

The Crack Tip Fields for Anti-plane Crack in Functionally Graded Magneto-electroelastic Materials

Guangtu SHEN, Xiao CHONG, Peng ZHANG

School of Noncommissioned Officer
The University of Astronautics engineering
Beijing, China
e-mail: chongxiao2005@163.com

Yao DAI

The department of mechanical engineering
The Academy of Armored Force Engineering
Beijing, China
e-mail: dai_yao@sina.com

Abstract—The problem of a anti-plane crack in functionally graded magneto-electroelastic materials is investigated. The material properties of the functionally graded magneto-electroelastic materials are assumed to be exponential function of y perpendicular to the crack. To make the analysis tractable, the crack surface condition is assumed to be electrically and magnetically impermeable. The high order crack tip fields are obtained by the method of eigen-expansion method. This study has fundamental significance as Williams' solution.

Keywords—component; crack tip fields; eigen-expansion method; Williams' solution; functionally graded magneto-electroelastic materials

I. INTRODUCTION

Due to magneto-electroelastic coupling behavior, the composite materials are extensively used to design actuators, sensors and other electronic products in modern technology, particularly in smart materials/intelligent structures that are cable of responding to internal and environment changes. With the applications of smart devices such as actuators, sensors and other adaptive control units in aeronautics, astronautics, communication, biomedical industry and so on, the research on the smart materials in these devices has been receiving increasing attentions nowadays. However, under the mechanical and/or electric and/or magnetic loading, these materials are usually predisposed to fracture owing to the existing microscopic defects. In recent years, an area of increasing interest is the fracture mechanics of magneto-electroelastic materials, which combine the ferromagnetic and ferroelectric phases. For inclusions and cracks embedded in such magneto-electroelastic solids, magneto-electroelastic behaviors have been analyzed by many researchers including Wang et al. [1], Wan et al. [2], Zhong and Li [3], Huang et al. [4], Gao et al. [5], Sih et al. [6], Li and Lee [7], Athanasius and Ang [8], Bhargava and Sharma [9], Tupholme [10], Wang and Mai [11], Zheng et al. [12], Ma et al. [13] and Li et al. [14]. It can be found that only the crack tip singular field is involved in the researches mentioned above. This is because that the eigen-functions or the higher-order crack tip fields of the functionally graded magneto-electroelastic materials as Williams' solutions have not been available upto now. Therefore, the main effort of this paper is to find out the higher-order crack tip fields of the crack of functionally graded magneto-electroelastic materials by the eigen-expansion method.

II. BASIC EQUATIONS

Consider the functionally graded magneto-electroelastic material containing a crack. For anti-plane problems, only the out-of-plane displacement, the in-plane electric fields and the in-plane magnetic fields are of interest. Therefore, only the anti-plane deformation is coupled with the in-plane magneto-electric fields, and the constitutive relation of the magneto-electroelastic material to reduced to

$$\tau_r = \mathbf{M} \frac{\partial \mathbf{w}}{\partial r} \quad \tau_\theta = \mathbf{M} \frac{\partial \mathbf{w}}{r \partial \theta} \quad (1)$$

where $\tau_k = [\tau_{kz} \ D_k \ B_k]^T$ ($k = r, \theta$) is the generalized stress and τ, D, B are the anti-plane stress, electric displacement, magnetic induction, respectively. $\mathbf{w} = [w \ \phi \ \varphi]^T$ is the generalized displacement and w, ϕ, φ are the anti-plane mechanical displacement, electric potential, magnetic potential, respectively.

The property matrix

$$\mathbf{M} = \begin{bmatrix} c_{44} & e_{15} & h_{15} \\ e_{15} & -\epsilon_{11} & -d_{11} \\ h_{15} & -d_{11} & -\mu_{11} \end{bmatrix} \quad (2)$$

where $c_{44}, e_{15}, h_{15}, \epsilon_{11}, d_{11}, \mu_{11}$ are the shear modulus, piezoelectric coefficient, piezomagnetic coefficient, dielectric coefficient, magneto-electric coefficient, and magnetic permeability, respectively. The present work employs exponential functions to describe the continuous variations of material properties,

$$\mathbf{M} = \mathbf{M}_0 e^{\beta r \sin \theta} \quad (3)$$

$$\mathbf{M}_0 = \begin{bmatrix} c_{440} & e_{150} & h_{150} \\ e_{150} & -\epsilon_{110} & -d_{110} \\ h_{150} & -d_{110} & -\mu_{110} \end{bmatrix} \quad (4)$$

where $c_{44}, e_{15}, h_{10}, \epsilon_{11}, \epsilon_{33}, \epsilon_{13}, \mu_{11}, \mu_{33}, \mu_{13}$ are the shear modulus, piezoelectric coefficient, piezomagnetic coefficient, dielectric coefficient, magnetoelectric coefficient, and magnetic permeability at $y = 0$.

