Airport Security Analysis
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Abstract. Nowadays, airplane is the fastest transportation tool, and more and more passengers choose it when they are traveling. So, Airport security analysis can help to upgrade Airport security system. For analyzing Airport security system, the Model of M / M / c queuing system in Queuing Theory is built to explore the flow of passengers in security screening system. The data obtained from reality in reference are linear fitted and the Gray Model is employed to analyze the data, and the average service rate of different Zones is obtained. In addition, added the principle of human traffic to make the model more complete.

Introduction

Nowadays, airplane is the fastest transportation tool, and more and more passengers choose it when they are traveling. But what will happen if the passengers spend more time on waiting, meaning that the time cost on plane more than train? It is obvious that the passengers will take other traffic tools, not the planes. At the same time, the airport security has been significantly enhanced throughout the world to avoid the potential terrorist attacks. How to shorten the waiting time under the premise of security? In order to solve the global problems, the Queuing Theory is employed to establish Model to analyzing the Airport security system.

The mathematical model based on Queuing Theory

Airport Security Screening Process Analysis. Referring to literatures and our boarding experience at airport, we make the interpretations of Figure 1 as below:

- The ID Check channel is in one-to-one relationship with the subsequent detection channel. That is, the ID Check channel can’t be detected by another subsequent detection channel.
- And then, Passengers are ready to do Millimeter Wave Scan and X-ray Scan, where they should remove the package of electronic products and shoes, belts, jackets in advance.
- (Pre-Check travelers would not remove the shoes, belts, jackets)
- During the Millimeter Wave Scan and X-ray, passengers should go for a tap check in Zone D if there is a threat.
- If there is not a threat, passengers could pass the TSA Security Screening, and then waiting for their plane.
We divide the TSA Security Screening System into two parts. From figure 2, we know that the first part includes queue 1 and ID check, and the second part includes queue 2 and follow-up inspection (including millimeter-wave scan and X-ray scan). As the inspection of part 1 and part 2 is carried out at the same time, we assume that there will be just one queue at part 2.

**Queuing Theory.** We assume that the passengers’ arrival follow those conditions:

- a. The time between successive arrivals is independent of the past.
- b. In each small time interval of length $\Delta t$ the occurrence of an arrival is equally likely. In other words, Poisson arrivals occur completely random in time.
- c. For sufficiently small time $\Delta t$, The probability of two or more passengers arriving in the time interval $(t, t+\Delta t)$ is so tiny that we ignore it.

So the number of passengers obeys Poisson distribution with $\lambda$, and the time between arrivals is exponentially distributed with $\lambda$. As the third assumption mentioned above, we consider that the average service rate of each windows is constant.

According to Queuing Theory, we have:

$$
\begin{align*}
\mu P_1 &= \lambda P_0 \\
(n + 1)\mu P_{n+1} + \lambda P_{n-1} &= (\lambda + n\mu)P_n \quad (1 \leq n \leq c) \\
c\mu P_{n+1} + \lambda P_{n-1} &= (\lambda + c\mu)P_n \quad (n > c)
\end{align*}
$$

(1)
Where:
\( \lambda \) is average arrival rate.
\( \mu \) is average service rate.
\( P_i \) is state probability (i=0~n).
c is number of desks.
\( \rho = \frac{\lambda}{c\mu}, \sum_{i=0}^{\infty} P_i = 1, \rho \leq 1 \), \( \rho \) is service intensity.

Using the recursive method to solve the difference equation, we have:

\[
\begin{align*}
    P_0 &= \left[ \sum_{k=0}^{c-1} \frac{1}{k!} \left( \frac{\lambda}{\mu} \right)^k + \frac{1}{c!} * \frac{1}{1-\rho} * \left( \frac{\lambda}{\mu} \right)^c \right]^{-1} \\
    P_n &= \begin{cases} \\
        \left( \frac{1}{n!} \left( \frac{\lambda}{\mu} \right)^n P_0 \quad (n \leq c) \\
        \left( \frac{1}{c!^{n-c}} \left( \frac{\lambda}{\mu} \right)^n P_0 \quad (n > c) \\
    \end{cases}
\end{align*}
\]

Solving the Eq.2, we obtain:

\[
L_q = \sum_{n=c+1}^{\infty} (n-c)P_n = \frac{(c\rho)^c\rho}{c!(1-\rho)^2} P_0
\]

\[
W_q = \frac{L_q}{\lambda}
\]

Where:
\( L_q \) is the average length of queue.
\( W_q \) is average waiting time.

