

H_∞ Control for Markovian Jump Delay Systems with Distributed Delay

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Abstract—This paper aims to design the H_∞ controller for continuous Markovian jump systems with distributed delay. First, a new integral inequality is proposed. Next, based on the constructed Lyapunov functional, the new introduced inequality is used to investigate the delay-dependent stability condition for the systems. Third, a delay-dependent condition is derived for the solvability of the H_∞ control problem, and the desired controller can be obtained. Finally, a numerical example is provided to demonstrate the effectiveness and the less conservatism of the method.

Keywords—new integral inequality; H_∞ control; Markovian jump delay systems with distributed delay

I. INTRODUCTION

Markovian jump systems (MJSs) belong to the category of stochastic hybrid systems with state and jump mode modeled by differential equations. Applications may be found in many processes, such as fault-tolerant systems, biology systems, distributed network systems, robotic manipulator systems and wireless communication systems [1-3]. On the hypothesis of known transition probabilities, fruitful results on stability, stabilization, sliding mode control, H₂ and H_∞ control are reported in [1-3] and the references therein.

Among those results, the stability analysis of MJSs with delay-dependent has attracted much interest [4, 5]. To obtain delay-dependent criterion via the Lyapunov-Krasovskii functional method, a challenging problem is how to cope with the integral term. For this problem, recently, the descriptor model transformation technique [6], together with Park or Moon inequality [7], is applied to handle the integral term. However, extra dynamics and conservativeness may be caused by the model transformation. To render less conservative results, the free-weighting-matrix approach is proposed by [6] and [7]. Unfortunately, introduced slack variables bring heavy computation complexity. Recently, the Jensen's like inequalities technique could give less conservativeness and slack matrices, but the majority of Jensen's like inequalities are related to single integral inequalities and double integral inequalities, a very few inequalities are triple integral inequalities which contain lots of information. On the other hand, H_∞ control for continues-time MJSs with distributed time delay and has not been fully investigated in the literature yet. Using triple integral inequalities to study H_∞ control for continues-time MJSs with distributed time delay, have motivated this paper.

In this paper, we further consider the delay-dependent H_∞ state feedback control of MJSs. Firstly, we will introduce one improved Jensen's like inequalities based on the optimization theory and construction techniques. Then, we investigate the distributed delay systems with markovian jump to illustrate the advantages of applying these inequalities to obtain delay-dependent stability criterion and to design the H_∞ controller. Finally, a numerical example is provided to demonstrate the validity of the established results.

II. PROBLEM STATEMENT AND PRELIMINARIES

In this section, we consider the following continuous-time Markovian jump systems with distributed delay described as

$$\begin{cases} \dot{x}(t) = A(r_t)x(t) + A_1(r_t)x(t-h) \\ + A_2(r_t)\int_{t-h}^t x(s)ds + B(r_t)\omega(t) + C(r_t)u(t) \\ z(t) = D(r_t)x(t) + E(r_t)\omega(t) + F(r_t)u(t) \\ x(t) = \varphi(t), t \in [-h, 0], r(0) = r_0, \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector of the system; $u(t) \in \mathbb{R}^m$ is control input; $z(t) \in \mathbb{R}^p$ is control output; $\omega(t) \in \mathbb{R}^q$ is the noise signal which is assumed to be an arbitrary signal; h is the time-delay; $\varphi(t)$ is vector-valued initial continuous function and belongs to $[-h, 0]$. $A(r_t)$, $A_1(r_t)$, $A_2(r_t)$, $B(r_t)$, $C(r_t)$, $D(r_t)$, $E(r_t)$ and $F(r_t)$ are system matrices. r_t is a continuous Markov process and takes values in $\mathcal{S} = \{1, 2, \dots, s\}$ and satisfies

$$\Pr\{r_{t+\Delta} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), i \neq j \\ 1 + \pi_{ii}\Delta + o(\Delta), i = j \end{cases} \quad (2)$$

where $h > 0$, $\pi_{ij} \geq 0$ for $i \neq j$ and $\pi_{ii} = -\sum_{j=1, j \neq i}^s \pi_{ij}$ for each mode i , $\lim_{\Delta \rightarrow 0} o(\Delta)/\Delta = 0$.

As a sequence, the corresponding transition probability matrix is

$$\begin{bmatrix} \pi_{11} & \pi_{12} & \cdots & \pi_{1s} \\ \pi_{21} & \pi_{22} & \cdots & \pi_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ \pi_{s1} & \pi_{s2} & \cdots & \pi_{ss} \end{bmatrix}.$$

For notation simplicity, when $r_i = i$, the system matrices of the i -th mode can be simplified as A_i , A_{ii} , A_{2i} , B_i , C_i , D_i , E_i and F_i .

