Stabilization for Markovian Jump Systems with Additive Time-varying Delays

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Abstract—In this paper, the problem of stabilization for Markovian jump systems with additive time-varying delays is considered. First of all, by constructing a new Lyapunov functional with triple integral terms, using new inequalities, convex combination technique and combining with other analytical techniques, the stochastic stability and stabilization conditions of the Markovian jump systems in terms of linear matrix inequalities with lower conservatism are obtained. Then, the controller of the closed-loop system is designed by the transformation technique of inequalities. Finally, a numerical example is given to verify the effectiveness of the provided method and the superiority of the results.

Keywords—markovian jump systems; additive time-varying delays; stochastically stabilization; linear matrix inequalities

I. INTRODUCTION

In recent years, a large number of attentions have been focused on the study of the Markov jump systems (MJSs), since this family of systems were firstly introduced by Krasovskii and Lidskii[1] in 1961. Because of their advantage of modeling many practical dynamic systems, lots of works on stability analysis, controller synthesis and filter design have been focused on the study for MJSs [2-8]. Meanwhile, as a special kind of hybrid systems with finite operation modes whose structures are subject to random abrupt changes, which may result from abrupt phenomena such as random failures and repairs of the components, sudden environmental changes, changes in interconnections of subsystems, etc. So, the research about the synthesis problem for MJSs will become more complex and difficult. In addition, delay systems are frequently encountered in various practical systems, such as engineering, biology, economics, neural networks, and network control, [9-16], however, this class system is instability and performance deterioration in many cases. Thus, recent past decades have witnessed extensive research on delay systems in the literature, including stability analysis, stabilization H^∞ controller design, robust filtering analysis, and model reduction or simplification [17-26].

Especially, the delay systems with additive time-varying delay component were firstly proposed in [27], which has a strong application background in remote control and networked control (see, for instance [18,28,29] and the references therein). Take a state-feedback networked control, for example. Some time delays will appear inevitably when signals are transmitted

from one device to another, due to the physical plant, controller, sensor and actuator are located at different places. Among those delays, there are two network-induced delays, one from sensor to controller and the other from controller to actuator. With the two delays considered, the closed-loop system will appear with two additive time delays in the state. Unfortunately, the two delays are generally time-varying with different properties, because the network transmission conditions is different. This implies that the systems with additive time-varying delays are more complicated and challenged. Therefore, the studies on the stability analysis and controller synthesis for MJSs with additive time-varying delay components are of theoretical and practical importance.

Fortunately, several results about Markov jump neural networks with additive time-varying delay components have been proposed in the past years. For instance, in [30], the authors investigated the synchronization of singular Markovian jumping complex dynamical networks with two additive time-varying delay components by using the pinning control. In [31,32], the problem of dissipativity and passivity of Markovian jump stochastic neural networks involving two additive time-varying delays was investigated, several sufficient conditions are derived for verifying the dissipativity criteria of neural networks. [33] Concerned with the problem of finite-time boundedness, L_2 gain analysis and control of Markovian jump switched neural networks with additive time-varying delays. [34] Focused on the problem of delay-dependent stability and dissipativity analysis of generalized neural networks with Markovian jump parameters and two delay components. [35] Mainly concerned on stability problem of Markovian jump neural networks with mode-dependent two additive time-varying delays based on quadratic convex combination approach. Despite these efforts, however, to the best knowledge of our authors, the stabilization problem for MJSs with additive time-varying delay components has not been investigated fully in the literature.

Motivated by the aforementioned concerns, in this paper, we mainly investigate the stabilization for MJSs with additive time-varying delays. At first, we provide a novel double integral inequality, which is less conservative than the same type inequality found in the literature. Secondly, some new Lyapunov functionals with triple integral terms are constructed via considering more information about the two additive time-varying delays of system, such as $x(t - d_1(t))$, $x(t - d_1)$, then the

stochastically stable condition of the Markovian jump systems in terms of linear matrix inequalities with lower conservatism is derived by using new inequalities, convex combination technique and combining with other analytical techniques. At last but not least, based on the obtained stochastically stable condition, the controller is designed later. Finally, a numerical example is given to verify the effectiveness of the proposed method and the superiority of the results.

Notation: Throughout this paper, R^n denotes the n -dimensional Euclidean space, $R^{m \times n}$ is the set of all $m \times n$ real matrices. $\|\cdot\|$ refers to the Euclidean vector norm. For a real symmetric matrix X , $X > 0$ ($X \geq 0$) means that X is positive definite (semi-positive definite). The notation "*" in a block matrix always represents the symmetric terms. The superscript "T" stands for the transpose of a matrix or a vector. $diag\{\dots\}$ for a block-diagonal matrix, and $sym\{M\} = M + M^T$. The identity and zero matrices of appropriate dimensions are denoted by I and O , respectively. The following nomenclature is used to simplify vector and matrix representations: $e_i = (0 \dots 0, \underset{i}{I}, 0 \dots 0), (i = 1, \dots, 21)$.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider the following MJSs with additive time-varying delays:

