Robust L_1 Model Reduction for Linear Parameter-Varying Systems with Parameter-Varying Delays

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Abstract—In this paper, we investigate the problem of robust L_1 model reduction for linear parameter-varying (LPV) systems with parameter-varying delays. It is essential that the design of reduced-order system guarantees LPV error system to be asymptotically stable and satisfy peak-to-peak performance constraint with respect to all bounded peak value input signals. By using parameter-dependent Lyapunov function (PDLF), the sufficient conditions for the existence of the peak-to-peak criterion are established for error system with time delays for the first time, which realize the decoupling between the system matrices and PDLF matrices by introducing a slack matrix and applying Projection lemma. Under the conditions, the reduced-order system can be obtained in terms of linear matrix inequality (LMI) technology. Based on the approximate basis function and the gridding technique, the design problem of reduced-order system is cast into convex optimization problem subject to parameter LMI constraints. Finally, the validity of the proposed design method is illustrated by a numerical example.

Keywords- robust L_1 model reduction; linear parameter-varying (LPV) systems; parameter-dependent Lyapunov function (PDLF)

I. INTRODUCTION

Compared with general time-varying systems, linear parameter-varying (LPV) systems have been assumed that the trajectory of the parameters is not known a priori although the bounds on parameter values and the rate of variation of these parameters are available in advance. In addition, time delays are often encountered in many dynamic systems, whose existence has been generally regarded as an important source of instability and poor performance. Therefore, time delays have to be taken into account in the design of control laws. Now, the analysis and synthesis problems for the delay-dependent systems have been solved [1, 2]. In recent years, progress has been made in the studies of LPV systems with delay-dependent case [3, 4]. In many fields of engineering, high-order mathematical models are often used to describe physical systems. Thus model reduction method was proposed in order to find a reduced-order system to approximate a high-order system, while it remains main feature of the original system. It is an effective way and plays a key role for analysis and synthesis of control system if high order models can be replaced by reduced-order systems without incurring too much error. In recent decades, model reduction has been a popular research area and has attracted considerable attention. Various model reduction methods [5-8] have been proposed. It has been introduced to solve the model reduction problem for

different classes of systems including linear systems [9], stochastic systems [10] and LPV systems [11].

In this paper, we introduce L_1 performance constraint under the LMI into reduced-order system synthesis. Although the model reduction problem have been studied for time-delayed LPV systems under the H_∞ criterion in, it seems that little effort has been made towards solving the delay-dependent L_1 model reduction for LPV systems. Thus the study on L_1 optimization problems for LPV systems is truly a significant work.

The goal of the paper is to investigate the problem of robust L_1 model reduction for time-delayed LPV systems. We establish L_1 performance criterion for time-delayed LPV systems for the first time which ensures the error system is asymptotically stable and minimizes the worst case peak-to-peak gain for all time-varying parameters and delays. On the basis of the above results, the sufficient conditions for the existences of reduced-order LPV system are proposed by using parameter LMI technology. The effectiveness of the suggested method is illustrated by a numerical example.

II. THE PROBLEM DESCRIPTION

Consider the following LPV system with time-varying delays

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) + A_d(\rho(t))x(t - h(\rho(t))) + B(\rho(t))\omega(t) \\ y(t) &= C(\rho(t))x(t) + C_d(\rho(t))x(t - h(\rho(t))) + D(\rho(t))\omega(t) \\ x(\theta) &= \Phi(\theta), \quad \forall \theta \in [-\tilde{h}, 0] \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the state vector; $y(t) \in R^p$ is the measured output; $\omega(t) \in R^q$ is the disturbance input which belongs to $\omega(t) \in L_\infty[0, \infty]$; The state-space matrices $A(\cdot)$, $A_d(\cdot)$, $B(\cdot)$, $C(\cdot)$, $C_d(\cdot)$, $D(\cdot)$ and the delay $h(\cdot)$ are assumed to be bounded continuous functions of a time-varying parameter vector $\rho(t)$, satisfying