The generalized equilibrium equations can be written as

$$\tau_{r,r} + \frac{1}{r} \tau_{\theta,\theta} + \frac{1}{r} \tau_r = 0 \quad (5)$$

Substituting (1) and (2) into (5) yields the governing equations

$$\mathbf{M} \nabla^2 \mathbf{w} + \frac{\partial \mathbf{M}}{\partial r} \frac{\partial \mathbf{w}}{\partial r} + \frac{1}{r^2} \frac{\partial \mathbf{M}}{\partial \theta} \frac{\partial \mathbf{w}}{\partial \theta} = 0 \quad (6)$$

where $\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2}$ is the two-dimensional Laplace operator.

III. THE HIGHER ORDER CRACK-TIP FIELDS

The displacement component w , electric potential ϕ and magnetic potential φ can be expanded as follows

$$\mathbf{w} = \sum_{n=1}^{\infty} r^{\frac{n}{2}} \mathbf{w}_n(\theta) \quad (7)$$

where, $\mathbf{w}_n(\theta) = [w_n(\theta) \quad \phi_n(\theta) \quad \varphi_n(\theta)]^T$ and $w_n(\theta)$, $\phi_n(\theta)$, $\varphi_n(\theta)$ are eigen-functions.

Substitute (7) into (6). According to the linear independence of $r^{-3/2}$, r^{-1} , $r^{-1/2}$, ..., $r^{i/2-2}$, ..., the eigen-equations are obtained. The system of ordinary differential equations are

$$\begin{cases} \frac{1}{4} \frac{d \mathbf{w}_1(\theta)}{d \theta} + \frac{d^2 \mathbf{w}_1(\theta)}{d \theta^2} = 0 \\ \mathbf{w}_2(\theta) + \frac{d^2 \mathbf{w}_2(\theta)}{d \theta^2} = 0 \\ \frac{9}{4} \mathbf{w}_3(\theta) + \frac{d^2 \mathbf{w}_3(\theta)}{d \theta^2} + \frac{1}{2} \beta \sin \theta \mathbf{w}_1(\theta) + \beta \cos \theta \frac{d \mathbf{w}_1(\theta)}{d \theta} = 0 \\ 4 \mathbf{w}_4(\theta) + \frac{d^2 \mathbf{w}_4(\theta)}{d \theta^2} + \beta \sin \theta \mathbf{w}_2(\theta) + \beta \cos \theta \frac{d \mathbf{w}_2(\theta)}{d \theta} = 0 \\ \frac{25}{4} \mathbf{w}_5(\theta) + \frac{d^2 \mathbf{w}_5(\theta)}{d \theta^2} + \frac{3}{2} \beta \sin \theta \mathbf{w}_3(\theta) + \beta \cos \theta \frac{d \mathbf{w}_3(\theta)}{d \theta} = 0 \\ 9 \mathbf{w}_6(\theta) + \frac{d^2 \mathbf{w}_6(\theta)}{d \theta^2} + 2 \beta \sin \theta \mathbf{w}_4(\theta) + \beta \cos \theta \frac{d \mathbf{w}_4(\theta)}{d \theta} = 0 \\ \dots \end{cases} \quad (8)$$

In the case of electrically and magnetically impermeable crack, the boundary conditions are

$$\tau_r \Big|_{\theta=\pm\pi} = 0 \quad (9)$$

Further, they can be expressed as

$$\frac{d \mathbf{w}_n(\theta)}{d \theta} \Big|_{\theta=\pm\pi} = 0 \quad (10)$$

Solving the system of ordinary differential equations, we can obtain the results

$$\begin{cases} \mathbf{w}_1(\theta) = \mathbf{A}_{11} \sin \frac{\theta}{2} \\ \mathbf{w}_2(\theta) = \mathbf{A}_{21} \cos \theta \\ \mathbf{w}_3(\theta) = \mathbf{A}_{31} \sin \frac{3\theta}{2} - \frac{1}{12} \beta \mathbf{A}_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta \mathbf{A}_{11} \cos \frac{\theta}{2} \\ \mathbf{w}_4(\theta) = \mathbf{A}_{41} \cos 2\theta \\ \mathbf{w}_5(\theta) = \mathbf{A}_{51} \sin \frac{5\theta}{2} + \frac{1}{20} \beta \mathbf{A}_{31} \cos \frac{5\theta}{2} + \frac{1}{32} \beta^2 \mathbf{A}_{11} \sin \frac{3\theta}{2} + \\ \quad \frac{1}{48} \beta^2 \mathbf{A}_{11} \sin \frac{\theta}{2} - \frac{1}{4} \beta \mathbf{A}_{31} \cos \frac{\theta}{2} \\ \mathbf{w}_6(\theta) = \mathbf{A}_{61} \cos 3\theta - \frac{1}{12} \beta \mathbf{A}_{41} \sin 3\theta + \frac{1}{4} \beta \mathbf{A}_{41} \cos 3\theta \\ \dots \end{cases} \quad (11)$$

where $\mathbf{A}_{ij} = [A_{ij} \quad B_{ij} \quad C_{ij}]$ and A_{ij} , B_{ij} , C_{ij} are the undetermined coefficients.

