

An Energy Method-based Damage Constitutive Law of Concrete for SRC

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Abstract: Based on stochastic damage model, a new damage variate with two parameters is proposed to describe the damage constitutive relationship of concrete. The damage evolution laws of high-strength and high-performance concrete are obtained by a large number of experiments on its mechanical properties. The damage constitutive equation is established by boundary conditions. Through comparisons of theoretical calculations and a large number of mechanical test results of concrete, showing that the damage constitutive equation can reasonably reflect the damage characteristics of concrete. This paper's research identifies a certain theoretical basis for further studying the dynamic and stochastic damage constitutive relationship of concrete.

Introduction

As the complexity and stochasticness of concrete inner structure, although the existing concrete constitutive relationship can reflect the mechanical characteristics, it was still different from the actual mechanical properties of concrete^[1]. In the recent 20 years, the research shows us that the damage variate on the selection of concrete still has not been established a uniform standard. But to be able to better reflect the changes in concrete damage, you need to select a reasonable damage variate. Therefore, the research of concrete for the combination of steel reinforced concrete structures has certain practical significance. In this paper, it is based on the damage detection test of concrete, by using the changes of initial elastic modulus and the final and initial elastic modulus ratio as a damage variate to establish its' fitting curves^[2]. Proposed a damage function with two parameters to describe the concrete damage evolution^{[3] [4]}, combined with the experimental stress-strain relationship curves to derive the two parameters. Comparisons between the established constitutive relationship and experimental results is made to verify its' reasonableness^[5]. This paper research provide some references for the study of stochastic damage constitutive relationship.

Concrete damage detection test

In this paper, the $D = \Delta E / E$ formula is used as damage indicators for concrete. E is the initial elastic modulus of concrete. The "damage freezing" experiments designed by Hu Shi-sheng means that the damage formation and development of test piece has been "fixed" in a determined strain in compression^[6]. The identified strain can be obtained mainly by controlling the height of steel rims outside the specimen. The installation is shown in Figure (1).

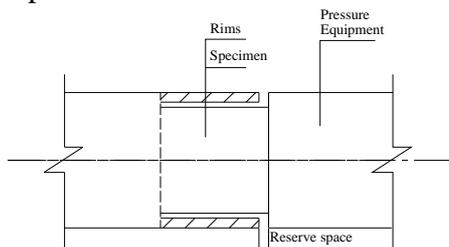


Fig. 1 Damage to "freeze" Experimental schematic diagram

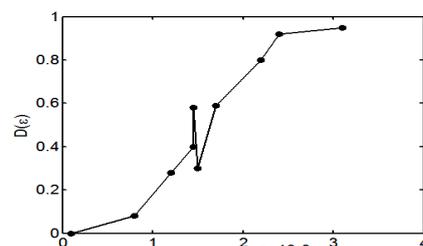


Fig. 2 fitting curve of concrete damage(Strain $\times 10^{-3}$)

Three set of experiments about the concrete of C100 has been made for testing its' damage evolution. Each group contains five specimens, the loading rate was controled in the $2 \times 10^{-4} \text{min}^{-1}$ and the strain was controled from 10^{-3} to 3×10^{-3} . Then these specimens were tested in the MTS machine to obtain the distribution of damage variate defined by equation (1), shown in Figure (2).

Experimental stress-strain curves

The stress-strain curve of concrete obtained in the experiment is shown in Figure (3). In the figure, C1, C2 and C3 are designed by literature [6]. The specimen's size is 100mm×100mm, the average compressive strength is 119.0MPa. The following characteristics are observed, according to the analysis of this figure: compared with ordinary concrete stress-strain curves, its ascent stage have been from the curve tends to a straight line, and the higher is the linear it more remarkable, and descending more steep, which is coincide with the mechanical properties of concrete.

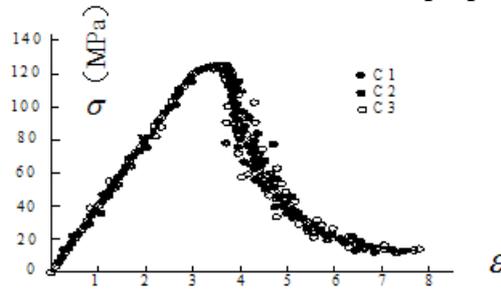


Fig.3. 119Mpa Concrete compressive stress-strain curves(Strain×10⁻³)

The Constitutive law based on a new damage variate

According to the analysis of stress-strain curve and the distribution of the damage variate obtained by experiment, the damage variate is defined as follow:

$$D(e) = \frac{2}{p} \arctan(ae^b) \tag{2}$$

where *a* and *b* are two parameters. Substituting Equation (2) into the classical damage mechanics constitutive equation available:

$$s(e) = Ee(1-D) \Big|_{D=\frac{2}{p}\arctan(ae^b)} = Ee(1 - \frac{2}{p} \arctan(ae^b)) \tag{3}$$

This equation is the constitutive law based on the new damage variate.