$$0 < h(\cdot) \leq \tilde{h} < \infty, \quad 0 \leq \dot{h}(\cdot) \leq h_d < 1, \quad \forall t \geq 0 \quad (2)$$

the time-varying parameter vector $\rho(t) = [\rho_1(t), \rho_2(t), \dots, \rho_s(t)]^T$ satisfies that $\rho_i(t) \in [\underline{\rho}_i, \bar{\rho}_i]$ is measurable in real time and parameter variation rates $\dot{\rho}(t) \in [\underline{\tau}_i, \bar{\tau}_i]$. For convenience, ρ and ρ_i denote

respectively the vector of time-varying parameter $\rho(t)$ and $\rho_i(t)$ in the latter description.

Here, we are interested in approximating system (1) by a reduced-order system described by

$$\begin{aligned}\dot{x}(t) &= \mathbf{A}_r(\rho)x_r(t) + \mathbf{A}_{rd}(\rho)x_r(t-h(\rho)) + \mathbf{B}_r(\rho)\omega(t) \\ y_r(t) &= \mathbf{C}_r(\rho)x_r(t) + \mathbf{C}_{rd}(\rho)x_r(t-h(\rho)) + \mathbf{D}_r(\rho)\omega(t) \\ x_r(\theta) &= \psi(\theta), \quad \forall \theta \in [-\tilde{h}, 0]\end{aligned} \quad (3)$$

where $x_r(t) \in \mathbb{R}^k$ ($k < n$) is the state vector; $\mathbf{A}_r(\cdot), \mathbf{A}_{rd}(\cdot), \mathbf{B}_r(\cdot), \mathbf{C}_r(\cdot), \mathbf{C}_{rd}(\cdot), \mathbf{D}_r(\cdot)$ are the unknown parameter matrices of the reduced-order system.

Considering the systems (1) and (3), we get the following error system

$$\begin{aligned}\dot{\xi}(t) &= \bar{\mathbf{A}}(\rho)\xi(t) + \bar{\mathbf{A}}_d(\rho)\xi(t-h(\rho)) + \bar{\mathbf{B}}(\rho)\omega(t) \\ e(t) &= \bar{\mathbf{C}}(\rho)\xi(t) + \bar{\mathbf{C}}_d(\rho)\xi(t-h(\rho)) + \bar{\mathbf{D}}(\rho)\omega(t) \\ \xi(\theta) &= \{\Phi^T(\theta), \psi^T(\theta)\}^T, \quad \forall \theta \in [-\tilde{h}, 0]\end{aligned} \quad (4)$$

where $\xi(t) = [x^T(t) \ x_r^T(t)]^T$ is the state vectors of error system, $e(t) = y(t) - y_r(t)$ is error signal and

$$\bar{\mathbf{A}}(\rho) = \begin{bmatrix} \mathbf{A}(\rho) & 0 \\ 0 & \mathbf{A}_r(\rho) \end{bmatrix}, \bar{\mathbf{A}}_d(\rho) = \begin{bmatrix} \mathbf{A}_d(\rho) & 0 \\ 0 & \mathbf{A}_{rd}(\rho) \end{bmatrix}, \bar{\mathbf{B}}(\rho) = \begin{bmatrix} \mathbf{B}(\rho) \\ \mathbf{B}_r(\rho) \end{bmatrix} \quad (5)$$

$$\bar{\mathbf{C}}(\rho) = [\mathbf{C}(\rho) \ -\mathbf{C}_r(\rho)], \bar{\mathbf{C}}_d(\rho) = [\mathbf{C}_d(\rho) \ -\mathbf{C}_{rd}(\rho)], \bar{\mathbf{D}}(\rho) = \mathbf{D}(\rho) - \mathbf{D}_r(\rho)$$

Before proceeding further, we recall the following idea and a lemma, which will be used during the proof of our main results.