Before ending this section, we will introduce a novel integral inequality, which play an important role in proof of the main results.

Lemma 2.1: For given a symmetric positive definite matrix $R > 0$ and any differentiable function $x : [a, b] \rightarrow \mathbb{R}^n$, then the following inequality holds:

$$\begin{aligned} & \frac{1}{b-a} \int_a^b \int_\theta^b \int_u^b \dot{x}^T(s) R \dot{x}(s) ds d\theta d\theta \\ & \geq 6\Omega_9^T R \Omega_9 + 10\Omega_{10}^T R \Omega_{10} + 14\Omega_{11}^T R \Omega_{11} \end{aligned} \quad (3)$$

Where

$$\begin{aligned} \Omega_9 &= \frac{1}{2} x(b) - \frac{1}{(b-a)^2} \int_a^b \int_\theta^b x(s) ds d\theta \\ \Omega_{10} &= \frac{1}{2} x(b) + \frac{3}{(b-a)^2} \int_a^b \int_\theta^b x(s) ds d\theta \\ & \quad - \frac{12}{(b-a)^3} \int_a^b \int_\theta^b \int_u^b x(s) ds d\theta d\theta \\ \Omega_{11} &= \frac{1}{2} x(b) - \frac{10}{(b-a)^2} \int_a^b \int_\theta^b x(s) ds d\theta \\ & \quad + \frac{60}{(b-a)^3} \int_a^b \int_\theta^b \int_u^b x(s) ds d\theta d\theta - \frac{180}{(b-a)^4} \int_a^b \int_\theta^b \int_u^b \int_v^b x(s) ds dv d\theta d\theta \end{aligned}$$

Proof. For $f(s) \in R$ and an integrable function $w(s)$ in

$$[a, b] \rightarrow \mathbb{R}^n, \text{ we define } p_i = \int_a^b \int_\theta^b \int_u^b f_i^2(s) ds d\theta d\theta,$$

$$\Omega_i(w) = \int_a^b \int_\theta^b \int_u^b f_i(s) w(s) ds d\theta d\theta.$$

$$\text{Letting } V = \int_a^b \int_\theta^b \int_u^b w^T(s) R w(s) ds d\theta d\theta \quad \text{and}$$

$$z(s) = \sum_{i=1}^{\infty} \frac{1}{p_i} f_i(s) \Omega_i(w). \text{ If}$$

$$\int_a^b \int_\theta^b \int_u^b f_i(s) f_j(s) ds d\theta d\theta = 0 \quad (4)$$

for $i = 1, 2, 3, \dots$ and $i \neq j$, it is obvious that

$$\int_a^b \int_\theta^b \int_u^b [w(s) - z(s)]^T R [w(s) - z(s)] ds d\theta d\theta$$

$$\begin{aligned} &= \int_a^b \int_\theta^b \int_u^b w^T(s) R w(s) ds d\theta d\theta - 2 \int_a^b \int_\theta^b \int_u^b z^T(s) R w(s) ds d\theta d\theta \\ & \quad + \int_a^b \int_\theta^b \int_u^b z^T(s) R z(s) ds d\theta d\theta = \int_a^b \int_\theta^b \int_u^b w^T(s) R w(s) ds d\theta d\theta \\ & \quad - 2 \sum_{i=1}^{\infty} \frac{1}{p_i} \Omega_i^T(w) R \int_a^b \int_\theta^b \int_u^b f_i(s) w(s) ds d\theta d\theta \\ & \quad + \int_a^b \int_\theta^b \int_u^b \left[\sum_{i=1}^{\infty} \frac{1}{p_i^2} f_i^2(s) \Omega_i^T(w) R \Omega_i(w) \right] ds d\theta d\theta \\ &= V - \sum_{i=1}^{\infty} \frac{1}{p_i} \Omega_i^T(w) R \Omega_i(w) \geq 0 \end{aligned}$$

which gives

$$\int_a^b \int_\theta^b \int_u^b w^T(s) R w(s) ds d\theta d\theta \geq \sum_{i=1}^{\infty} \frac{1}{p_i} \Omega_i^T(w) R \Omega_i(w). \quad (5)$$

Let

$$f_1(s) = 1, \quad f_2(s) = \frac{4s - 3b - a}{4},$$

$$f_3(s) = \frac{15s^2 - 10(a + 2b)s + a^2 + 8ab + 6b^2}{15}. \quad (6)$$

It is easy to see that $f_i(s) (i = 1, 2, 3)$ in (6) satisfy (4), and hence we can easily derive

$$p_1 = \frac{(b-a)^3}{6}, \quad p_2 = \frac{(b-a)^5}{160}, \quad p_3 = \frac{(b-a)^7}{3150}.$$

