Sieve Method: Sieve the Forward and Reverse In One Time

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Abstract—This article carries on "Sieve Method: Sieve the Forward and Reverse In One Time" through establishing the dual element set and forms a new fundamental mathematical theory to sieve odd prime number pairs. The conclusion verifies the formula that calculating the total number of pairs that any even number is the sum of two odd prime numbers, and the minimum is that when an even number is extracted of root, the total number of odd prime number pairs is never less than one, so the Goldbach Conjecture is solved.

Keywords—sieve Method; sieve the forward and reverse in one time; prime number pairs; dual element set

I. POSITIVE AND NEGATIVE SCREENING

The only assumption is that "prime" is the only element in the mathematics of mass energy (matter and energy).

Example: take a W=26 even after prescribing the prime number of prime sieve to 2, 3, 5 (red):

Positive subsets: +S = 0 1 ② ③ 4 ⑤ 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 (positive energy of superscript)

Reverse subsets: -S = 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6 ⑤ ④ ③ ② 1 0 (subscript negative energy)

(1) [theorem 1] Goldbach's conjecture for the exact solution of the formula:

\[ Z_w(w) = Z_{pq}(w) + 2Z_a(w) \cdots \cdots \ (1.1) \]

\[ Z_{pq}(w) = \sum_{k_1 \leq \sqrt{w}} 1 \left( \sum_{k_2 \leq \sqrt{w}} 1 \left( \sum_{k_3 \leq \sqrt{w}} 1 \left( \sum_{k_4 \leq \sqrt{w}} 1 \left( \sum_{k_5 \leq \sqrt{w}} 1 \left( \sum_{\Phi} 1 \right) \right) \right) \right) \right) +1 \] \cdots \cdots \ (1.2)

Checking calculation: Taking the minimum value =6 of the even number, the limit value.

Goldbach's conjecture

[theorem 1-2] is confirmed by the appraisal document of the United States intellectual properties bureau and the certificate of authorization of the 2012.4.4.

After 51 years (1962-2012) and 17 Chinese University professor... Hong Kong has Qiwen professor Su Dazhu, Ph.D., Southern China Normal University clock set column number of tutors, Xiamen University mathematician Li Wenging guiding the audit results. Opening an equation to solve two unknowns is an alternative mathematics.

If the formula (1.1) is used to calculate the "heavy prime" table, "double key lock password" "DNA sequence" "atomic energy meter"... All the "universe tables" can be accurately looked up! The realization of Ai Bernstein "... How to establish an aesthetic system that can be strictly expressed by a formula. " A mathematical description of the lifetime of the universe!
Figure II.
Identification Documents of the United States Intellectual Property Office

Take any even number, \( w = 26 \) say.

We look at the set of its non-negative integers:

Forward Subset: \( S^+ = 0, 1, 2, \ldots, 24, 25, 26 \)

Backward Subset: \( S^- = 26, 25, 24, \ldots, 2, 1, 0 \)

By combining the two subsets, we can see that the sum of any arbitrary pair of numbers is equal \( w = 26 \).

\( \sqrt{w} = \sqrt{26} \approx 5.099 \), which after rounding, the prime factors are 2, 3 and 5.

This gives us two pairs of odd primes of known elements: 3+23 and 23+3, along with three pairs of odd primes with unknown elements: 7+19, 13+13, and 19+7.

II. BACKGROUND

In a letter to Euler on June 7th 1742, Goldbach proposed the conjecture expressed in two forms:

(A) Every even integer \( \geq 6 \) can be written as the sum of two odd primes.

(B) Every odd integer \( \geq 9 \) can be written as the sum of three odd primes.

We call (A) the strong conjecture and (B) the weak conjecture.

III. THE THEORY

Mr. Hui uses his "Sieve Method: Sieve the Forward and Reverse In One Time" to analyze the strong form of Goldbach’s Conjecture, i.e. given any even number \( w > 5 \), there exists two odd primes \( p, q \) such that \( w = p + q \).

- Let \( Z_w(p, q) \) be the number of pairs \( (p, q) \) that satisfies \( w = p + q \).
  - Let
\[
A_{w-1} = \begin{cases} 
1, & \text{if } w-1 \text{ is prime (i.e. } (1, w-1) \text{ hasn't been sieved)} \\
0, & \text{if } w-1 \text{ is not a prime (i.e. } (1, w-1) \text{ has been sieved)} 
\end{cases}
\]

- Let \( Z_i(w) \) be the number of pairs \((p, w-p_i)\) where \( p_i \) is a prime between \([0, \sqrt{w}]\) and \( w-p_i \) is also prime.

