(t)) = 0 \text{ if and only if } e_{x,k}(t) = e_{y,k}(t) = 0. \quad (10)$$

If there exists a constant $\lambda > 0$ such that initial state vector $x_k(0) - x_{k+1}(0) = 0, y_k(0) - y_{k+1}(0) = 0$, and

$$\lim_{k \rightarrow +\infty} Q^k(t) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad (11)$$

where

$$Q(t) = \begin{pmatrix} \|I - (DM)^T\| & 0 \\ 0 & \|I - (DM)^T\| \end{pmatrix} + \begin{pmatrix} \|C^T\| & 0 \\ 0 & \|C^T\| \end{pmatrix} \cdot |\Phi(t)| \cdot \frac{B(t)}{\lambda}, \quad \Phi(t) \text{ is a}$$

basic solution matrix of the system

$$\begin{pmatrix} \|V_{k,1}(t)\| \\ \|V_{k,2}(t)\| \end{pmatrix}' \leq \begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix} \begin{pmatrix} \|V_{k,1}(t)\| \\ \|V_{k,2}(t)\| \end{pmatrix}, \quad B(t) = |\Phi^{-1}(t)| \begin{pmatrix} \varpi_{11} & \varpi_{12} \\ \varpi_{21} & \varpi_{22} \end{pmatrix},$$

then the system (1) can guarantee that $z_{x,k}(t), z_{y,k}(t)$ can track $z_{x,d}(t), z_{y,d}(t)$, respectively.

Proof From conditions (A), (B) and the above inference, we can obtain

$$\begin{pmatrix} \|V_{k,1}(t)\| \\ \|V_{k,2}(t)\| \end{pmatrix}' \leq \begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix} \begin{pmatrix} \|V_{k,1}(t)\| \\ \|V_{k,2}(t)\| \end{pmatrix} + \begin{pmatrix} \varpi_{11} & \varpi_{12} \\ \varpi_{21} & \varpi_{22} \end{pmatrix} \begin{pmatrix} \|V_1(e_{x,k}(t), e_{y,k}(t))\| \\ \|V_2(e_{x,k}(t), e_{y,k}(t))\| \end{pmatrix}. \quad (12)$$

From (1)-(7) and (12), the following conclusion

$$\begin{pmatrix} \|V_{k,1}(t)\| \\ \|V_{k,2}(t)\| \end{pmatrix} \leq \Phi(t) \cdot \int_0^t \Phi^{-1}(\rho) \begin{pmatrix} \varpi_{11} & \varpi_{12} \\ \varpi_{21} & \varpi_{22} \end{pmatrix} \begin{pmatrix} \|V_1(e_{x,k}(\rho), e_{y,k}(\rho))\| \\ \|V_2(e_{x,k}(\rho), e_{y,k}(\rho))\| \end{pmatrix} d\rho$$

is true because $x_k(0) - x_{k+1}(0) = 0, y_k(0) - y_{k+1}(0) = 0$.

$$\begin{pmatrix} \|V_1(e_{x,k+1}(t), e_{y,k+1}(t))\| \\ \|V_2(e_{x,k+1}(t), e_{y,k+1}(t))\| \end{pmatrix} \leq \begin{pmatrix} \|I - (DM)^T\| & 0 \\ 0 & \|I - (DM)^T\| \end{pmatrix} \begin{pmatrix} \|V_1(e_{x,k}(t), e_{y,k}(t))\| \\ \|V_2(e_{x,k}(t), e_{y,k}(t))\| \end{pmatrix} + \begin{pmatrix} \|C^T\| & 0 \\ 0 & \|C^T\| \end{pmatrix} \cdot \Phi(t) \cdot \int_0^t \Phi^{-1}(\rho) \begin{pmatrix} \varpi_{11} & \varpi_{12} \\ \varpi_{21} & \varpi_{22} \end{pmatrix} \begin{pmatrix} \|V_1(e_{x,k}(\rho), e_{y,k}(\rho))\| \\ \|V_2(e_{x,k}(\rho), e_{y,k}(\rho))\| \end{pmatrix} d\rho.$$