Then, we have from (5) that

$$V \geq \frac{1}{p_1} \Omega_1^T R \Omega_1 + \frac{1}{p_2} \Omega_2^T R \Omega_2 + \frac{1}{p_3} \Omega_3^T R \Omega_3. \quad (7)$$

Letting $w(s) = \dot{x}(s)$, we obtain

$$\Omega_1(\dot{x}) = (b-a)^2 \left(\frac{1}{2} v_1 - v_2 \right), \quad \Omega_2(\dot{x}) = \frac{(b-a)^3}{4} \left(\frac{1}{2} v_1 + 3v_2 - 12v_3 \right),$$

$$\Omega_3(\dot{x}) = \frac{(b-a)^4}{15} \left(\frac{1}{2} v_1 - 10v_2 + 60v_3 - 180v_4 \right),$$

with

$$v_1 = x(b), \quad v_2 = \frac{1}{(b-a)^2} \int_a^b \int_\theta^b x(s) ds d\theta,$$

$$v_3 = \frac{1}{(b-a)^3} \int_a^b \int_\theta^b \int_u^b x(s) ds d\theta d\theta,$$

$$v_4 = \frac{1}{(b-a)^4} \int_a^b \int_\theta^b \int_u^b \int_v^b x(s) ds dv d\theta d\theta.$$

Substituting the above into (7) yields (3). This completes the proof.

Remark 2.1: Lemma 2.1 is based on inequality (5) with special choices of $w(s) = \dot{x}(s)$ and $i = 1, 2, 3$. It is easy to see that the result of Lemma 2.1 improves that of Theorem 2 in [8]. In addition, different from the inequality of Lemma 2.3 in [9], this inequality is a triple integral inequality, which could be used to reduce the conservativeness of the delay-dependent stability conditions.

III. MAIN RESULTS

Theorem 3.1: Given scalar $h > 0$, the system described by (1) with $u(t) = 0$ is globally asymptotically stochastically stable, where $\gamma > 0$ is a prescribed scalar. If there exist symmetric positive definite matrices P_i , Q_{li} , R_{li} , Q , R , R_2 , R_3 , R_4 , such that the following linear matrix inequalities hold for all $i \in \mathcal{S}$:

$$[\Phi_i = \phi_{(m,n)i}]_{9 \times 9} < 0,$$

$$\sum_{j=1}^s \pi_{ij} Q_{lj} - Q < 0, \sum_{j=1}^s \pi_{ij} R_{lj} - R < 0 \quad (8)$$

where

$$\begin{aligned} \phi_{11} &= P_i A_i + A_i^T P_i + \sum_{j=1}^s \pi_{ij} P_j + Q_{li} + hQ - \frac{9}{h} R_{li} \\ &\quad - 12R_2 - \frac{15h}{2} R_3 + hR_4, \phi_{12} = P_i A_{li} + \frac{3}{h} R_{li}, \\ \phi_{22} &= -Q_{li} - \frac{9}{h} R_{li}, \phi_{13} = hP_i A_{2i} - \frac{24}{h} R_{li} + 12R_2, \phi_{23} = \frac{36}{h} R_{li}, \\ \phi_{33} &= -\frac{192}{h} R_{li} - 72R_2 - 9hR_4, \phi_{14} = P_i B_i, \phi_{44} = -\gamma^2 I, \\ \phi_{25} &= -\frac{60}{h} R_{li}, \phi_{15} = \frac{60}{h} R_{li} - 120R_2 + 58hR_3, \phi_{88} = -N, \\ \phi_{35} &= \frac{360}{h} R_{li} + 480R_2 + 36hR_4, \phi_{16} = 360R_2 - 360hR_3, \\ \phi_{55} &= -\frac{720}{h} R_{li} - 3600R_2 - 1496hR_3 - 192hR_4, \phi_{19} = D_i^T, \\ \phi_{36} &= -1080R_2 - 60hR_4, \phi_{56} = 8640R_2 + 8760hR_3 + 360hR_4, \\ \phi_{66} &= -21600R_2 - 51840hR_3 - 720hR_4, \phi_{17} = 1260hR_3, \\ \phi_{57} &= -25200hR_3, \phi_{67} = 151200hR_3, \phi_{77} = -1453600hR_3, \\ \phi_{18} &= A_i^T N, \phi_{28} = A_{li}^T N, \phi_{38} = hA_{2i}^T N, \phi_{48} = B_i^T N, \\ N &= \frac{h^2}{2} R + hR_{li} + \frac{h^2}{2} R_2 + \frac{h^3}{6} R_3, \phi_{49} = E_i^T, \phi_{99} = -I. \end{aligned}$$