- Let \( Z_{pq}(w) \) be the number of pairs \((p, q)\) that satisfies \( w = p + q \) but falls outside the categories of \( Z_i(w) \) and \( A_{w-1} \).

This covers all types of number pairs we will encounter,

\[
Z_{pq}(w) = w + 1 + \sum_{l+m \leq n} (-1)^{l+m} \sum_{1 \leq l_i < j, j \leq n} \left( \frac{w - p_i^{\phi_j - j}}{p_i \cdots p_j} + 1 \right) - 2A_{w-1}
\]

where \( p_i^{\phi_j - j} \) is the initial value of \( S_{j_1 \cdots j_m}^{p_i^{\phi_j - j}} \), and

\[
S_{j_1 \cdots j_m}^{p_i^{\phi_j - j}} = S^{p_i \cdots p_j} \cap \cdots \cap S^{p_j},
\]

with \( S^{p_i} \) and \( S^{p_j} \) being the resultant subsets that are obtained by sieving the top and bottom rows of \( S \) by \( p_i \) and \( p_j \) respectively.

The lower bound of \( Z_i(w) \), called \( Z'_w(w) \), is:

\[
Z'_w(w) = \pi(\sqrt{w}) - 1 + A_{w-1}
\]

where \( n = \pi(\sqrt{w}) \) is the number of primes between \([0, \sqrt{w}]\).

It is easy to show that \( \pi(\sqrt{w}) - 1 + A_{w-1} \) is always greater than or equal to 1 (as there are infinitely many primes), and thus by combining (1) and (3), Goldbach’s Conjecture can be verified. The author fully understands the implication of this statement and would like to clarify that the aims behind his theories are solely for the purpose of furthering any discussion in this field. This is because the basis of his proof is fundamentally based on his concept of a 2-Way Sieve, which he believes requires a much higher level of scrutiny than it currently receives. Otherwise, he is afraid that the concept would only be deemed worthy of ridicule by the worldwide community.

IV. PROOF

Here is how the proof roughly goes:

Take the set \( S \)

And divide by \( \sqrt{w} \) into three segments: AB, BC and CD. AB and CD are symmetrical. \( p_i \) is a prime between \([0, \sqrt{w}]\) which we use to sieve the top and bottom rows of \( S \) separately.

(a) Sieve the top row of \( S \) (i.e. the Forward Sieve)
using $p_i$, and call the resulting subset $S^{p_i}$ (note we will express $p_i$ as $p_i, \ldots, p_i$, $l \leq n$):

$$S^{p_i} = \left\{ \begin{array}{l} \left( \frac{0}{w} \right), \left( \frac{p_i}{w-p_i} \right), \left( \frac{2p_i}{w-2p_i} \right), \ldots, \left( \frac{x}{w-x} \right), \ldots \end{array} \right\}$$

The smallest $x$ could be is 0, which we’ll denote as $r^{p_i} = 0$, the initial value of $S^{p_i}$. Remove $S^{p_i}$ from $S$ and note that $\left( \frac{p_i}{w-p_i} \right)$ is also sieved.

(b) Take $p_j$ and sieve the bottom row of $S$ (the Backwards Sieve), expressing $p_j$ as $p_j, \ldots, p_j$, with $m \leq n$. We obtain the subset $S_{p_j}$:

$$S_{p_j} = \left\{ \begin{array}{l} \ldots, \left( \frac{w-x}{x} \right), \ldots, \left( \frac{w-2p_j}{w-2p_j} \right), \left( \frac{w-p_j}{p_j} \right), \left( \frac{w}{0} \right) \end{array} \right\}$$

Since $w-x = \lambda p_j$ (with $\lambda$ an integer), we get $w-x \equiv 0 \pmod{p_j}$, hence $x \equiv w \pmod{p_j}$. The minimum value of $x$ is $r_{p_j} = \left\{ \frac{w}{p_j}, \frac{w}{p_j} \right\}$ being the remainder of $w$ divided by $p_j$. As before this is the initial value of $S_{p_j}$, which we remove from $S$ and noting again that $\left( \frac{w-p_j}{p_j} \right)$ is also sieved.

By subtracting $S^{p_i}$, $S^{p_j}$, $S^{p_k}$, $S^{p_h} S^{p_r}$, $S_{p_j}$, $S_{p_k}$, $S_{p_r}$, $S_{p_h}$, $S_{p_m}$ from $S$, all elements of the form $\left( \frac{\lambda p_i}{w-\lambda p_i} \right)$ are sieved from segments $AB$, $BC$ and $CD$. This leaves us with pairs of odd primes $\left( \frac{p}{q} \right)$ in the BC, which we will denote as $Z_{pq}(w)$.