Now, our objective can be stated as follow: find a reduced-order system of the form given by (3) that guarantees the following two conditions:

(a) The error system (4) is asymptotically stable; (b) The error system (4) guarantees a prescribed L_1 noise attenuation level γ (that is, $\|T_{ew}\|_{L_1} < \gamma$) for all nonzero $\omega(t) \in L_\infty[0, \infty)$ under zero initial condition, where T_{ew} is the operator mapping input $\omega(t)$ to output $e(t)$, and define

$$\|T_{ew}\|_{L_1} = \sup_{\omega \in L_\infty} \frac{\|e(t)\|_{L_\infty}}{\|\omega(t)\|_{L_\infty}} = \sup_{\omega \in L_\infty} \left(\frac{\sup_t \|e(t)\|_2}{\sup_t \|\omega(t)\|_2} \right)$$

thus $\|T_{ew}\|_{L_1}$ corresponds to the maximum peak-to-peak gain of T_{ew} . The reduced-order system satisfying above two conditions is called the robust L_1 reduced-order system.

Lemma 1 Given matrices $\mathbf{G}, \mathbf{U}, \mathbf{V}$, there exists a matrix $\mathbf{X}$ such that $\mathbf{G} + \mathbf{UXV}^T + \mathbf{VX}^T\mathbf{U}^T < 0$, if and only if

$$\mathbf{N}_U^T \mathbf{G} \mathbf{N}_U < 0, \mathbf{N}_V^T \mathbf{G} \mathbf{N}_V < 0$$

hold, where $\mathbf{N}_U$ and $\mathbf{N}_V$ are orthogonal complement of $\mathbf{U}$ and $\mathbf{V}$ respectively.

III. DELAY-DEPENDENT L_1 PERFORMANCE CRITERION

In this subsection, we will provide the sufficient condition for the existence of a delay-dependent L_1 performance criterion which guarantees the error system to be asymptotically stable and have a worst case peak-to-peak performance constraint.

Theorem 1 Consider the error system (4), for arbitrary

measurable $\rho(t)$, given positive scalars $\alpha, \tilde{h}$ and $\beta \in (0, 1)$, this system is asymptotically stable and has a guaranteed L_1 noise attenuation level γ if there exist a continuously differentiable matrix $0 < \mathbf{P}(\rho) \in \mathbb{R}^{(n+k) \times (n+k)}$ and matrices $0 < \mathbf{Q}_1 \in \mathbb{R}^{(n+k) \times (n+k)}, 0 < \mathbf{Q}_2 \in \mathbb{R}^{(n+k) \times (n+k)}$ satisfying

$$\begin{bmatrix} \Phi_{11} & \mathbf{P}(\rho)\bar{\mathbf{A}}_d(\rho) & \mathbf{P}(\rho)\bar{\mathbf{B}}(\rho) \\ * & -(1 - \sum_{i=1}^s (\tau_i \frac{\partial h}{\partial \rho_i}))\mathbf{Q}_1 & 0 \\ * & * & -\alpha\mathbf{I} \end{bmatrix} < 0 \quad (6)$$

$$\alpha\mathbf{Q}_1 - (1 - \alpha\tilde{h})\mathbf{Q}_2 < 0 \quad (7)$$

$$\begin{bmatrix} -\alpha\beta\mathbf{P}(\rho) & 0 & 0 & \bar{\mathbf{C}}^T(\rho) \\ * & -\alpha(1 - \beta)\mathbf{P}(\rho) & 0 & \bar{\mathbf{C}}_d^T(\rho) \\ * & * & -(\gamma - \alpha)\mathbf{I} & \bar{\mathbf{D}}^T(\rho) \\ * & * & * & -\gamma\mathbf{I} \end{bmatrix} < 0 \quad (8)$$

Where

$$\Phi_{11} = \bar{\mathbf{A}}^T(\rho)\mathbf{P}(\rho) + \mathbf{P}(\rho)\bar{\mathbf{A}}(\rho) + \alpha\mathbf{P}(\rho) + \mathbf{Q}_1 + \tilde{h}\mathbf{Q}_2 + \sum_{i=1}^s (\tau_i \frac{\partial \mathbf{P}}{\partial \rho_i})$$