Proof. Choose the following Lyapunov-Krasovskii functional candidate for system (1) as:

$$V(x_i, t, i) = \sum_{m=1}^6 V_m(x_i, t, i), \quad (9)$$

where

$$\begin{aligned} V_1(x_i, t, i) &= x^T(t) P_i x(t), \\ V_2(x_i, t, i) &= \int_{t-h}^t x^T(s) Q_{li} x(s) ds + \int_{-h}^0 \int_{t+\theta}^t x^T(s) Q x(s) ds d\theta, \\ V_3(x_i, t, i) &= \int_{-h}^0 \int_{t+\theta}^t \dot{x}^T(s) R_{li} \dot{x}(s) ds d\theta \\ &\quad + \int_{t-h}^t \int_{\theta}^t \int_u^t \dot{x}^T(s) R \dot{x}(s) ds du d\theta, \\ V_4(x_i, t, i) &= \int_{t-h}^t \int_{\theta}^t \int_u^t \dot{x}^T(s) R_2 \dot{x}(s) ds du d\theta, \\ V_5(x_i, t, i) &= \int_{t-h}^t \int_{\theta}^t \int_u^t \int_v^t \dot{x}^T(s) R_3 \dot{x}(s) ds dv du d\theta, \\ V_6(x_i, t, i) &= \int_{-h}^0 \int_{t+\theta}^t x^T(s) R_4 x(s) ds d\theta. \end{aligned}$$

Resorting the following infinitesimal operator L in [7]

$$LV(x_i, t, r_i) = \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} [\mathcal{E}(V(x_{i+\Delta}, t + \Delta, r_{i+\Delta}) | x_i, r_i) - V(x_i, t, r_i)],$$

then the derivative of $V_m(x_i, t, i)$ is

$$LV_1(x_i, t, i) = 2x^T(t) P_i \dot{x}(t) + x^T(t) \sum_{j=1}^s \pi_{ij} P_j x(t),$$

$$\begin{aligned} LV_2(x_i, t, i) &= x^T(t) (Q_{li} + hQ) x(t) - x^T(t-h) Q_{li} x(t-h) \\ &\quad + \int_{t-h}^t x^T(s) \left(\sum_{j=1}^s \pi_{ij} Q_{lj} - Q \right) x(s) ds, \end{aligned}$$

$$\begin{aligned} LV_3(x_i, t, i) &= h\dot{x}^T(t) R_{li} \dot{x}(t) + \frac{h^2}{2} \dot{x}^T(t) R \dot{x}(t) - \int_{t-h}^t \dot{x}^T(s) R_{li} \dot{x}(s) ds \\ &\quad + \int_{t-h}^t \int_{\theta}^t \dot{x}^T(s) \sum_{j=1}^s \pi_{ij} R_{lj} \dot{x}(s) ds d\theta - \int_{t-h}^t \int_{\theta}^t \dot{x}^T(s) R \dot{x}(s) ds d\theta, \end{aligned}$$

$$LV_4(x_i, t, i) = \frac{h^2}{2} \dot{x}^T(t) R_2 \dot{x}(t) - \int_{t-h}^t \int_{\theta}^t \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta,$$

$$LV_5(x_i, t, i) = \frac{h^3}{6} \dot{x}^T(t) R_3 \dot{x}(t) - \int_{t-h}^t \int_{\theta}^t \int_u^t \dot{x}^T(s) R_3 \dot{x}(s) ds du d\theta,$$

$$LV_6(x_i, t, i) = h x^T(t) R_4 x(t) - \int_{t-h}^t x^T(s) R_4 x(s) ds.$$

Applying Lemmas 5.1 and 2.3 in [11] and [9] to the integral terms in $LV_3(x_i, t, i)$ and $LV_4(x_i, t, i)$ yields

$$-\int_{t-h}^t \dot{x}^T(s) R_{li} \dot{x}(s) ds \leq -\frac{1}{h} \bar{\Omega}_1^T R_{li} \bar{\Omega}_1 - \frac{3}{h} \bar{\Omega}_2^T R_{li} \bar{\Omega}_2 - \frac{5}{h} \bar{\Omega}_3^T R_{li} \bar{\Omega}_3$$

with

$$\bar{\Omega}_1 = x(t) - x(t-h) = (e_1 - e_2) \xi(t),$$

$$\bar{\Omega}_2 = x(t) + x(t-h) - \frac{2}{h} \int_{t-h}^t x(s) ds = (e_1 + e_2 - 2e_3) \xi(t),$$

$$\begin{aligned} \bar{\Omega}_3 &= x(t) - x(t-h) + \frac{6}{h} \int_{t-h}^t x(s) ds - \frac{12}{h^2} \int_{t-h}^t \int_{\theta}^t x(s) ds d\theta \\ &= (e_1 - e_2 + 6e_3 - 12e_4) \xi(t). \end{aligned}$$