$$\begin{cases} \dot{x}(t) = A(r_t)x(t) + B(r_t)x(t-d(t)) + C(r_t)u(t) \\ x(t_0 + \theta) = \varphi(\theta), \quad \forall \theta \in [-d, 0] \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ is the control input, $\varphi(\theta)$ is an initial condition function. The time delay $d(t) = d_1(t) + d_2(t)$, where $d_1(t)$ and $d_2(t)$ represent two time-varying delays in the state with different physical characteristics and satisfy:

$$\begin{aligned} 0 \leq d_1(t) \leq d_1 < \infty, 0 \leq d_2(t) \leq d_2 < \infty, \\ 0 \leq \dot{d}_1(t) \leq \mu_1 < \infty, 0 \leq \dot{d}_2(t) \leq \mu_2 < \infty, \end{aligned} \quad (2)$$

where d_1, d_2 and μ_1, μ_2 are known constants. Naturally, we denote $d = d_1 + d_2, \mu = \mu_1 + \mu_2$. $A(r_t), B(r_t), C(r_t)$ are known matrix functions of the random Markov jumping process. $\{r_t, t \geq 0\}$ is right-continuous Markov process on the probability space taking values in a finite state space $N = (1, 2, \dots, N)$ with generator $\Lambda = (\lambda_{ij}), i, j \in N$ given by

$$Pr\{r_{t+\Delta} = j \mid r_t = i\} = \begin{cases} \lambda_{ij}\Delta + o(\Delta), & j \neq i \\ 1 + \lambda_{ii}\Delta + o(\Delta), & j = i \end{cases}, \quad (3)$$

where $\Delta > 0, \lim_{\Delta \rightarrow 0} o(\Delta)/\Delta = 0, \lambda_{ij} \geq 0$, for $i \neq j$ is the transition rate from mode i at time t to mode j at time $t + \Delta$, $\lambda_{ii} = -\sum_{j=1, j \neq i}^N \lambda_{ij}$.

For the sake of simplicity, the solution $x(t, \varphi(\theta), r_0)$ with $r_0 \in N$ is denoted $x(t)$. It is known from [36] that $\{x(t), r_t\}$ is a Markov process with an initial state $\{\varphi(\theta), r_0\}$ and its weak infinitesimal generator, acting on function $V(x(t), t, r_t)$, is defined in [37]:

$$LV(x(t), t, r_t) = \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} \{E[V(x(t+\Delta), t+\Delta, r_{t+\Delta}) \mid x(t)] - V(x(t), t, r_t)\}. \quad (4)$$

Before presenting the main results, the following definition and lemmas are necessary, which will be used in the proof of our main results.

Definition 1[38]: The system (1) with $u(t) = 0$ is said to be stochastically stable if the following condition holds:

$$E\left\{\int_0^\infty \|x(t)\|^2 dt \mid \varphi, r_0\right\} \leq \infty, \quad (5)$$

for every initial condition $\varphi(\theta) \in R^n$ and $r_0 \in N$.

Lemma 1[39]: For a positive definite matrix $R \in R^{n \times n}$, and a differentiable function $\{x(u) \mid u \in [a, b]\}$, the following inequality holds:

$$\sigma \int_a^b \dot{x}^T(s) R \dot{x}(s) ds \geq Y_1^T R Y_1 + 3 Y_2^T R Y_2 + 5 Y_3^T R Y_3, \quad (6)$$

where $\sigma = (b-a)$, and

$$Y_1 = x(b) - x(a), Y_2 = x(b) + x(a) - 2\sigma^{-1} \int_a^b x(s) ds,$$

$$Y_3 = Y_1 + 6\sigma^{-1} \int_a^b x(s) ds - 12\sigma^{-2} \int_a^b \int_a^b x(s) ds d\theta$$

Lemma 2[40]: For a positive definite matrix $R \in R^{n \times n}$, matrices $N_1, N_2, N_3 \in R^{4n \times n}$ and any differentiable function $\{x(u) \mid u \in [a, b]\}$, the following inequality holds:

$$-\int_a^b \int_a^b \dot{x}^T(s) R \dot{x}(s) ds d\theta \leq \eta^T \Omega \eta, \quad (7)$$

where

$$\begin{aligned}\Omega &= 2^{-1}\sigma^2 N_1 R^{-1} N_1^T + 4^{-1}\sigma^2 N_2 R^{-1} N_2^T + 6^{-1}\sigma^2 N_3 R^{-1} N_3^T \\ &\quad + \sigma \text{sym}\{N_1 \Pi_1 + N_2 \Pi_2 + N_3 \Pi_3\}, \quad \Pi_1 = \bar{e}_1 - \bar{e}_2, \\ \Pi_2 &= \bar{e}_1 + 2\bar{e}_2 - 6\bar{e}_3, \quad \Pi_3 = \bar{e}_1 - 3\bar{e}_2 + 24\bar{e}_3 - 60\bar{e}_4, \\ \eta &= \left[x^T(b), \sigma^{-1} \int_a^b x^T(s) ds, \sigma^{-2} \int_a^b \int_\theta^b x^T(s) ds d\theta, \right. \\ &\quad \left. \sigma^{-3} \int_a^b \int_\theta^b \int_u^b x^T(s) ds du d\theta \right], \quad \bar{e}_1 = (I, 0, 0, 0), \\ \bar{e}_2 &= (0, I, 0, 0), \quad \bar{e}_3 = (0, 0, I, 0), \quad \bar{e}_4 = (0, 0, 0, I).\end{aligned}$$

III. MAIN RESULTS

Our purpose in this chapter is to study the stabilization and design the controller for MJSs (1).