Proof: we choose the following Lyapunov function for the error system (4)

$$V(t) = V_1(\xi, \rho) + V_2(\xi, \rho) \quad V_2(\xi, \rho) = \int_{t-\tilde{h}}^t \int_s^t \xi^T(\theta) \mathbf{Q}_2 \xi(\theta) d\theta ds$$

$$V_1(\xi, \rho) = \xi^T(t) \mathbf{P}(\rho) \xi(t) + \int_{t-h(\rho)}^t \xi^T(s) \mathbf{Q}_1 \xi(s) ds$$

where $\mathbf{P}(\rho)$ is matrix that depends on parameter $\rho(t)$ and is real symmetric positive definite; $\mathbf{Q}_1 > 0, \mathbf{Q}_2 > 0$. By taking the time derivative of $V(t)$, we obtain that Define $\varphi(t) = [\xi^T(t) \ \xi^T(t-h(\rho)) \ \omega^T(t)]^T$, we may get following result through some ordinary matrix computations By making use of (7) and the following inequality

$$\int_{t-\tilde{h}}^t \int_s^t \xi^T(\theta) (\mathbf{Q}_2 - \alpha\mathbf{Q}_1) \xi(\theta) d\theta ds \leq \tilde{h} \int_{t-\tilde{h}}^t \xi^T(s) (\mathbf{Q}_2 - \alpha\mathbf{Q}_1) \xi(s) ds \quad (9)$$

We obtain that

$$\dot{V}(t) < \varphi^T(t) \Delta \varphi(t) + \alpha\omega^T(t)\omega(t) - \alpha V(t)$$

where

$$\Delta = \begin{bmatrix} \Phi_{11} & \mathbf{P}(\rho)\bar{\mathbf{A}}_d(\rho) & \mathbf{P}(\rho)\bar{\mathbf{B}}(\rho) \\ * & -[1 - \sum_{i=1}^s (\tau_i \frac{\partial h}{\partial \rho_i})]\mathbf{Q}_1 & 0 \\ * & * & -\alpha\mathbf{I} \end{bmatrix} \quad (10)$$

Φ_{11} has been defined in (6). Since the inequality (6) guarantees $\Delta < 0$, we can get the following condition

$$\dot{V}(t) < \alpha \omega^T(t) \omega(t) - \alpha V(t) \quad (11)$$

For all $\varphi(t) \neq 0$ and $\omega(t) = 0$, the inequality (11) means $\dot{V}(t) < 0$. We can conclude that the error system (4) is asymptotically stable.

Define $\mathcal{V} = \{x: V(t) \leq 1\}$, for the state $\xi(t)$ of the error system in (4) with $\|\omega(t)\|_{L_\infty} \leq 1$ and zero initial condition (namely, $\Phi(\theta) = 0$ and $\psi(\theta) = 0$ for $\theta \in [-\tilde{h}, 0]$) then we may get $V(t)|_{t=0} = 0$, we will discuss (12)

$$f(t) = \begin{bmatrix} \xi(t) \\ \xi(t-h(\rho)) \\ \omega(t) \end{bmatrix}^T \Sigma \begin{bmatrix} \xi(t) \\ \xi(t-h(\rho)) \\ \omega(t) \end{bmatrix}$$

where

$$\Sigma = \begin{bmatrix} -\alpha\beta P(\rho) & 0 & 0 \\ * & -\alpha(1-\beta)P(\rho) & 0 \\ * & * & -(\gamma-\alpha)I \end{bmatrix} + \frac{1}{\gamma} \begin{bmatrix} \bar{C}^T(\rho) \\ \bar{C}_d^T(\rho) \\ \bar{D}^T(\rho) \end{bmatrix} \begin{bmatrix} \bar{C}^T(\rho) \\ \bar{C}_d^T(\rho) \\ \bar{D}^T(\rho) \end{bmatrix}^T$$

According to (8) and Schur complement, we know $f(t) < 0$, which can further lead to

$$e^T(t)e(t) < \gamma[\alpha\beta\xi^T(t)P(\rho)\xi(t) + (\gamma-\alpha)\omega^T(t)\omega(t) + \alpha(1-\beta)\xi^T(t-h(\rho))P(\rho)\xi(t-h(\rho))] \quad (12)$$

Since $\beta\xi^T(t)P(\rho)\xi(t) + (1-\beta)\xi^T(t-h(\rho))P(\rho)\xi(t-h(\rho)) < 1$

Here β is weighting factor. For all $\|\omega(t)\|_{L_\infty} \leq 1$, it can be concluded from (12) that $\sup_{\|\omega(t)\|_{L_\infty} \leq 1} \|e(t)\|_{L_\infty} < \gamma$. The proof is concluded.