and

$$-\int_{t-h}^t \int_{\theta}^t \dot{x}^T(s) R_2 \dot{x}(s) ds d\theta \leq -2\bar{\Omega}_4^T R_2 \bar{\Omega}_4 - 4\bar{\Omega}_5^T R_2 \bar{\Omega}_5 - 6\bar{\Omega}_6^T R_2 \bar{\Omega}_6$$

with

$$\bar{\Omega}_4 = x(t) - \frac{1}{h} \int_{t-h}^t x(s) ds = (e_1 - e_3) \xi(t),$$

$$\bar{\Omega}_5 = x(t) + \frac{2}{h} \int_{t-h}^t x(s) ds - \frac{6}{h^2} \int_{t-h}^t \int_{\theta}^t x(s) ds d\theta \\ = (e_1 + 2e_3 - 6e_4) \xi(t),$$

$$\bar{\Omega}_6 = x(t) - \frac{3}{h} \int_{t-h}^t x(s) ds + \frac{24}{h^2} \int_{t-h}^t \int_{\theta}^t x(s) ds d\theta \\ - \frac{60}{h^3} \int_{t-h}^t \int_{\theta}^t \int_u^t x(s) ds dv d\theta = (e_1 - 3e_3 + 24e_4 - 60e_5) \xi(t).$$

Similarly, by using inequality in Remark 4 of [11] and (3) in Lemma 2.1, we have

$$- \int_{t-h}^t x^T(s) R_4 x(s) ds \\ \leq - \frac{1}{h} \left(\int_{t-h}^t x(s) ds \right)^T R_4 \left(\int_{t-h}^t x(s) ds \right) - \frac{3}{h} \bar{\Omega}_7^T R_4 \bar{\Omega}_7 - \frac{5}{h} \bar{\Omega}_8^T R_4 \bar{\Omega}_8,$$

with

$$\bar{\Omega}_7 = \int_{t-h}^t x(s) ds - \frac{2}{h} \int_{t-h}^t \int_{\theta}^t x(s) ds d\theta = (he_3 - 2he_4) \xi(t), \\ \bar{\Omega}_8 = \int_{t-h}^t x(s) ds - \frac{6}{h} \int_{t-h}^t \int_{\theta}^t x(s) ds d\theta + \frac{12}{h^2} \int_{t-h}^t \int_{\theta}^t \int_u^t x(s) ds dv d\theta \\ = (he_3 - 6he_4 + 12he_5) \xi(t).$$

and

$$- \int_{t-h}^t \int_{\theta}^t \int_u^t \dot{x}^T(s) R_3 \dot{x}(s) ds dv d\theta \\ \leq -6h \bar{\Omega}_9^T R_3 \bar{\Omega}_9 - 10h \bar{\Omega}_{10}^T R_3 \bar{\Omega}_{10} - 14h \bar{\Omega}_{11}^T R_3 \bar{\Omega}_{11},$$

with

$$\bar{\Omega}_9 = \frac{1}{2} x(t) - \frac{1}{h^2} \int_{t-h}^t \int_{\theta}^t x(s) ds d\theta = \left(\frac{1}{2} e_1 - e_4 \right) \xi(t), \\ \bar{\Omega}_{10} = \frac{1}{2} x(t) + \frac{3}{h^2} \int_{t-h}^t \int_{\theta}^t x(s) ds d\theta - \frac{12}{h^3} \int_{t-h}^t \int_{\theta}^t \int_u^t x(s) ds dv d\theta \\ = \left(\frac{1}{2} e_1 + 3e_4 - 12e_5 \right) \xi(t), \\ \bar{\Omega}_{11} = \frac{1}{2} x(t) - \frac{10}{h^2} \int_{t-h}^t \int_{\theta}^t x(s) ds d\theta + \frac{60}{h^3} \int_{t-h}^t \int_{\theta}^t \int_u^t x(s) ds dv d\theta \\ - \frac{180}{h^4} \int_{t-h}^t \int_{\theta}^t \int_u^t \int_v^t x(s) ds dv d\theta \\ = \left(\frac{1}{2} e_1 - 10e_4 + 60e_5 - 180e_6 \right) \xi(t).$$