Upon letting $2Z_a(w)$ be the number of pairs between $AB$ and $CD$ where $p_i$ and $w-p_i$ are both prime, we get:

$$Z_w(w) = Z_{pq}(w) + 2Z_a(w)$$

We begin by looking at the size of the sets:

$$|S| = w + 1, \quad |S^{p_i}| = \left[ \frac{w-r^{p_i}}{p_i} + 1 \right], \quad |S_{p_j}| = \left[ \frac{w-r_{p_j}}{p_j} + 1 \right]$$

Suppose we know what $\pi(\sqrt{w})$ is. If we now consider the number of primes between $BD$, we obtain the following:

$$\pi(w) - \pi(\sqrt{w}) = (w + 1) - 1 - \sum_{i=1}^{\alpha} \left[ \frac{w-r^{p_i}}{p_i} + 1 \right] + \sum_{0 \leq i < j \leq n} \left[ \frac{w-r^{p_i}p_j}{p_i \cdot p_j} + 1 \right] - L + (-1)^\alpha \left[ \frac{w-r^{p_1 \cdots p_N}}{p_1 \cdot p_2 \cdots p_N} + 1 \right]$$

Which simplifies to
\[ \pi(w) - 1 = \left[ \pi(\sqrt{w}) - 1 \right] + w + \sum_{1 \leq i \leq n} (-1)^i \sum_{1 \leq i_1 < \cdots < i_l \leq n} \left[ \frac{w - r_{i_1 \cdots i_l}}{p_{i_1} \cdots p_{i_l}} \right] \] (2.1)

Now,

\[ \begin{align*}
Z_{pq}^m(w) &= |S| + \sum_{0 \leq i, j \leq n} \left( -1 \right)^{i+j} \sum_{0 \leq k \leq n} |S_{pq}^{i+j+k} S_{pq}^{i+j+k}|

&= |S| + \sum_{0 \leq i, j \leq n} \left| S_{pq}^{i+j+k} S_{pq}^{i+j+k} \right| - \sum_{0 \leq i, j \leq n} \left| S_{pq}^{i+j+k} S_{pq}^{i+j+k} \right| + L + \left( -1 \right)^{i+j} \left| S_{pq}^{i+j+k} S_{pq}^{i+j+k} \right|

&= w + 1 - \sum_{0 \leq i, j \leq n} \left[ \frac{w - r_{i,j}^1}{p_{i,j}} + 1 \right] + \sum_{0 \leq i, j \leq n} \left[ \frac{w - r_{i,j}^2}{p_{i,j}} + 1 \right] - \sum_{0 \leq i, j \leq n} \left[ \frac{w - r_{i,j}^3}{p_{i,j}} + 1 \right] + L + \left( -1 \right)^{i+j} \left[ \frac{w - r_{i,j}^4}{p_{i,j}} + 1 \right]

&= w + 1 + \sum_{0 \leq i, j \leq n} \left( -1 \right)^{i+j} \sum_{0 \leq i, j \leq n} \left[ \frac{w - r_{i,j}^4}{p_{i,j}} + 1 \right] - 2A_{w-1}

Where

\[ \begin{align*}
x &= 0 \pmod{p_i} \\
x &= 0 \pmod{p_j} \\
&\ldots\\n\prod_{i \in I \setminus \{j \}} p_i &= 0 \pmod{p_j}
\end{align*} \]

\[ \prod_{i \in I \setminus \{j \}} p_i = 0 \pmod{p_j} \]

Moving on, we now show

\[ Z_w(w) = \left[ \pi(\sqrt{w}) - 1 \right] + A_{w-1} \] (3)
Prior to sieving, we can alter (2.1) slightly to express the sum of odd prime pairs \( Z_\omega(w) \) and pairs that contain a single odd prime \( Z_p(w) \) as:

\[
2Z_w(w) + Z_p(w) = 2\left\{ \pi(\sqrt{w}) - 1 \right\} + w + \sum_{1 \leq i < j \leq m} (-1)^j \sum_{1 \leq i < j \leq m} \left( \frac{w - r_{i:j}^{k:j}}{P_i \cdots P_j} + 1 \right) \right\)
\]

(2.2)

After applying the 2-Way Sieve, segments AB and BD are removed, leaving us with \( Z_{pq}(w) \) in segment BC, i.e.