IV. THE DECOUPLING L_1 PERFORMANCE CRITERION

By introducing a slack matrix and applying Lemma 1, we obtain the following theorem 2, which will play an important role for designing an admissible L_1 reduced-order system.

Theorem 2 Consider the error system (4), given scalars $\alpha, \tilde{h} \in R^+$, $\beta \in (0,1)$ this system is asymptotically stable and has a guaranteed L_1 noise attenuation level γ if there exist a continuously differentiable matrix $0 < P(\rho) \in R^{(n+k) \times (n+k)}$ and matrices $0 < Q_1, Q_2, W \in R^{(n+k) \times (n+k)}$, satisfying (7), (8) and the following (13) for all parameter variation trajectories.

$$\begin{bmatrix} -(W+W^T) & P(\rho)+W^T\bar{A}(\rho) & W^T\bar{A}_d(\rho) & W^T\bar{B}(\rho) & W^T \\ * & -P(\rho)+\Omega_{11} & 0 & 0 & 0 \\ * & * & -[1-\sum_{i=1}^s(\tau_i\frac{\partial h}{\partial \rho_i})]Q_1 & 0 & 0 \\ * & * & * & -\alpha I & 0 \\ * & * & * & * & -P(\rho) \end{bmatrix} < 0 \quad (13)$$

where $\Omega_{11} = \sum_{i=1}^s(\tau_i\frac{\partial P}{\partial \rho_i}) + \alpha P(\rho) + \tilde{h}Q_2 + Q_1$

Proof: according to the Lemma 1, inequality (13) is equivalent to the following inequality

$$\Pi + U^T W V + V^T W^T U < 0 \quad (14)$$

where

$$\Pi = \begin{bmatrix} 0 & P(\rho) & 0 & 0 & 0 \\ * & -P(\rho)+\Omega_{11} & 0 & 0 & 0 \\ * & * & -[1-\sum_{i=1}^s(\tau_i\frac{\partial h}{\partial \rho_i})]Q_1 & 0 & 0 \\ * & * & * & -\alpha I & 0 \\ * & * & * & * & -P(\rho) \end{bmatrix}$$

$$U = [-I \quad \bar{A}(\rho) \quad \bar{A}_d(\rho) \quad \bar{B}(\rho) \quad I], V = [I \quad 0 \quad 0 \quad 0 \quad 0]$$

thus the null-space of U and V are $\mathbb{N}_U$ and $\mathbb{N}_V$, namely

$$\mathbb{N}_U = \begin{bmatrix} \bar{A}(\rho) & \bar{A}_d(\rho) & \bar{B}(\rho) & I \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}, \mathbb{N}_V = \begin{bmatrix} 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}$$

By Lemma 1, the inequalities (7), (8) and (13) can guarantee the error system is asymptotically stable and has a prescribed L_1 noise attenuation level γ .

V. ROBUST L_1 MODEL REDUCTION

In this subsection, based on theorem 2, we present an approach to solve robust L_1 model reduction problem for time-delayed LPV systems. First, we give the following theorem.3.