Taking λ -norm, we have

$$\begin{pmatrix} \|V_1(e_{x,k+1}(t), e_{y,k+1}(t))\|_\lambda \\ \|V_2(e_{x,k+1}(t), e_{y,k+1}(t))\|_\lambda \end{pmatrix} \leq \begin{pmatrix} \|E - (DM)^T\| & 0 \\ 0 & \|E - (DM)^T\| \end{pmatrix} \begin{pmatrix} \|V_1(e_{x,k}(t), e_{y,k}(t))\|_\lambda \\ \|V_2(e_{x,k}(t), e_{y,k}(t))\|_\lambda \end{pmatrix} + \begin{pmatrix} \|C^T\| & 0 \\ 0 & \|C^T\| \end{pmatrix} \cdot |\Phi(t)| \cdot \frac{B(t)}{\lambda} \begin{pmatrix} \|V_1(e_{x,k}(t), e_{y,k}(t))\|_\lambda \\ \|V_2(e_{x,k}(t), e_{y,k}(t))\|_\lambda \end{pmatrix} = Q(t) \cdot \begin{pmatrix} \|V_1(e_{x,k}(t), e_{y,k}(t))\|_\lambda \\ \|V_2(e_{x,k}(t), e_{y,k}(t))\|_\lambda \end{pmatrix}.$$

when the condition (11) holds, we have

$$\lim_{k \rightarrow +\infty} \begin{pmatrix} \|V_1(e_{x,k}(t), e_{y,k}(t))\|_\lambda \\ \|V_2(e_{x,k}(t), e_{y,k}(t))\|_\lambda \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}. \quad \text{That implies}$$

$$\lim_{k \rightarrow +\infty} \|V_j(e_{x,k}(t), e_{y,k}(t))\|_\lambda = 0, j=1,2, \lim_{k \rightarrow +\infty} \|V_j(e_{x,k}(t), e_{y,k}(t))\| = 0, \text{ i.e.}$$

$$\lim_{k \rightarrow +\infty} e_{x,k}(t) = 0, \lim_{k \rightarrow +\infty} e_{y,k}(t) = 0 \text{ from the condition (10).}$$

Corollary 3 Suppose the operator F, G in system (1) satisfy $F(t, 0, 0) = G(t, 0, 0) = 0$. There are operators $V_i(e_{x,k}(t), e_{y,k}(t)) = \alpha_i e_{x,k}(t) + \beta_i e_{y,k}(t), i=1,2$, such that conditions (A), and

$$\alpha_i e_{x,k}(t) + \beta_i e_{y,k}(t) = 0, i=1,2, \text{ if and only if } e_{x,k}(t) = e_{y,k}(t) = 0.$$

If there exists a constant $\lambda > 0$ such that initial state vector $x_k(t) - x_{k+1}(t) = 0, y_k(t) - y_{k+1}(t) = 0$, and

$$\lim_{k \rightarrow +\infty} Q^k(t) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad (13)$$

where

$$Q(t) = \begin{pmatrix} \|I - (DM)^T\| & 0 \\ 0 & \|I - (DM)^T\| \end{pmatrix} + \begin{pmatrix} \|C^T\| & 0 \\ 0 & \|C^T\| \end{pmatrix} \cdot |\Phi(t)| \cdot \frac{B(t)}{\lambda},$$

$\Phi(t)$ is a basic solution matrix of the system

$$\begin{pmatrix} \|V_{k,1}(t)\| \\ \|V_{k,2}(t)\| \end{pmatrix}' \leq \begin{pmatrix} p_{11}(t) & p_{12}(t) \\ p_{21}(t) & p_{22}(t) \end{pmatrix} \begin{pmatrix} \|V_{k,1}(t)\| \\ \|V_{k,2}(t)\| \end{pmatrix},$$

$B(t) = |\Phi^{-1}(t)|$, then the system (1) can guarantee that $z_{x,k}(t), z_{y,k}(t)$ can track $z_{x,d}(t), z_{y,d}(t)$, respectively.