Then, we can get

$$LV(x_t, t, i) = \sum_{m=1}^6 LV_m x_t, t, i \leq \xi^T(t) \Theta_i \xi(t) < 0, \quad (10)$$

where

$$\xi(t) = \begin{pmatrix} x^T(t), x^T(t-h), \frac{1}{h} \int_{t-h}^t x^T(s) ds, \frac{1}{h^2} \int_{t-h}^t \int_{\theta}^t x^T(s) ds d\theta, \\ \frac{1}{h^3} \int_{t-h}^t \int_{\theta}^t \int_u^t x^T(s) ds dv d\theta, \frac{1}{h^4} \int_{t-h}^t \int_{\theta}^t \int_u^t \int_v^t x^T(s) ds dv d\theta \end{pmatrix}^T \\ \sum_{j=1}^s \pi_{ij} Q_{1j} - Q < 0, \quad \sum_{j=1}^s \pi_{ij} R_{1j} - R < 0, \\ e_k = [0 \cdots \underset{k}{1} \cdots 0] \in \mathbb{R}^{6n \times n} \quad (k = 1, \dots, 6)$$

$$\Theta_i = 2e_1^T P_i e_0 + e_1^T \sum_{j=1}^s \pi_{ij} P_j e_1 + e_1^T Q_{1i} e_1 - e_2^T Q_{1i} e_2 + he_1^T Q e_1 + he_0^T R_{1i} e_0 \\ + \frac{h^2}{2} e_0^T R e_0 + \frac{h^2}{2} e_0^T R_2 e_0 + \frac{h^3}{6} e_0^T R_3 e_0 + he_1^T R_4 e_1 - \frac{1}{h} \bar{\Omega}_1^T R_{1i} \bar{\Omega}_1 \\ - \frac{3}{h} \bar{\Omega}_2^T R_{1i} \bar{\Omega}_2 - \frac{5}{h} \bar{\Omega}_3^T R_{1i} \bar{\Omega}_3 - 2\bar{\Omega}_4^T R_2 \bar{\Omega}_4 - 4\bar{\Omega}_5^T R_2 \bar{\Omega}_5 - 6\bar{\Omega}_6^T R_2 \bar{\Omega}_6 \\ - he_3^T R_4 e_3 - 3h \bar{\Omega}_7^T R_4 \bar{\Omega}_7 - 5h \bar{\Omega}_8^T R_4 \bar{\Omega}_8 - 6h \bar{\Omega}_9^T R_3 \bar{\Omega}_9 \\ - 10h \bar{\Omega}_{10}^T R_3 \bar{\Omega}_{10} - 14h \bar{\Omega}_{11}^T R_3 \bar{\Omega}_{11} < 0,$$

$$\text{with } e_0 = A_1 e_1 + A_{11} e_2 + h A_{21} e_3.$$

Next, we will show the system (1) with $u(t) = 0$ satisfies Definition 2 in [10] for any $\omega(t) \in L_2[0, \infty]$. Set

$$J(T) = E \left\{ \int_0^T z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) dt \right\}. \quad (11)$$

Applying (9) to system (1) with $u(t) = 0$, we can know that

$$J(T) = E \left\{ \int_0^T z^T(t) z(t) - \gamma^2 \omega^T(t) \omega(t) \right. \\ \left. + LV(x_t, t, i) dt \right\} \leq \bar{\xi}^T(t) \Phi_i \bar{\xi}(t), \quad (12)$$

where

$$\bar{\xi}(t) = \begin{pmatrix} x^T(t), x^T(t-h), \frac{1}{h} \int_{t-h}^t x^T(s) ds, \omega^T(t), \\ \frac{1}{h^2} \int_{t-h}^t \int_{\theta}^t x^T(s) ds d\theta, \frac{1}{h^3} \int_{t-h}^t \int_{\theta}^t \int_u^t x^T(s) ds dv d\theta, \\ \frac{1}{h^4} \int_{t-h}^t \int_{\theta}^t \int_u^t \int_v^t x^T(s) ds dv d\theta \end{pmatrix}^T,$$

By using Schur complement, we have that $\Phi_i < 0$ is described as matrix (8), this implies that system (1) with $u(t) = 0$ are H_∞ stable, and the proof is completed.

Based on Theorem 3.1, an LMI-based state-feedback controller design method for known transition probabilities is established in the following theorem.

Theorem 3.2: Given scalars $\gamma > 0$ and $h > 0$, the closed-loop system in model (1) with $u(t) = K_i x(t)$ is stochastically stable, if there exist real matrices Y_i , symmetric positive definite matrices X_i , Q_i , R_i , Q , R , R_2 , R_3 , R_4 , such that the following linear matrix inequalities hold:

$$[\Psi_i = \varphi_{(m,n)i}]_{12 \times 12} < 0, \quad (13)$$

$$\sum_{j=1}^s \pi_{ij} \tilde{Q}_{1j} - \tilde{Q} < 0, \quad \sum_{j=1}^s \pi_{ij} \tilde{R}_{1j} - \tilde{R} < 0. \quad (14)$$