Theorem 3 Consider time-delayed LPV system (1). For given scalars $\alpha, \tilde{h} \in R^+$ and $\beta \in (0,1)$, there exists an admissible L_1 reduced-order LPV system in the form of (3) such that the error system (4) is asymptotically stable and has a guaranteed L_1 noise attenuation level γ , if there exist a continuously differentiable matrix $0 < \bar{P}(\rho), \bar{Q}_1, \bar{Q}_2 \in R^{(n+k) \times (n+k)}$ and matrices $R_1 \in R^{n \times n}, R_2 \in R^{k \times k}, R_3 \in R^{n \times k}$, $\bar{A}_r(\rho), \bar{A}_{rd}(\rho) \in R^{k \times k}$, $\bar{B}_r(\rho) \in R^{r \times m}$, $\bar{C}_r(\rho) \in R^{p \times k}$, $\bar{C}_{rd}(\rho) \in R^{p \times k}$, $\bar{D}_r(\rho) \in R^{p \times m}$, for all parametric variation trajectories, satisfy the following the inequalities (15), (16) and (17).

VI. NUMERICAL EXAMPLE

Consider a time-delay LPV system in (1) with the following parameter matrices

$$A_d(\rho) = \begin{bmatrix} 1 & 0.5 & 0 & 1 & 0 & -1 + 0.1\rho_1(t) \\ 0 & -1 & 2 & 0 & 0.1 & 0 \\ 0.5 & -1 & 1 & 0.2 & 0 & 0.2 + 0.2\rho_1(t) \\ 1 & 0 & 0 & 1 & 0 & -1 \\ -0.5 & 0 & 0.2 & 0 & -1 & 0 \\ 1 & 0 & -1 & 0 & 0 & -1 \end{bmatrix}, B(\rho) = \begin{bmatrix} 0.5 & 0 & 0.5 \\ 0 & 0.5 & 1 \\ 0 & 1 & 0.5 \\ -0.5 & 0 & 0.5 \\ 0.5 & 0 & 0.5 \\ 1 & 0 & 0.5 \end{bmatrix}$$

$$C_d(\rho) = \begin{bmatrix} 0 & 0.3 & 1 & 1 & -1 & 1 \\ 1 & 0 & 0 & 1 & -1 & 0 \end{bmatrix}, D(\rho) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}, C(\rho) = \begin{bmatrix} 0.5 & 0.1\rho_1(t) \\ 0 & 0 \\ 0.5 & 0.5 \\ 0 & 0.5 \\ 0 & 0 \\ 0.5 & 0 \end{bmatrix}$$

$$A(\rho) = \begin{bmatrix} -11 + 0.1 \times \rho_1(t) & 1 & 1 & 1 & 0 & 0 \\ 1 & -10 & -2 & 1 & 0.3 & 0 \\ -1 & 0 & -7 & 1 + 0.2 \times \rho_1(t) & 0 & 1 \\ 0 & 1 & 0 & -4 & 0 & 0.5 \\ 0 & 0.5 & 0 & 0 & -3 & 0 \\ 1 & 0 & -1 & 0 & 0 & -10 + 0.1 \times \rho_1(t) \end{bmatrix}$$

where $\rho_1(t) = \sin(t)$, $\rho_2(t) = |\cos(t)|$, $\dot{\rho}_1(t), \dot{\rho}_2(t) \in [-1, 1]$ are time-varying parameters. The time-varying delay is $h(\rho) = 0.2\rho_2(t)$ whose maximum value is 0.2. Making use of approximate basis function and gridding technique, we can transform infinite parameter LMI (15) and (17) in theorem 3 into finite parameter LMI. We choose basis function

$$f_1(\rho) = 1, f_2(\rho) = \rho_1(t), f_3(\rho) = \rho_2(t)$$

So we can get $Y(\rho) = Y_1 + \rho_1(t)Y_2 + \rho_2(t)Y_3$

Table 1 shows the suboptimal L_1 performance for different order of the reduced systems when $\alpha = 0.9, \beta = 0.47$

TABLE I. NOISE ATTENUATION LEVEL γ FOR DIFFERENT ORDER

γ noise attenuation level	Order Number
2.3390	1
1.5518	2
1.1056	3

We can obtain the following parameter matrices of the first-order system when $\gamma = 2.3390$. Suppose the disturbance input $\omega(t) = [\sin(0.2t) \ 0.5\cos(0.3t) \ 1]^T$, the output trajectories of the original system (solid line) and the first-order reduced system (dotted line) are compared in Figure 1 and Figure 2. The output errors of the first-order reduced system are shown in Figure 3. From these figures, we can see that the reduced-order system approximate the original system very well.

