

Analytical Solution of Plastoelastic Buckling of Rectangular Section Column Strengthened by Carbon Fibre Sheets

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Abstract. Based on the Ježek method of computing elastic-plastic buckling of compression-bending member, analytical solutions of buckling ultimate load for rectangular column strengthened by carbon fibre sheets are derived. A new method of theoretical research on elastic-plastic buckling of steel member is provided. Some conclusions are obtained by analyzing the stress of rectangular column before and after being pasted with carbon fiber sheets. The effects of different thickness of composites on the ultimate load of rectangular column are discussed, the theoretical analysis is proved correct by the numerical simulation test.

Introduction

The plastic bearing capacity of steel members strengthened by carbon fibre sheets can be improved. It is necessary to study the plastic instability state of steel structure pasted with carbon fiber sheets. Analytical expressions of ultimate load of buckling about neutral axis with maximum moment of inertia for rectangular column that its upper and lower surfaces are pasted with carbon fibre sheets are derived, it is confirmed that carbon fibre sheets is effective to enhance the plastic buckling of steel members.

Unilateral or bilateral plastic zones may appear in the tension and compression zone of cross section of compression-bending member^[1], simply supported structure at both ends with plastic zone only in compression-side is discussed in the paper.

Elastic-plastic buckling formula for column with plastic zone only in compression-side

Formula derivation under these basic assumptions: ①The member is only subjected to axial compressive load P and bending moment M , both ends are hinged (Fig.1); ②The member is an ideal elastic plastic body^[2]; ③Carbon fibre has brittle elastic constitutive relation and always maintains good line elasticity before being broke; ④Carbon fibre is affixed to the upper and lower surfaces of the member, both are well bonded and no relative slip; ⑤The bending deformation of the member is half wave sine curve; ⑥The member deformation is a small deformation; ⑦Bending occurs only on the strong axis of the member^[3].

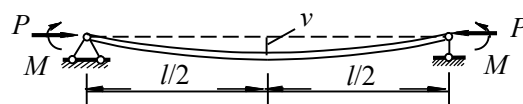


Fig. 1 Simple supported beam under axial compressive load and bending moment

Now, $P_y = AS_y$ expresses full section yield pressure of the member under axial compressive load, According to the balance condition of the axial force of the member section^[4],

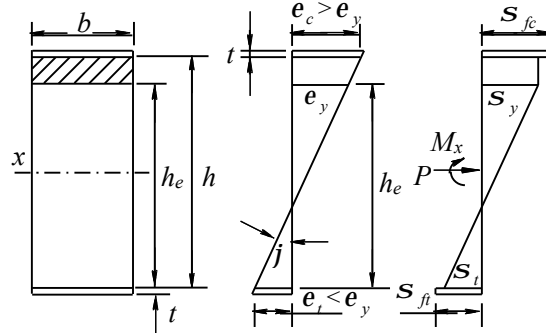


Fig. 2 The central section just bears the strain and stress of the compressive yield zone

$$P = s_y A - \frac{1}{2}(s_y + s_t)bh_e + E_f(e_c - e_t)bt \quad (1)$$

In the equation, s_y is member yield strength; A is cross sectional area of member; s_t is lower surface stress of the member, less than s_y ; h_e is elastic core height of member cross section (Fig. 2, shadow part is plastic zone); E_f is elastic modulus of carbon fibre; e_c is the strain of upper surface of member; e_t is the strain lower surface of member and carbon fibre^[5]; t is the thickness of carbon fibre.

From Fig.2,

$$e_c = \frac{h}{h_e}(e_y + e_t) - e_t \quad (2)$$

Where e_y is yield strain.

Put Eq. (2) into Eq.(1), we get

$$P = P_y - \frac{1}{2}(s_y + s_t)bh_e + \frac{E_f}{E} \left[\frac{h}{h_e}(s_y + s_t) - 2s_t \right] bt \quad (3)$$

According to the balance condition of moment,

$$M_x = M + Pv = \frac{1}{2}(s_y + s_t)bh_e \left(\frac{h}{2} - \frac{h_e}{3} \right) + E_f(e_c + e_t)bt \frac{h}{2} \quad (4)$$

Put Eq. (2) into Eq.(4), and according to $e = \frac{S}{E}$, then

$$M + Pv = \frac{1}{2}(s_y + s_t)bh_e \left(\frac{h}{2} - \frac{h_e}{3} \right) + \frac{E_f}{E} \frac{h}{h_e}(s_y + s_t)bt \frac{h}{2} \quad (5)$$

From Fig.2,

$$j = \frac{e_y + e_t}{h_e} = \frac{s_y + s_t}{Eh_e} \quad (6)$$

According to the hypothesis of deformation curve, deflection curve is

$$y = v \sin \frac{px}{l} \quad (7)$$

Curvature of the central cross section is

$$j = -y''\left(\frac{l}{2}\right) = \frac{vp^2}{l^2} = \frac{s_y + s_t}{Eh_e} \quad (8)$$

From Eq.(8),

$$s_t = \frac{vp^2}{l^2} Eh_e - s_y \quad (9)$$

Where,

$$k = \frac{p^2}{l^2} E, \quad l = \frac{E_f}{E} \quad (10)$$

Then,

$$s_t = k v h_e - s_y \quad (11)$$

Put Eq. (11) into Eq.(3) and (5),

$$P = P_y - \frac{1}{2} k v b h_e^2 + l k v b t h - 2 l k v b t h_e + 2 l s_y b t \quad (12)$$

$$M + P v = \frac{1}{2} k v b h_e^2 \left(\frac{h}{2} - \frac{h_e}{3} \right) + l k v b t \frac{h^2}{2} \quad (13)$$