Theorem 1. The MJSs (1) with $u(t)=0$ is stochastically stable if there exist symmetric positive definite matrix $P_i \in R^{n \times n}$, $W \in R^{6n \times 6n}$, $Q_i, R_i \in R^{n \times n}$, ($i=1,2,3,4,5,6$; $l_2=1,2,3,4,5$) and any matrices $N_i, M_i \in R^{4n \times n}$, ($i=1,2,3$), $X, Y, Z, K \in R^{12n \times n}$ such that

$$Q_1 - Q_2 > 0, Q_2 - Q_3 > 0, Q_3 - Q_4 > 0, \quad (8)$$

$$\Psi_{20 \times 20} = \begin{pmatrix} \Psi_{12 \times 12} & \hat{N} & \hat{M} & \hat{\Xi}_l \\ * & -\hat{R}_1 & 0 & 0 \\ * & * & -\hat{R}_2 & 0 \\ * & * & * & -\hat{R}_{3l} \end{pmatrix} < 0, \quad (9)$$

where ($l=1,2,3,4$) and

$$\begin{aligned}\Psi_{12 \times 12} &= e_1^T \Pi e_1 + e_1^T (Q_1 + Q_2) e_1 - (1 - \mu_1) e_2^T (Q_1 - Q_2) e_2 \\ &\quad - (1 - \mu) e_3^T (Q_2 - Q_3) e_3 - (1 - \mu_1) e_4^T (Q_3 - Q_4) e_4 \\ &\quad - e_5^T (Q_4 + Q_6) e_5 - e_5^T (Q_5 - Q_6) e_9 - H_{11}^T R_4 H_{11} - 3 \\ &\quad H_{12}^T R_4 H_{12} - 5 H_{13}^T R_4 H_{13} - H_{21}^T R_5 H_{21} - 3 H_{22}^T R_5 H_{22} \\ &\quad - 5 H_{23}^T R_5 H_{23} + \text{sym}\{\zeta^T X (e_1 - e_2) + \zeta^T Y (e_2 - e_3) \\ &\quad + \zeta^T Z (e_3 - e_4) + \zeta^T K (e_4 - e_5)\} + \text{sym}\{e_{st}^T P_i e_1 \\ &\quad + \eta^T W \eta\} + d_2 \text{sym}\{\zeta_{10}^T N_1 \zeta_{11} + \zeta_{10}^T N_2 \zeta_{12} + \zeta_{10}^T N_3 \zeta_{13}\} \\ &\quad + d_1 \text{sym}\{\zeta_{20}^T M_1 \zeta_{21} + \zeta_{20}^T M_2 \zeta_{22} + \zeta_{20}^T M_3 \zeta_{23}\} + e_{st}^T \hat{R} e_{st},\end{aligned}$$

$$\Pi = \sum_{j=1}^N \lambda_{ij} P_j, \hat{N} = (N_1^T, N_2^T, N_3^T), \hat{M} = (M_1^T, M_2^T, M_3^T),$$

$$\hat{R}_1 = \text{diag}\{2d_2^{-2} R_1, 4d_2^{-2} R_1, 6d_2^{-2} R_1\}, \hat{\Xi}_1 = (Z, K), \hat{\Xi}_2 = (X, Z),$$

$$\hat{R}_2 = \text{diag}\{2d_2^{-2} R_2, 4d_2^{-2} R_2, 6d_2^{-2} R_2\}, \hat{\Xi}_3 = (Y, K), \hat{\Xi}_4 = (X, Y),$$

$$\hat{R}_{31} = \text{diag}\{d_2^{-1} R_3, d_1^{-1} R_3\}, \hat{R}_{32} = \text{diag}\{d_1^{-1} R_3, d_2^{-1} R_3\},$$

$$\hat{R}_{33} = \text{diag}\{d_2^{-1} R_3, d_1^{-1} R_3\}, \hat{R}_{34} = \text{diag}\{d_1^{-1} R_3, d_2^{-1} R_3\},$$

$$\hat{R} = (2^{-1} d_2^2 R_1 + 2^{-1} d_1^2 R_2 + d R_3 + d_2^2 R_4 + d_1^2 R_5), e_{st} = A_i e_1 + B_i e_3,$$