$$\begin{bmatrix} -(R_1 + R_1^T) & -(E^T R_2 + R_3) & Y_{13} & Y_{14} & R_1^T A_d(\rho) \\ * & -(R_2 + R_2^T) & Y_{23} & Y_{24} & R_2^T A_d(\rho) \\ * & * & Y_{33} & Y_{34} & 0 \\ * & * & * & Y_{44} & 0 \\ * & * & * & * & Y_{55} \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \\ * & * & * & * & * \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} E^T \bar{A}_{rd}(\rho) & Y_{17} & R_1^T & E^T R_2 \\ \bar{A}_{rd}(\rho) & Y_{27} & R_3^T & R_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Y_{56} & 0 & 0 & 0 \\ Y_{66} & 0 & 0 & 0 \\ * & -\alpha I & 0 & 0 \\ * & * & -\bar{P}_1(\rho) & -\bar{P}_2(\rho) \\ * & * & * & -\bar{P}_3(\rho) \end{bmatrix} < 0 \quad (15)$$

$$\alpha \begin{bmatrix} \bar{Q}_{111} & \bar{Q}_{112} \\ * & \bar{Q}_{122} \end{bmatrix} - (1 - \alpha \tilde{h}) \begin{bmatrix} \bar{Q}_{211} & \bar{Q}_{212} \\ * & \bar{Q}_{222} \end{bmatrix} < 0 \quad (16)$$

$$\begin{bmatrix} -\alpha\beta\bar{P}_1(\rho) & -\alpha\beta\bar{P}_2(\rho) & 0 & 0 & 0 & C^T(\rho) \\ * & -\alpha\beta\bar{P}_3(\rho) & 0 & 0 & 0 & -\bar{C}_r^T(\rho) \\ * & * & -\alpha(1-\beta)\bar{P}_1(\rho) & -\alpha(1-\beta)\bar{P}_2(\rho) & 0 & C_d^T(\rho) \\ * & * & * & -\alpha(1-\beta)\bar{P}_3(\rho) & 0 & -\bar{C}_{rd}^T(\rho) \\ * & * & * & * & -(\gamma-\alpha)I & D^T(\rho) - \bar{D}_r^T(\rho) \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \quad (17)$$

where $Y_{13} = R_1^T A(\rho) + \bar{P}_1(\rho)$, $Y_{23} = R_3^T A(\rho) + \bar{P}_2(\rho)$

$$Y_{14} = E^T \bar{A}_r(\rho) + \bar{P}_2(\rho), Y_{24} = \bar{A}_r(\rho) + \bar{P}_3(\rho)$$

$$Y_{17} = R_1^T B(\rho) + E^T \bar{B}_r(\rho), Y_{27} = R_3^T B(\rho) + \bar{B}_r(\rho)$$

$$Y_{55} = -(1 - \sum_{i=1}^s (\tau_i \frac{\partial h}{\partial \rho_i})) \bar{Q}_{111}, Y_{56} = -(1 - \sum_{i=1}^s (\tau_i \frac{\partial h}{\partial \rho_i})) \bar{Q}_{112}$$

$$Y_{66} = -(1 - \sum_{i=1}^s (\tau_i \frac{\partial h}{\partial \rho_i})) \bar{Q}_{122}, E = [I_{k \times k} \ 0_{k \times (n-k)}]$$

$$Y_{34} = -\bar{P}_2(\rho) + \alpha \bar{P}_2(\rho) + \sum_{i=1}^s (\tau_i \frac{\partial \bar{P}_2(\rho)}{\partial \rho_i}) + \bar{Q}_{112} + \tilde{h} \bar{Q}_{212}$$

$$Y_{44} = -\bar{P}_3(\rho) + \alpha \bar{P}_3(\rho) + \sum_{i=1}^s (\tau_i \frac{\partial \bar{P}_3(\rho)}{\partial \rho_i}) + \bar{Q}_{122} + \tilde{h} \bar{Q}_{222}$$

$$Y_{33} = -\bar{P}_1(\rho) + \alpha \bar{P}_1(\rho) + \sum_{i=1}^s (\tau_i \frac{\partial \bar{P}_1(\rho)}{\partial \rho_i}) + \bar{Q}_{111} + \tilde{h} \bar{Q}_{211}$$