where

$$\begin{aligned} \varphi_{11} &= [A_i X_i + C_i Y_i] + [A_i X_i + C_i Y_i]^T + \sum_{j=1}^s \pi_{ij} \tilde{P}_j \\ &\quad + \tilde{Q}_{1i} + h \tilde{Q} - \frac{9}{h} \tilde{R}_{1i} - 12 \tilde{R}_2 - \frac{15h}{2} \tilde{R}_3 + h \tilde{R}_4, \\ \varphi_{12} &= A_i X_i + \frac{3}{h} \tilde{R}_{1i}, \quad \varphi_{22} = -\tilde{Q}_{1i} - \frac{9}{h} \tilde{R}_{1i}, \quad \varphi_{23} = \frac{36}{h} \tilde{R}_{1i}, \\ \varphi_{13} &= h A_{2i} X_i - \frac{24}{h} \tilde{R}_{1i} + 12 \tilde{R}_2, \quad \varphi_{33} = -\frac{192}{h} \tilde{R}_{1i} - 72 \tilde{R}_2 - 9h \tilde{R}_4, \\ \varphi_{14} &= B_i X_i, \quad \varphi_{44} = -\gamma^2 I, \quad \varphi_{25} = -\frac{60}{h} \tilde{R}_{1i}, \quad \varphi_{29} = \varphi_{210} = h X_i A_{1i}^T, \\ \varphi_{15} &= \frac{60}{h} \tilde{R}_{1i} - 120 \tilde{R}_2 + 58h \tilde{R}_3, \quad \varphi_{35} = \frac{360}{h} \tilde{R}_{1i} + 480 \tilde{R}_2 + 36h \tilde{R}_4, \\ \varphi_{35} &= -\frac{720}{h} \tilde{R}_{1i} - 3600 \tilde{R}_2 - 1496h \tilde{R}_3 - 192h \tilde{R}_4, \\ \varphi_{16} &= 360 \tilde{R}_2 - 360h \tilde{R}_3, \quad \varphi_{36} = -1080 \tilde{R}_2 - 60h \tilde{R}_4, \\ \varphi_{56} &= 8640 \tilde{R}_2 + 8760h \tilde{R}_3 + 360h \tilde{R}_4, \quad \varphi_{39} = \varphi_{310} = h^2 X_i A_{2i}^T, \\ \varphi_{66} &= -21600 \tilde{R}_2 - 51840h \tilde{R}_3 - 720h \tilde{R}_4, \quad \varphi_{49} = \varphi_{410} = h B_i^T, \\ \varphi_{17} &= 1260h \tilde{R}_3, \quad \varphi_{57} = -25200h \tilde{R}_3, \quad \varphi_{67} = 151200h \tilde{R}_3, \\ \varphi_{77} &= -1453600h \tilde{R}_3, \quad \varphi_{18} = h[A_i X_i + C_i Y_i]^T, \quad \varphi_{211} = h^2 X_i A_{1i}^T, \\ \varphi_{28} &= h X_i A_{1i}^T, \quad \varphi_{38} = h^2 X_i A_{2i}^T, \quad \varphi_{48} = h B_i^T, \quad \varphi_{311} = h^3 X_i A_{2i}^T, \\ \varphi_{88} &= h(He(-X_i) + \tilde{R}_{1i}), \quad \varphi_{19} = h[A_i X_i + C_i Y_i]^T, \quad \varphi_{411} = h^2 B_i^T, \\ \varphi_{99} &= 2(He(-X_i) + \tilde{R}), \quad \varphi_{110} = h[A_i X_i + C_i Y_i]^T, \quad \varphi_{412} = E_i^T, \\ \varphi_{1010} &= 2(He(-X_i) + \tilde{R}_2), \quad \varphi_{111} = h^2[A_i X_i + C_i Y_i]^T, \quad \varphi_{1212} = -I, \\ \varphi_{1111} &= 6h(He(-X_i) + \tilde{R}_3), \quad \varphi_{112} = [D_i X_i + F_i Y_i]^T. \end{aligned}$$

Moreover, the controller gain matrices K_i can be calculated as $K_i = Y_i X_i^{-1}$.

Proof. Based on the conditions of Theorem 3.1. Letting $A_i = A_i + C_i K_i$ and $D_i = D_i + F_i K_i$, we will obtain a new matrix inequality $\bar{\Phi}_i < 0$. Then, pre-multiplying and post-multiplying $\bar{\Phi}_i$ by the diagonal matrix

$$\text{diag} \begin{bmatrix} P_i^{-1}, P_i^{-1}, P_i^{-1}, I, P_i^{-1}, P_i^{-1}, \\ P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}, P_i^{-1}, I \end{bmatrix}.$$