IV EXAMPLE

Considering the system

$$\dot{x}_k = 0.2e^{-t}x_k + 0.3y_k \sin t + u_{x,k},$$

$$z_{x,k} = 0.03x_k + 0.3u_{x,k},$$

$$u_{x,k+1} = u_{x,k} + 3e_{x,k},$$

$$\dot{y}_k = 0.3x_k \sin t + 0.2e^{-t}y_k + u_{y,k},$$

$$z_{y,k} = 0.03y_k + 0.3u_{y,k},$$

$$u_{y,k+1} = u_{y,k} + 3e_{y,k}.$$

We take $T=3$, that is $t \in [0, 3]$, and $z_{x,d}(t) = \sin t, z_{y,d}(t) = \cos t$. It is easy to verify this example satisfies the conditions of Theorem 2 when $V_1(e_{x,k}(t), e_{y,k}(t)) = (e_{x,k}(t) + e_{y,k}(t))^2$,

$V_2(e_{x,k}(t), e_{y,k}(t)) = (e_{x,k}(t) - e_{y,k}(t))^2$, and satisfies the conditions of Corollary 3 when $V_1(e_{x,k}(t), e_{y,k}(t))$

$= \alpha e_{x,k}(t) + \beta e_{y,k}(t), V_2(e_{x,k}(t), e_{y,k}(t)) = \beta e_{x,k}(t) + \alpha e_{y,k}(t)$. In following Figure I and Figure II, the output errors $e_{x,k}(t), e_{y,k}(t)$ are exhibited at iteration $k=4, k=5$, respectively.

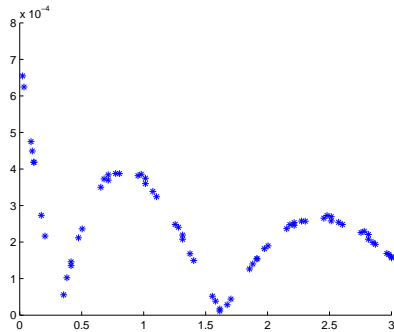


FIGURE I. ERROR $e_{x,k}(t)$ AFTER ITERATION 4

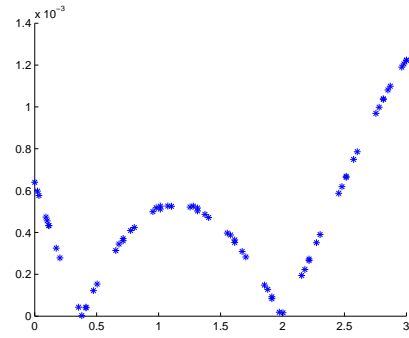


FIGURE II. ERROR $e_{y,k}(t)$ AFTER ITERATION 4

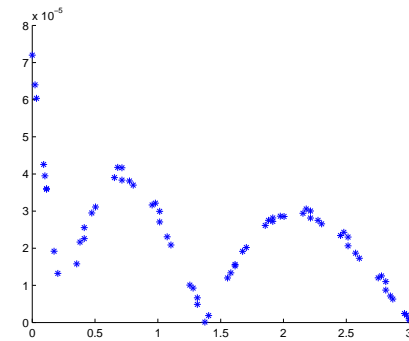


FIGURE III. ERROR $e_{x,k}(t)$ AFTER ITERATION 5

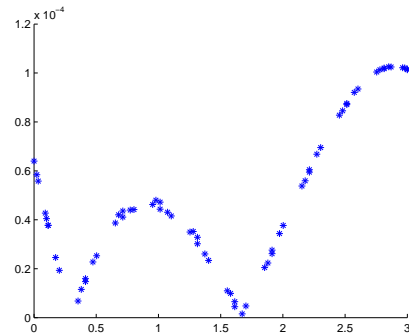


FIGURE IV. ERROR $e_{y,k}(t)$ AFTER ITERATION 5

V CONCLUSION

In this paper, considering the iterative learning control problem for a class of systems, and combining with vector Lyapunov function, the novel controllers, which can guarantee the robust convergence of the tracking error, are designed. From Figs.1 and 2 of the given example, we find that the output errors $e_{x,k}(t), e_{y,k}(t)$ downsize almost 10 times from iteration 4 to iteration 5. So it is made known that the proposed method is effective.

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