Moreover, under the above conditions, an admissible L_1 reduced-order system is given by

$$\begin{bmatrix} A_r(\rho) & A_{rd}(\rho) & B_r(\rho) \\ C_r(\rho) & C_{rd}(\rho) & D_r(\rho) \end{bmatrix} = \begin{bmatrix} R_2^{-1} & 0 \\ 0 & I \end{bmatrix} \times \begin{bmatrix} \bar{A}_r(\rho) & \bar{A}_{rd}(\rho) & \bar{B}_r(\rho) \\ \bar{C}_r(\rho) & \bar{C}_{rd}(\rho) & \bar{D}_r(\rho) \end{bmatrix} \quad (18)$$

Proof: the method of proof is omitted.

FIGURE I FIRST OUTPUT TRAJECTORY

FIGURE II SECOND OUTPUT TRAJECTORY

FIGURE III. OUTPUT ERRORS of FIRST-ORDER SYSTEM

VII. CONCLUSIONS

In this paper, the problem of robust L_1 model reduction for LPV systems with parameter-varying delays has been studied. At first, delayed-dependent L_1 performance criterion is established where there exists the coupling between system matrices and Lyapunov function matrices. By introducing a slack matrix, we can apply Projection Lemma to the decoupling between them and construct reduced-order system for given time-delayed LPV system that guarantees error system to be asymptotically stable and satisfy L_1 noise attenuation level γ . Finally, a numerical example is provided to demonstrate the effectiveness of the proposed design method.

REFERENCES

- [1] F. Yasumasa, D. Fabrizio, T. Roberto. Probabilistic design of LPV control systems. *Automatica*, 2003, 39(8):1323–1337..
- [2] He Y., Liu G. P., Rees D., *et al.* H_∞ filtering for discrete-time systems with time-varying delay. *IEEE Trans. Signal Processing*, 2009, 89(3): 275-282.
- [3] Velni J. M., Grigoriadis K. M. Delay-dependent H_∞ filtering for time-delayed LPV systems. *Systems & Control Lett.* 2008, 57(4): 290-299.
- [4] Wu L. G., Shi P., Wang C. H., *et al.* Delay-dependent robust H_∞ and L_2 - L_∞ filtering for LPV systems with both discrete and distributed delays. *IEE Proc. Control Theory Appl.*, 2006, 153(4): 483-492.
- [5] Lastman G. J. Reduced-order aggregated models for bilinear time-invariant dynamical systems. *IEEE Trans. Automat. Control*, 1984, 29: 359-361.
- [6] Zhang L., Lam J., Huang B., *et al.* On gramians and balanced truncation of discrete-time bilinear system. *Int. J. Control*, 2003, 76: 414-427.
- [7] Tatjana Styket. Balanced truncation model reduction for semidiscretized stokes equation. *Linear Algebra and its Applications*, 2006, 415(2-3): 262-189.
- [8] Wang J., Xu W., Chen W. W. Optimal Hankel-norm reduction of active suspension model with application in suspension multiobjective control. *International Journal of Vehicle Design*, 2006 40(1-3): 175-195.
- [9] Ebihara Y., Hiral Y., Hagiwara T. On H_∞ model reduction for discrete-time linear time-invariant systems using linear matrix inequalities. *Asian Journal of Control*, 2008, 10(3): 291-300.
- [10] Li Y. H., Lam J., Luo X. L. Hankel norm model reduction of uncertain neutral stochastic time-delay systems. *International Journal of Innovative and Computing Information and Control*, 2009, 5(9): 2819-2828.
- [11] Li Y. H., Qu Y. C., Gao H. J., *et al.* Robust L_1 model reduction for uncertain stochastic systems with state delay. *American Control Conference, Portland, Oregon*, 2005: 2602-2607.