$$H_{11} = (e_9 - e_5), H_{12} = (e_9 + e_5 - 2e_6), H_{13} = (e_9 - e_5 + 6e_6 - 12e_7),$$

$$H_{21} = (e_1 - e_9), H_{22} = (e_1 + e_9 - 2e_{10}), H_{23} = (e_1 - e_9 + 6e_{10} - 12e_{11}),$$

$$\zeta = (e_1, e_2, \dots, e_{11}, e_{12}), \zeta_{10} = (e_9, e_6, e_7, e_8), \zeta_{20} = (e_1, e_{10}, e_{11}, e_{12}),$$

$$\zeta_{11} = (e_9 - e_6), \zeta_{12} = (e_9 + 2e_6 - 6e_7), \zeta_{13} = (e_9 - 3e_6 + 24e_7 - 60e_8),$$

$$\zeta_{21} = (e_1 - e_{10}), \zeta_{22} = (e_1 + 2e_{10} - 6e_{11}), \zeta_{23} = (e_1 - 3e_{10} + 24e_{11} - 60e_{12}),$$

$$\eta = (e_1, d_2 e_6, d_2^2 e_7, d_2^3 e_8, d_1 e_{10}, d_1^2 e_{11}, d_1^3 e_{12}), \dot{\eta} = (e_9 - e_5, d_2 (e_9 - e_6),$$

$$2^{-1} d_2^2 (e_9 - 2e_7), e_1 - e_9, d_1 (e_1 - e_{10}), 2^{-1} d_1^2 (e_1 - 2e_{11})).$$

Proof. Constructing a stochastic Lyapunov functional candidate as

$$V(x(t), t, r_t) = \sum_{j=1}^5 V_j(x(t), t, r_t),$$

where

$$V_1(x(t), t, r_t) = x^T(t) P(r_t) x(t) + \eta^T(t) W \eta(t), \quad (10)$$

$$\begin{aligned}V_2(x(t), t, r_t) &= \int_{t-d_1(t)}^t x^T(s) Q_1 x(s) ds + \int_{t-d(t)}^{t-d_1(t)} x^T(s) Q_2 x(s) ds \\ &\quad + \int_{t-d_1(t)-d_2}^{t-d(t)} x^T(s) Q_3 x(s) ds + \int_{t-d}^{t-d_1(t)-d_2} x^T(s) Q_4 x(s) ds \\ &\quad + \int_{t-d_1}^t x^T(s) Q_5 x(s) ds + \int_{t-d}^{t-d_1} x^T(s) Q_6 x(s) ds,\end{aligned} \quad (11)$$

$$\begin{aligned}V_3(x(t), t, r_t) &= \int_{-d}^{-d_1} \int_u^{-d_1} \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta du \\ &\quad + \int_{-d_1}^0 \int_u^0 \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta du,\end{aligned} \quad (12)$$

$$V_4(x(t), t, r_t) = \int_{-d}^0 \int_{t+\theta}^t \dot{x}^T(s) R_3 \dot{x}(s) ds d\theta, \quad (13)$$

$$V_5(x(t), t, r_i) = d_2 \int_{-d}^{-d_1} \int_{t+\theta}^t \dot{x}^T(s) R_4 \dot{x}(s) ds d\theta \\ + d_1 \int_{-d_1}^0 \int_{t+\theta}^t \dot{x}^T(s) R_5 \dot{x}(s) ds d\theta, \quad (14)$$

where $P(r_i), W, Q_{i_1}, R_{i_2}$ have been defined in Theorem 1 and are all positive matrices. Therefore, combining the inequality (8) can we know that $V(x(t), t, r_i) > 0$.

Then, for given $r_i = i \in N, P_i = P_i$ and the infinitesimal operator L of the stochastic process $\{x(t), r_i\}$ along the evolution of $V(x(t), t, r_i)$ are given as

$$LV_1(x(t), t, i) = \xi^T [\text{sym}\{\dot{x}^T(t) P_i x(t) + \dot{\eta}^T W \eta\} + e_1^T \Pi e_1] \xi, \quad (15)$$

$$LV_2(x(t), t, i) \leq \xi^T [e_1^T (Q_1 + Q_2) e_1 - (1 - \mu_1) e_2^T (Q_1 - Q_2) e_2 \\ - (1 - \mu) e_3^T (Q_2 - Q_3) e_3 - (1 - \mu_1) e_4^T (Q_3 - Q_4) e_4 \\ - e_5^T (Q_4 + Q_6) e_5 - e_9^T (Q_5 - Q_6) e_9] \xi, \quad (16)$$

$$LV_3(x(t), t, i) = \dot{x}^T(t) (2^{-1} d_2^2 R_1 + 2^{-1} d_1^2 R_2) \dot{x}(t) \\ - \int_{t-d}^{t-d_1} \int_{\theta}^{t-d_1} \dot{x}^T(s) R_1 \dot{x}(s) ds d\theta - \int_{t-d_1}^t \int_{\theta}^t \dot{x}^T(s) S_2 \dot{x}(s) ds d\theta, \quad (17)$$