\[
2Z_w(w) + 2Z_a(w) = 2\left\{ \pi(\sqrt{w}) - 1 \right\} + w + \sum_{1 \leq i < j \leq m} (-1)^j \sum_{1 \leq i < j \leq m} \left( \frac{w - r_{i:j}^{k:j}}{P_i \cdots P_j} + 1 \right) - Z_p(w) - 2\pi(\sqrt{w}) - 2A_{w-1}
\]

And substituting in equation (1), we get:

\[
2Z_{pq}(w) + Z_p(w) = 2\left\{ w + \sum_{1 \leq i < j \leq m} (-1)^j \sum_{1 \leq i < j \leq m} \left( \frac{w - r_{i:j}^{k:j}}{P_i \cdots P_j} + 1 \right) \right\} - 2A_{w-1}
\]

(2.3)

So equations (2.2) and (2.3) must be satisfied before and after sieving. Thus we can solve these simultaneously to get:

\[
2Z_w(w) - 2Z_{pq}(w) = 2\pi(\sqrt{w}) - 1 + 2A_{w-1}
\]

\[
Z_w(w) = \pi(\sqrt{w}) - 1 + A_{w-1} + Z_{pq}(w)
\]

Now suppose that \( Z_{pq}(w) = 0 \), then we are left with the minimum value of \( Z_w(w) \), which is:

\[
Z'_w(w) = \pi(\sqrt{w}) - 1 + A_{w-1}
\]

V. SOME BRIEF EXAMPLES

1) Take \( w = 10 \).

\[
Z_{pq}(w) = 10 + 1 - \left( \left\lfloor \frac{10 - 0}{2} \right\rfloor + \left\lfloor \frac{10 - 1}{3} \right\rfloor + \left\lfloor \frac{10 - 1}{3'} \right\rfloor + 1 \right) - \left( \left\lfloor \frac{10 - 4}{2} \right\rfloor + \left\lfloor \frac{10 - 4}{2 \cdot 3'} \right\rfloor + 1 \right) = 0
\]

and \( 2Z_a(w) = 2 \) (since we have 7+3 and 3+7).

\[ \therefore Z_w(w) = Z_{pq}(w) + 2Z_a(w) = 1 + 2 = 3 \] (the three pairs being 3+7, 5+5 and 7+3).

\[
Z'_w(w) = \pi(\sqrt{w}) - 1 + A_{w-1} = 2 - 1 + 0 = 1
\]
is also correct.

2) Take \( w = 12 \).

\[
Z_{pq}(w) = 12 + 1 - \left( \left\lfloor \frac{12 - 0}{2} \right\rfloor + \left\lfloor \frac{12 - 1}{3} \right\rfloor + \left\lfloor \frac{12 - 1}{3'} \right\rfloor + 1 \right) - \left( \left\lfloor \frac{12 - 4}{2} \right\rfloor + \left\lfloor \frac{12 - 4}{2 \cdot 3'} \right\rfloor + 1 \right) = 0
\]

and \( 2Z_a(w) = 2 \) (since we have 7+3 and 3+7).

\[ \therefore Z_w(w) = Z_{pq}(w) + 2Z_a(w) = 2 + 3 = 5 \] (the three pairs being 3+7, 5+5 and 7+3).

\[
Z'_w(w) = \pi(\sqrt{w}) - 1 + A_{w-1} = 2 - 1 + 0 = 1
\]
\[ Z_{pq}(w) = 12 + 1 - \left( \left\lfloor \frac{12}{2} + 1 \right\rfloor + \left\lfloor \frac{12}{3} + 1 \right\rfloor \right) + \left\lfloor \frac{12}{2.3} + 1 \right\rfloor - 2 = 13 - (7 + 5) + 3 - 2 = 2 \]

\[ \therefore Z_w(w) = Z_{pq}(w) + 2Z_a(w) = 2 + 0 = 2 \]

(the two pairs being 5+7 and 7+5)

\[ Z'_w(w) = [\pi(\sqrt{w}) - 1] + A_{w-1} = 2 - 1 + 1 = 2 \]

is also correct.

Calculation:

\[ Z'_w(w) = [\pi(\sqrt{w}) - 1] + A_{w+1} \quad (1.4) \]

REFERENCES

[1] 2001-2004, the vice professor Song Fugao of Shen Zhen university calculated \( W = 6 \rightarrow 10^0 \) conformed with (1.4)

[2] 2002, professor Zhu Lie of Su Zhou university calculated \( W < 20001 \) conformed with (1.4)

[3] 2006, professor Xu Zuoming of Liao Ning university calculated \( W = 6 \rightarrow 10^{11} \) conformed with (1.4)