**The principle of human traffic.** If people need to pass continuously through two or more service systems, the unequal average service rate of these several systems will make the average human traffic unequal between the last and the next. It is regarded that the average human traffic is related to the average flow rate and average human traffic of the previous one.

We draw Figure 3 after improving the Figure 2:

![Figure 3. The principle applied to Security System](image)

The average human traffic in part 1 is \( \lambda_1 \), namely, is the total human traffic in airport. Besides, the average human traffic in part 2 is \( \lambda_2 \), the numbers of people arriving within a minute is \( k_1 \). As the thirteenth assumption mentioned above, we assume that the arrival of passengers obeys to Poisson distribution.

Figure 3 shows the probability of the number of passengers arrived part 1 in a minute is:

\[
P(n = k_1) = \frac{\lambda_1^{k_1}e^{-\lambda_1}}{k_1!}
\]

Then we calculate the average length of queue in part 1\( (L_{q1}) \). What’s more, we consider that the service rate of part 1 is more than the service rate of part 2, so we do not take \( L_{q1} \) into account. Meanwhile, we find that the influence is quite small.

- When \( k_1 \leq \mu_1 \), we consider that passengers can pass part 1 directly. This means that, part 1 would not influence the velocity of flow of passengers. Where \( k_2 = k_1 \).
- When \( k_1 > \mu_1 \), we consider that passengers could only pass the number of the value of \( \mu_1 \) in a minute. Where \( k_2 = \mu_2 \).

From these analyses, we get:
\[ P(n = k_2) = \begin{cases} \frac{\lambda_2^{k_2} e^{-\lambda_1}}{k_2!} & (k_2 \leq \mu_1) \\ \sum_{k=\lceil \mu_1 \rceil+1}^{\infty} \frac{\lambda_2^{k} e^{-\lambda_1}}{k!} & (k_2 > \mu_1) \end{cases} \] (6)

From Eq.6, we find that the value of \( k_1 \) is the same as \( \lambda_2 \). However, in order to solve the Eq.2, we should know the value of \( \mu_1 \) and \( \mu_2 \). Firstly, we use Grey Model to work out \( T_{\text{ID}} \). Studying the data of ID Check by two different TSA officers, we stack them by transforming time to time stamps. Furthermore, using the Least Square Method to calculate its slope, we find that the ID check processing time of officer 1 is about 10.8 s/people, whereas the ID check processing time of another Officer is about 12.1 s/people. Taking the average of those two data, we get the average service rate of part 1. Besides, there remains modification if more data was given.

Using Gray Model and computer software MATLAB, we analyze the average service rate of part 2, we obtain statistic results as follows:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>time(s/people)</th>
<th>Symbol</th>
<th>time(people/min)</th>
<th>Symbol</th>
<th>time(people/min)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_x )</td>
<td>7.97</td>
<td>( \mu_1 )</td>
<td>5.263</td>
<td>( \lambda_r )</td>
<td>4.096</td>
</tr>
<tr>
<td>( t_m )</td>
<td>12.28</td>
<td>( \mu_2r )</td>
<td>0.6771</td>
<td>( \lambda_p )</td>
<td>6.092</td>
</tr>
<tr>
<td>( t_b )</td>
<td>28.37</td>
<td>( \mu_2p )</td>
<td>1.2341</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to Table 1, we take the average of service time as the data be given is not enough. What’s worst, we couldn’t ensure the efficiency of airport officers. After analyzing the average service rate of part 2, we find that the \( t_m \) and \( t_x \) are shorter than \( t_b \). So we consider that the passengers have passed the millimeter scanning before the luggage going through the scanning process. Thinking about the pre-check process, we consider that regular passengers will cost 60 s while pre-check passengers will cost 20 s to pass the Security System. Furthermore, the passing time should be modified with objective data. According to the first two columns of data, we use the least square method to match the slope of curve. In other words, it is the average arrival rate of the measuring time, which can be used to test model and give the optimal allocation of security resources.

**Conclusion**

Based on queuing theory, this paper deeply analyzes the airport security system. In this paper, I established the principle of human traffic, and use it to analyze the relationship between the waiting time and the arrival rate of two related service systems in the airport. Readers can use the text method to analyze other queuing theory models.

**Reference**


1252-1260.