From Lemma 2, it follows that

$$LV_3(x(t), t, i) \leq \dot{x}^T(t) (2^{-1} d_2^2 R_1 + 2^{-1} d_1^2 R_2) \dot{x}(t) + \zeta_{10}^T (2d_2^{-2} N_1 \\ \times R_1^{-1} N_1^T + 4d_2^{-2} N_2 R_1^{-1} N_2^T + 6d_2^{-2} N_3 R_1^{-1} N_3^T) \zeta_{10} + \zeta_{20}^T (2d_2^{-2} \\ \times M_1 R_2^{-1} M_1^T + 4d_2^{-2} M_2 R_2^{-1} M_2^T + 6d_2^{-2} M_3 R_2^{-1} M_3^T) \zeta_{20} \\ + d_2 \text{sym}\{\zeta_{10}^T N_1 \zeta_{11} + \zeta_{10}^T N_2 \zeta_{12} + \zeta_{10}^T N_3 \zeta_{13}\} \\ + d_1 \text{sym}\{\zeta_{20}^T M_1 \zeta_{21} + \zeta_{20}^T M_2 \zeta_{22} + \zeta_{20}^T M_3 \zeta_{23}\}, \quad (18)$$

$$LV_4(x(t), t, i) = \dot{x}^T(t) d R_3 \dot{x}(t) - \int_{t-d_1(t)}^t \dot{x}^T(s) R_3 \dot{x}(s) ds \\ - \int_{t-d(t)}^{t-d_1(t)} \dot{x}^T(s) R_3 \dot{x}(s) ds - \int_{t-d_1(t)-d_2}^{t-d(t)} \dot{x}^T(s) R_3 \dot{x}(s) ds \\ - \int_{t-d}^{t-d_1(t)-d_2} \dot{x}^T(s) R_3 \dot{x}(s) ds, \quad (19)$$

Now, we estimate the upper bound of the last four terms on the right-hand side in inequality (19). For any matrix $X, Y, Z, K \in \mathbb{R}^{12 \times n}$, we can get the following inequalities:

$$-\int_{t-d_1(t)}^t \dot{x}^T(s) R_3 \dot{x}(s) ds \leq \xi^T [d_1(t) \zeta^T X R_3^{-1} X^T \zeta + 2\zeta^T X (e_1 - e_2)] \xi, \\ -\int_{t-d(t)}^{t-d_1(t)} \dot{x}^T(s) R_3 \dot{x}(s) ds \leq \xi^T [d_2(t) \zeta^T Y R_3^{-1} Y^T \zeta + 2\zeta^T Y (e_2 - e_3)] \xi,$$

$$-\int_{t-d_1(t)-d_2}^{t-d(t)} \dot{x}^T(s) R_3 \dot{x}(s) ds \leq \xi^T [(d_2 - d_2(t)) \zeta^T Z R_3^{-1} Z^T \zeta \\ + 2\zeta^T Z (e_3 - e_4)] \xi, \quad -\int_{t-d}^{t-d_1(t)-d_2} \dot{x}^T(s) R_3 \dot{x}(s) ds \\ \leq \xi^T [(d_1 - d_1(t)) \zeta^T K R_3^{-1} K^T \zeta + 2\zeta^T K (e_4 - e_5)] \xi$$

$$LV_5(x(t), t, i) = \dot{x}^T(t) (d_2^2 S_4 + d_1^2 S_5) \dot{x}(t) \\ - d_2 \int_{t-d}^{t-d_1} \dot{x}^T(s) S_4 \dot{x}(s) ds - d_1 \int_{t-d_1}^t \dot{x}^T(s) S_5 \dot{x}(s) ds, \quad (20)$$

From Lemma 1, it follows that

$$LV_5(x(t), t, i) \leq \dot{x}^T(t) (d_2^2 R_4 + d_1^2 R_5) \dot{x}(t) - \xi^T \{H_{11}^T R_4 H_{11} - 3H_{12}^T R_4 \\ H_{12} - 5H_{13}^T R_4 H_{13} - H_{21}^T R_5 H_{21} - 3H_{22}^T R_5 H_{22} - 5H_{23}^T R_5 H_{23}\} \xi. \quad (21)$$