Based on the experimental stress-strain curve characteristics of concrete, introduce the boundary conditions of stress-strain curves at the origin and the ultimate compressive strength, just as follows :

$$s(e)|_{e=0} = 0, \quad \frac{ds}{de} \Big|_{e=0} = E, \quad s(e)|_{e=e_0} = f_c, \quad \frac{ds}{de} \Big|_{e=e_0} = 0 \tag{4}$$

Where *E* and *f_c* is the initial elastic modulus of concrete and broken strength of concrete respectively, *e₀* is ultimate compressive strain of concrete. Substituting Equation (3) into expression (4), the *a* and *b* will be written as follow:

$$a = \tan A \times e_0^{-b} \tag{5}$$

$$b = \frac{pf_c \times (\tan A^2 + 1)}{2 \times Ee_0 \tan A} \tag{6}$$

$$A = 1.5708 \times (1 - \frac{f_c}{Ee_0}) \tag{7}$$

For the Eq.(2) and Eq. (3), when *e* is a stochastic variate which has a connection with concrete damage and damage evolution, assuming the stochastic distribution probability density function is

$f(e)$, the above constitutive relationship turns to be a stochastic damage one, its mean and variance expressions is shown as follows.

$$m_s(e) = \int s(e)f(e)de \tag{8}$$

$$\text{var}_s(e) = M[s(e)]^2 - M^2[s(e)] \tag{9}$$

Validation of the Equations

For the concrete with the strength of 119MPa and 95Mpa, the respective Eq. (3) and Eq. (4) may be rewritten as:

for the 119Mpa:

$$s(e) = 40 \cdot e \cdot \left\{ 1 - \frac{2}{p} \arctan \left[(7.3318e-006)e^{8.0229} \right] \right\} \tag{10}$$

$$D(e) = \frac{2}{p} \arctan \left\{ (7.3318e-006)e^{8.0229} \right\} \tag{11}$$

for 95Mpa:

$$s(e) = 40e \left\{ 1 - \frac{2}{p} \arctan \left[(3.9631e-005) \times e^{8.4705} \right] \right\} \tag{12}$$

$$D(e) = \frac{2}{p} \arctan \left[(3.9631e-005)e^{8.47058} \right] \tag{13}$$

To verify the validity, the numerical calculating results of the Eq.(10), Eq.(11), Eq.(12) and Eq.(13) are compared with the experimental records of strain-stress curves of concrete, as shown in Fig. (4) and Fig. (5).

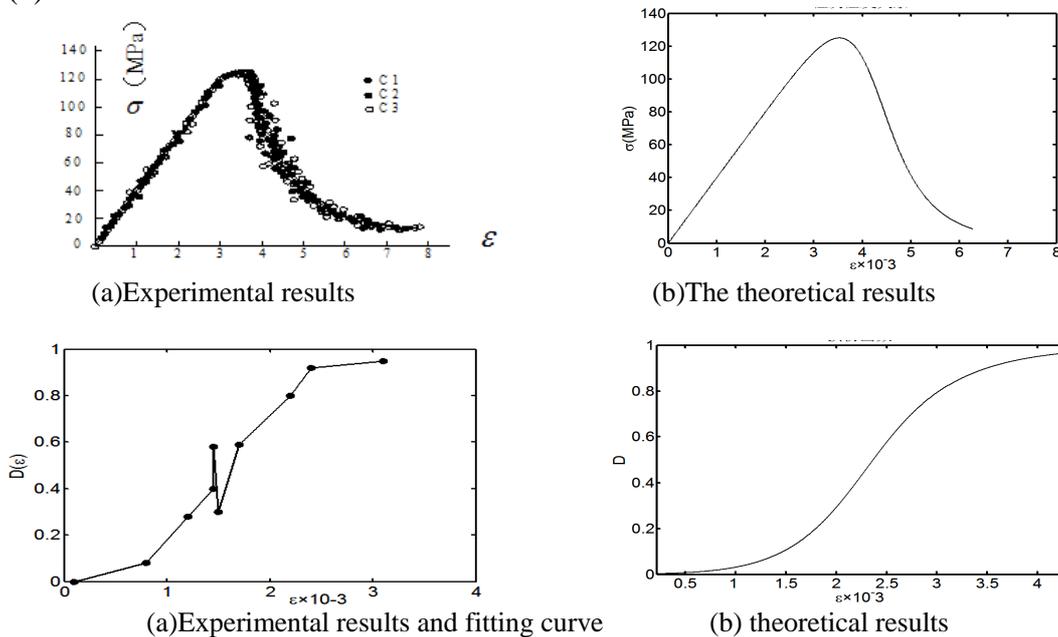


Fig. 4. experimental and theoretical results of 119MPa concrete

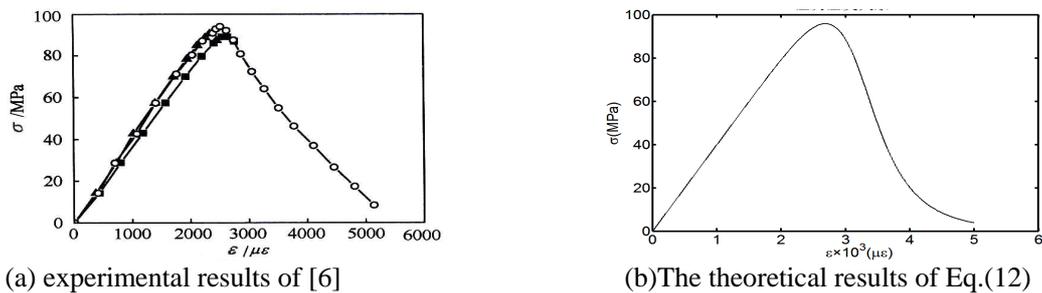


Fig. 5. experimental and theoretical results of 95MPa concrete

The comparisons of experimental and theoretical results show that the damage function established in this paper simulate concrete damage evolution well.

Conclusion

By proposing a new damage function to simulate damage evolution of concrete, then established the damage constitutive relationship of concrete by the definition of this damage function. The damage function and the stress-strain relations are simulated by computer. Comparisons between experimental results and theoretical results of damage variate and stress-strain relations verify that the constitutive damage equations are coincide with the experimental results basically. The damage function is proposed in this paper can be a good simulation of concrete for damage evolution, it can establish a certain foundation for stochastic damage constitutive relationship of concrete and apply in the actual engineering.

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