Besides, we also need to define

$$P_i^{-1} = X_i, \quad K_i X_i = Y_i, \quad X_i P_j X_i = \tilde{P}_j, \quad X_i Q_{1i} X_i = \tilde{Q}_{1i},$$

$$X_i Q X_i = \tilde{Q}, \quad X_i R_{1i} X_i = \tilde{R}_{1i}, \quad X_i R X_i = \tilde{R}, \quad X_i R_2 X_i = \tilde{R}_2,$$

$$X_i R_3 X_i = \tilde{R}_3, \quad X_i R_4 X_i = \tilde{R}_4, \quad R^{-1} = X_i \tilde{R}^{-1} X_i,$$

$$R_n^{-1} = X_i \tilde{R}_n^{-1} X_i (n = 1i, 2, 3). \quad (15)$$

It follows from the Theorem 3.1 in [12] and makes use of the Schur complement, we can get $\Psi_i < 0$. Accordingly, system (1) with $u(t) = K_i x(t)$ is stochastically stable with the required H_∞ performance once (13) and (14) hold.

In this case, a suitable stabilizing state-feedback controller can be chosen as $u(t) = Y_i X_i^{-1} x(t)$.

Remark 3.1: The nonlinear terms are contained in conditions (15). Note that $XW^{-1}X^T \geq He(X) - W$ holds for any matrix W (W denotes \tilde{R} , and $\tilde{R}_n (n = 1i, 2, 3)$ in this paper) with $He(X) = X + X^T$. Thus, the H_∞ controller can be formulated as follows Theorem 3.1 in [12]. The H_∞ controller design problem in (13) and (14) of Theorem 3.2 is a optimization problem with a sets of LMIs constraints, which can be solved by using the LMI toolbox of Matlab.

IV. NUMERICAL EXAMPLE

The numerical example in this section demonstrates the effectiveness of our method.

EXAMPLE 1. Consider the system(1) with $u(t) = K_i x(t)$ and two modes and the following parameters:

$$\begin{aligned} A_1 &= \begin{bmatrix} -1.2460 & -1.4410 \\ -1.5937 & -1.9289 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 1 & 0.9 \\ 1 & 1 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 1 & 0.8 \\ 1 & 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1.2460 & -0.4410 \\ 0.5937 & -1.9289 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0.0403 & 0.0403 \\ 0.6771 & 0.6771 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 0.0103 & 0.0103 \\ 0.5771 & 0.6771 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 1 & 0.9 \\ 1 & 1 \end{bmatrix}, \\ \Pi &= \begin{bmatrix} -0.5 & 0.5 \\ 0.8 & -0.8 \end{bmatrix}, \quad A_{11} = \begin{bmatrix} -0.7098 & 0 \\ 0 & -1.2025 \end{bmatrix}, \quad A_{12} = \begin{bmatrix} -1.7098 & 0 \\ 0 & -2.2025 \end{bmatrix}, \\ A_{21} &= \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} -2.4898 & 0.2895 \\ -1.3396 & -0.0211 \end{bmatrix}, \end{aligned}$$

$$E_1 = \begin{bmatrix} 0.1184 & 0.1184 \\ 0.1184 & 0.1184 \end{bmatrix}, E_2 = \begin{bmatrix} 0.3184 & 0.3184 \\ 0.3184 & 0.3184 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -0.3775 & -0.2959 \\ -0.3775 & -0.2959 \end{bmatrix}, D_2 = \begin{bmatrix} -1.4751 & -0.2340 \\ -1.4751 & -0.2340 \end{bmatrix}.$$

By solving the conditions given in Theorem 3.2, one can obtain the maximum allowable upper bounds on h for different γ , which is listed in Table 1, respectively. Moreover, controller gain matrices for $\gamma=0.7$ and $h=0.7086$, and $\gamma=0.8$ and $h=0.7786$ are calculated as

$$K_1 = \begin{bmatrix} 0.3601 & 0.2214 \\ 0.0646 & 0.0500 \end{bmatrix}, K_2 = \begin{bmatrix} 3.1081 & -0.7351 \\ -2.2300 & 0.5275 \end{bmatrix}.$$

$$K_1 = \begin{bmatrix} 0.4188 & 0.2660 \\ 0.0949 & 0.0611 \end{bmatrix}, K_2 = \begin{bmatrix} 1.6849 & -0.2004 \\ -0.8539 & 0.1017 \end{bmatrix}.$$

TABLE I. OBTAINED h FOR γ DIFFERENT IN EXAMPLE 1

γ	0.7	0.8	0.9	1
THEOREM 3.2	0.7086	0.7786	0.7934	0.8017

V. CONCLUSION

H_∞ control of Markovian jump linear systems with delay-dependent is discussed in this paper. Resorting to the up-to-date time-delay technique, sufficient conditions for the closed-loop systems to be stochastically stable with the prescribed H_∞ performance level γ are established in the framework of LMIs. Since a scaling method is developed to deal with controller design, how to reduce the induced scaling conservativeness needs further research.

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