So, combining the inequality (16) to (21), it obtain that

$$LV(x_i, t, i) = \sum_{j=1}^5 LV_j(x_i, t, i) \leq \xi^T \Psi \xi,$$

where

$$\Psi = \Psi_{12 \times 12} + \hat{N} \hat{R}_1^{-1} \hat{N}^T + \hat{M} \hat{R}_2^{-1} \hat{M}^T + d_1(t) X R_3^{-1} X^T + d_2(t) Y R_3^{-1} Y^T \\ + (d_2 - d_2(t)) Z R_3^{-1} Z^T + (d_1 - d_1(t)) K R_3^{-1} K^T + \Gamma^T \hat{R} \Gamma, \\ \xi = (x(t), x(t - d_1(t)), x(t - d(t)), x(t - d_1(t) - d_2(t)), \\ x(t - d), d_2^{-1} \int_{t-d}^{t-d_1} x(s) ds, d_2^{-2} \int_{t-d}^{t-d_1} \int_{\theta}^{t-d_1} x(s) ds d\theta, \\ d_2^{-3} \int_{t-d}^{t-d_1} \int_u^{t-d_1} \int_{\theta}^{t-d_1} x(s) ds d\theta du, d_1^{-1} \int_{t-d_1}^t x(s) ds, \\ d_1^{-2} \int_{t-d_1}^t \int_{\theta}^t x(s) ds d\theta, d_1^{-3} \int_{t-d_1}^t \int_u^t \int_{\theta}^t x(s) ds d\theta du)^T.$$

By utilizing the Schur complement and the convex combination technique given in [42], it is known that $\Psi < 0$ is equivalent to the inequality (9) holds which imply that $LV(x_i, t, i) < 0$. Therefore, using the Dynkin's formulatur can we gain

$$\mathcal{E} \left\{ \int_0^\infty \|x(t)\|^2 dt \mid \varphi, r_0 \right\} \leq \infty,$$

which means that system (1) is stochastic stable following Definition 1. The proof is completed.

Remark 1. The Lyapunov functionals we constructed in this paper consider more information about $d_1(t), d_2(t), d(t)$, such as $x(t - d_1(t)), x(t - d_1(t) - d_2(t)), x(t - d(t))$ and $x(t - d_1), x(t - d)$. In fact, the maximum of the two delays $d_1(t)$ and $d_2(t)$ may not be reached at same time, and their property may have sharply different properties, so we consider the process of the variation of delay generated in system is from $d_1(t)$ to $d(t)$, then to $d_1(t) + d_2$, and then to d , therefore the first four items in $V_2(x(t), t, i)$ are constructed, and the relation between the upper bound d_1 of $d_1(t)$ and the upper bound d of $d(t)$ is also taken into account in the last two items of $V_2(x(t), t, i)$. Meanwhile, the integral items $\int_{t-d}^t \dot{x}^T(s) R_3 \dot{x}(s) ds$ in $LV_4(x(t), t, i)$ is decomposed into the form like inequality (19), it is important to

reduce the conservation of the resulting stochastic stable conditions.

Remark 2. It is noted that the upper bound d of $d(t)$ is divided into two interval $[0, d_1]$ and $[d_1, d]$, so we construct the Lyapunov functional form like $V_3(x(t), t, i)$ and $V_5(x(t), t, i)$, and then the less conservative stochastic stable criteria is derived by utilizing the novel double integral inequality (7) and a single integral inequality (6) with less conservation. Furthermore, to increase the cross product item between the augmented vector, the last item in $V_1(x(t), t, i)$ is constructed, which is very useful to improve the upper bound of delay and reduce the conservation of the resulting stochastic stable criterions.

The approach of Theorem 1 can be used as a useful tool for the stabilization problem of system (1) with $u(t) = J_i x(t)$. Now we are in position to design the controller by convex optimization method.

Theorem 2. The system (1) with $u(t) = J_i x(t)$ is stochastically stable if there exist symmetric positive definite matrices $\bar{P}_i, \bar{Q}_i \in R^{n \times n}, (i = 1, 2, 3, 4, 5, 6)$, $\bar{R}_i \in R^{n \times n}, (i = 1, 2, 3, 4, 5)$, $\bar{W} \in R^{6n \times 6n}$, and any matrices $\bar{N}_i, \bar{M}_i \in R^{4n \times n}, (i = 1, 2, 3)$, $\bar{X}, \bar{Y}, \bar{Z}, \bar{K} \in R^{12n \times n}$, and a scalar $\lambda > 0$, such that

$$\bar{Q}_1 - \bar{Q}_2 > 0, \bar{Q}_2 - \bar{Q}_3 > 0, \bar{Q}_3 - \bar{Q}_4 > 0, \quad (22)$$

$$\begin{pmatrix} \bar{\Psi}_{20 \times 20} & \tilde{\Gamma}^T \\ * & -2\lambda^{-1}\bar{P} + \lambda^{-2}\bar{R} \end{pmatrix} < 0, \quad (23)$$

where

$$\bar{\Psi}_{20 \times 20} = \begin{pmatrix} \bar{\Psi}_{12 \times 12} & \bar{N} & \bar{M} & \bar{\Xi}_i \\ * & -\bar{R}_1 & 0 & 0 \\ * & * & -\bar{R}_2 & 0 \\ * & * & * & -\bar{R}_{3l} \end{pmatrix},$$