Eq.(12) is a quadratic equation with one unknown, we get

$$h_e = -2 l t + 2 l t \sqrt{1 + \frac{h}{2 l t} + \frac{s_y}{l k v t} - \frac{P - P_y}{2 l^2 k v b t^2}} \quad (14)$$

Take the derivative of v in Eq.(14) and use the extremal condition $\frac{dP}{dv} = 0$,

$$\frac{dh_e}{dv} = \frac{P - P_y - 2 l s_y b t}{\sqrt{4 l^2 k^2 v^4 b^2 t^2 + 2 l k^2 v^4 b^2 h t + 4 l k s_y v^3 b^2 t - 2 k v^3 b (P - P_y)}} \quad (15)$$

Then take the derivative of v in Eq.(13),

$$P = \frac{1}{2} k b h_e^2 \left(\frac{h}{2} - \frac{h_e}{3} \right) + \frac{1}{2} k v b (h h_e - h_e^2) \frac{dh_e}{dv} + l k b t \frac{h^2}{2} \quad (16)$$

Take Eq.(14) into Eq.(13), take Eq.(14) and (15) into Eq.(16), so we can obtain two equations of $P \sim v$ relationship, we can draw up their relation curve in the same coordinate system, then their

intersection is ultimate load of compression-flexure member.

Calculation example

One steel column of rectangular section, cross section width $b=100\text{mm}$, section height $h=200\text{mm}$, elastic modulus $E=206\times 10^9\text{Pa}$, material yield limit $s_y=235\times 10^6\text{Pa}$, Elastic modulus of composite $E_f=235\times 10^9\text{Pa}$, paste thickness $t=0.835\text{mm}$.

According to equation $l_0 = p\sqrt{EI/s_y A}$, critical length $l_0=5.37\text{m}$ is acquired, Elastic limit moment $M_e = s_y W = 0.157\times 10^6\text{Nm}$, plastic limit bending moment $M_u = 1.5M_e = 0.235\times 10^6\text{Nm}$.

Through table1 and table2, the enhancement of composites becomes greater and the plastic zone of steel member becomes smaller with the increasing of bending moment, it indicates that the plastic bearing capacity of steel member pasted with composites can be enhanced. In addition, the maximum stress of composites only is hundreds Mpa and far less than the breaking strength of composites.

Table1 Ultimate load before and after pasting composites $P_u(\text{KN})$

$M(\times 10^6)$	0.01	0.05	0.1	0.15
before pasting carbon fiber	1899	1311	845	444
After pasting carbon fiber	1957	1364	898	500
rise (%)	3.1	4.0	6.3	12.6

Table2 Elastic core height before and after pasting composites $h_e(\text{mm})$

$M(\times 10^6)$	0.01	0.05	0.1	0.15
before pasting carbon fiber	193	170	108	70.6
After pasting carbon fiber	193	170	147	118
add (%)	0.0	0.0	36.1	67.1

As is shown in Fig.3, there is a basic linear relationship between the thickness of composites pasted on member and the increasing of ultimate load of steel member.

In order to prove the correctness of the theoretical calculation formula, $P_u - M$ curve obtained by theoretical calculation formula and computing result obtained by the finite element software ANSYS were compared (see Fig.4), it is testified that theoretical calculation method and numerical solution are very consistent.

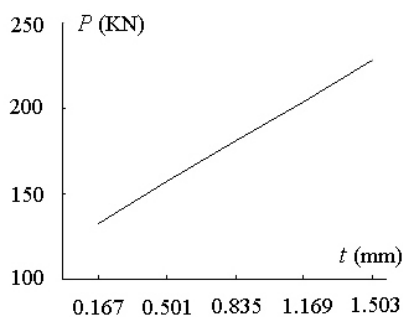


Fig.3 Effects of composites thickness on ultimate load

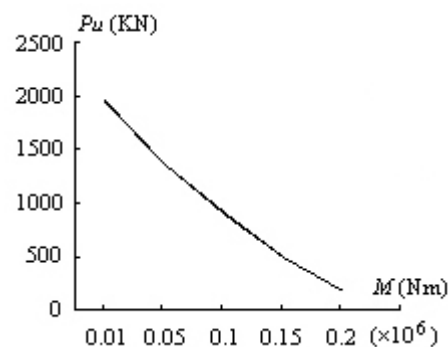


Fig.4 Comparison of analytical solutions and finite element solutions of ultimate loads (after pasting carbon fiber)

Conclusions

(1) The theoretical formula in this paper can accurately calculate the ultimate load of buckling about neutral axis with maximum moment of inertia for perfect elastic-plastic rectangular column pasted with composites. The study have a very good reference value to the theoretical research of steel member reinforced by composites.

(2) The calculation example shows that the member reinforced by composites can obviously improve the behaviour of compression member that in plastic state and increase the plastic buckling bearing capacity of member. The stiffness of the member reinforced by composites is almost unchanged, the elastic core height of member is markedly increased, it indicates that steel member pasted with composites can greatly enhance the strength margin of member.

The elastoplastic instability formula of member with plastic zone both in compression-side and tension side will be researched in another paper.

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