$$\begin{aligned} \bar{\Psi}_{12 \times 12} = & e_1^T \bar{\Pi} e_1 + e_1^T (\bar{Q}_1 - \bar{Q}_2) e_1 - (1 - \mu_1) e_2^T (\bar{Q}_1 - \bar{Q}_2) e_2 \\ & - (1 - \mu) e_3^T (\bar{Q}_2 - \bar{Q}_3) e_3 - (1 - \mu_1) e_4^T (\bar{Q}_3 - \bar{Q}_4) e_4 \\ & - e_5^T (\bar{Q}_4 + \bar{Q}_6) e_5 - e_9^T (\bar{Q}_5 - \bar{Q}_6) e_9 - H_{11}^T \bar{R}_4 H_{11} \\ & - 3H_{12}^T \bar{R}_4 H_{12} - 5H_{13}^T \bar{R}_4 H_{13} - H_{21}^T \bar{R}_5 H_{21} - 3H_{22}^T \bar{R}_5 H_{22} \\ & - 5H_{23}^T \bar{R}_5 H_{23} + \text{sym}\{\zeta^T \bar{X} (e_1 - e_2) + \zeta^T \bar{Y} (e_2 - e_3) \\ & + \zeta^T \bar{Z} (e_3 - e_4) + \zeta^T \bar{K} (e_4 - e_5)\} + \text{sym}\{e_{si}^T \bar{P} e_i + \dot{\eta}^T \bar{W} \eta\} \\ & + d_2 \text{sym}\{\zeta_{10}^T \bar{N}_1 \zeta_{11} + \zeta_{10}^T \bar{N}_2 \zeta_{12} + \zeta_{10}^T \bar{N}_3 \zeta_{13}\} \\ & + d_1 \text{sym}\{\zeta_{20}^T \bar{M}_1 \zeta_{21} + \zeta_{20}^T \bar{M}_2 \zeta_{22} + \zeta_{20}^T \bar{M}_3 \zeta_{23}\}, \end{aligned}$$

where $(l = 1, 2, 3, 4)$, $\zeta, \zeta_{10}, \zeta_{11}, \zeta_{12}, \zeta_{13}, \zeta_{20}, \zeta_{21}, \zeta_{22}, \zeta_{23}$ have been defined in Theorem 1. And

$$\bar{\Pi} = \sum_{j=1}^s \lambda_{ij} \bar{P}_j, \bar{N} = (\bar{N}_1^T, \bar{N}_2^T, \bar{N}_3^T), \bar{M} = (\bar{M}_1^T, \bar{M}_2^T, \bar{M}_3^T),$$

$$\bar{R}_1 = \text{diag}\{2d_2^{-2}\bar{R}_1, 4d_2^{-2}\bar{R}_1, 6d_2^{-2}\bar{R}_1\}, \bar{\Xi}_1 = (\bar{Z}, \bar{K}), \bar{\Xi}_2 = (\bar{X}, \bar{Z}),$$

$$\bar{R}_2 = \text{diag}\{2d_2^{-2}\bar{R}_2, 4d_2^{-2}\bar{R}_2, 6d_2^{-2}\bar{R}_2\}, \bar{\Xi}_3 = (\bar{Y}, \bar{K}), \bar{\Xi}_4 = (\bar{X}, \bar{Y}),$$

$$\bar{R}_{31} = \text{diag}\{d_2^{-1}\bar{R}_3, d_1^{-1}\bar{R}_3\}, \hat{R}_{32} = \text{diag}\{d_1^{-1}\bar{R}_3, d_2^{-1}\bar{R}_3\},$$

$$\bar{R}_{33} = \text{diag}\{d_2^{-1}\bar{R}_3, d_1^{-1}\bar{R}_3\}, \hat{R}_{34} = \text{diag}\{d_1^{-1}\bar{R}_3, d_2^{-1}\bar{R}_3\},$$

$$\bar{R} = (2^{-1}d_2^2\bar{R}_1 + 2^{-1}d_1^2\bar{R}_2 + d\bar{R}_3 + d_2^2\bar{R}_4 + d_1^2\bar{R}_5),$$

$$H_{11} = (e_9 - e_5), H_{12} = (e_9 + e_5 - 2e_6), H_{13} = (e_9 - e_5 + 6e_6 - 12e_7),$$

$$H_{21} = (e_1 - e_9), H_{22} = (e_1 + e_9 - 2e_{10}), H_{23} = (e_1 - e_9 + 6e_{10} - 12e_{11}),$$

$$\zeta = (e_1, e_2, \dots, e_{11}, e_{12}), \zeta_{10} = (e_9, e_6, e_7, e_8), \zeta_{20} = (e_1, e_{10}, e_{11}, e_{12}),$$

$$\zeta_{11} = (e_9 - e_6), \zeta_{12} = (e_9 + 2e_6 - 6e_7), \zeta_{13} = (e_9 - 3e_6 + 24e_7 - 60e_8),$$

$$\zeta_{21} = (e_1 - e_{10}), \zeta_{22} = (e_1 + 2e_{10} - 6e_{11}), \zeta_{23} = (e_1 - 3e_{10} + 24e_{11} - 60e_{12}),$$

$$\eta = (e_1, d_2 e_6, d_2^2 e_7, d_2^3 e_8, d_1 e_{10}, d_1^2 e_{11}, d_1^3 e_{12}), \dot{\eta} = (e_9 - e_5, d_2(e_9 - e_6),$$

$$2^{-1}d_2^2(e_9 - 2e_7), e_1 - e_9, d_1(e_1 - e_{10}), 2^{-1}d_1^2(e_1 - 2e_{11})).$$

$$\tilde{\Gamma} = (A_i \bar{P}_i + C_i L, 0, B_i \bar{P}_i, 0, \dots, 0)_{1 \times 20}^T,$$

Furthermore, a desired controller gain matrix is given by

$$J_i = L(\bar{P}_i)^{-1}. \quad (24)$$

Proof. When the controller $u(t) = J_i x(t)$, then the system (1) is formulated as follows:

$$\dot{x}(t) = (A_i + C_i J_i)x(t) + B_i x(t - d_1(t) - d_2(t)).$$

Replaced A_i with $(A_i + C_i J_i)$ and utilizing the Schur complement in Theorem 1, the inequality (11) can be expressed as:

$$\Psi_{20 \times 20} + \tilde{\Gamma}^T \hat{R} \tilde{\Gamma} < 0, \quad (25)$$

where $\hat{R}$, $\Psi_{20 \times 20}$ has been defined in Theorem 1.

Let block diagonal matrices $L_1 = \text{diag}\{P_i^{-1}, P_i^{-1}, \dots, P_i^{-1}\}$, $L_2 = \text{diag}\{P_i^{-1}, P_i^{-1}, \dots, P_i^{-1}\}$ with 20 and 6 dimensions, respectively. And $\bar{P}_i = P_i^{-1}$, $\bar{\Delta} = \bar{P}_i \Delta \bar{P}_i$. With these notations and (24) in mind, performing a congruence transformation to (8), (9) and $W \in R^{6n \times 6n}$ by $\bar{P}_i, L_1$, and L_2 respectively. Then, one can get $\bar{W} > 0$, inequality (25), and the following inequality by using Schur complements,

$$\begin{pmatrix} \bar{\Psi}_{20 \times 20} & \tilde{\Gamma} \\ * & -\hat{R}^{-1} \end{pmatrix} < 0, \quad (26)$$

where $\bar{\Psi}_{20 \times 20}$, $\tilde{\Gamma}$ and $\hat{R}$ are defined in Theorem 2. On the hand, as we all know, for any $\lambda > 0$, one can note that

$$(\lambda \bar{P}_i - \bar{R}) \bar{R}^{-1} (\lambda \bar{P}_i - \bar{R}) \geq 0,$$

it is easy to see that

$$-\bar{P}_i \bar{R}^{-1} \bar{P}_i \leq -2\lambda^{-1} \bar{P}_i + \lambda^{-2} \bar{R} \quad (27)$$

Therefore, (22)-(23) hold if (25)-(27) hold. The proof is completed.

Remark 3. It is very important to introduce a positive scalar λ to the effectiveness of Theorem 2. Otherwise, it is a high load for $-P_i$ to stabilize the matrices $\bar{R}$ in (25) following the technique in [43].

IV. NUMERICAL EXAMPLES

In order to show the reduced conservatism and the effectiveness of approaches presented in this paper, in this section, one numerical example is provided.

Example 1. To show the effectiveness of Theorem 2, consider the dynamical system (1) with $N = 2$ and satisfies

$$A_1 = \begin{pmatrix} -2 & 0 \\ 0 & -0.9 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -2 & 0 \\ 0 & -9 \end{pmatrix}, \quad C_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}, \quad B_2 = \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}, \quad C_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

If we set $\mu_1 = 0.1, \mu_2 = 0.8, d_1 = 1, d_2 = 0.988$ and $\lambda = 1/19$. Meanwhile, the relevant transition rate matrix Λ is considered as $\Lambda = \begin{pmatrix} 0.95 & 0.05 \\ 0.005 & 0.995 \end{pmatrix}$. According to the Theorem 2, some matrices are obtained by computing in the MATLAB as follows

$$J_1 = \begin{pmatrix} -1.9804 & 0.0255 \\ -3.9108 & -3.5470 \end{pmatrix}, \quad J_2 = \begin{pmatrix} -1.9035 & 0.1437 \\ -1.0928 & -1.8495 \end{pmatrix}.$$

It should be pointed out that the system (1) will not be stochastic stable with the controller designed by utilizing the approach in [43], ie., when $\lambda = 1$ in (27). Therefore, it is clearly that the approach in [14] for designing controller is more flexible, and this example shows the effectiveness of our methods presented in this paper.

V. CONCLUSION

In this paper, we study the problem of stabilization for MJSs with additive time-varying delays. Firstly, the stabilization condition of the systems is given by constructing a new Lyapunov functional with triple integral terms, using new inequalities, convex combination technique and combining with other analytical techniques. Then, the the controller of the closed-loop system is designed by the transformation technique of inequalities. Moreover, our future research topic is to obtain the less stabilization conditions of MJSs with the additive time-varying delays with fewer decision variables.

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