Substituting (11) into (7), the generalized displacement is obtained.

$$\begin{aligned} \mathbf{w} = & r^{\frac{1}{2}} \mathbf{A}_{11} \sin \frac{\theta}{2} + r \mathbf{A}_{21} \cos \theta + r^{\frac{3}{2}} \left(\mathbf{A}_{31} \sin \frac{3\theta}{2} - \frac{1}{12} \beta \mathbf{A}_{11} \cdot \right. \\ & \left. \cos \frac{3\theta}{2} - \frac{1}{4} \beta \mathbf{A}_{11} \cos \frac{\theta}{2} \right) + r^2 \mathbf{A}_{41} \cos 2\theta + r^{\frac{5}{2}} \left(\mathbf{A}_{51} \cdot \right. \\ & \left. \sin \frac{5\theta}{2} + \frac{1}{20} \beta \mathbf{A}_{31} \cos \frac{5\theta}{2} + \frac{1}{32} \beta^2 \mathbf{A}_{11} \sin \frac{3\theta}{2} + \right. \\ & \left. \frac{1}{48} \beta^2 \mathbf{A}_{11} \sin \frac{\theta}{2} - \frac{1}{4} \beta \mathbf{A}_{31} \cos \frac{\theta}{2} \right) + r^3 \left(\mathbf{A}_{61} \cos 3\theta - \right. \\ & \left. \frac{1}{12} \beta \mathbf{A}_{41} \sin 3\theta + \frac{1}{4} \beta \mathbf{A}_{41} \cos 3\theta \right) + \dots \end{aligned} \quad (12)$$

Then, the stress, electric displacement and magnetic induction components can be obtained

$$\begin{aligned} \tau_{rz} = c_{44} & \left[\frac{1}{2} r^{-\frac{1}{2}} A_{11} \sin \frac{\theta}{2} + A_{21} \cos \theta + \frac{3}{2} r^{\frac{1}{2}} (A_{31} \sin \frac{3\theta}{2} - \right. \\ & \frac{1}{12} \beta A_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta A_{11} \cos \frac{\theta}{2} + 2r A_{41} \cos 2\theta + \\ & \frac{5}{2} r^{\frac{3}{2}} (A_{51} \sin \frac{5\theta}{2} + \frac{1}{20} \beta A_{31} \cos \frac{5\theta}{2} + \frac{1}{32} \beta^2 A_{11} \sin \frac{3\theta}{2} + \\ & \frac{1}{48} \beta^2 A_{11} \sin \frac{\theta}{2} - \frac{1}{4} \beta A_{31} \cos \frac{\theta}{2}) + 3r^2 (A_{61} \cos 3\theta - \\ & \frac{1}{12} \beta A_{41} \sin 3\theta + \frac{1}{4} \beta A_{41} \cos 3\theta) \left. \right] + e_{15} \left[\frac{1}{2} r^{-\frac{1}{2}} B_{11} \sin \frac{\theta}{2} + \right. \\ & B_{21} \cos \theta + \frac{3}{2} r^{\frac{1}{2}} (B_{31} \sin \frac{3\theta}{2} - \frac{1}{12} \beta B_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta \cdot \\ & B_{11} \cos \frac{\theta}{2}) + 2r B_{41} \cos 2\theta + \frac{5}{2} r^{\frac{3}{2}} (B_{51} \sin \frac{5\theta}{2} + \frac{1}{20} \beta \cdot \\ & B_{31} \cos \frac{5\theta}{2} + \frac{1}{32} \beta^2 B_{11} \sin \frac{3\theta}{2} + \frac{1}{48} \beta^2 B_{11} \sin \frac{\theta}{2} - \\ & \frac{1}{4} \beta B_{31} \cos \frac{\theta}{2}) + 3r^2 (B_{61} \cos 3\theta - \frac{1}{12} \beta B_{41} \sin 3\theta + \\ & \frac{1}{4} \beta B_{41} \cos 3\theta) \left. \right] + h_{15} \left[\frac{1}{2} r^{-\frac{1}{2}} C_{11} \sin \frac{\theta}{2} + C_{21} \cos \theta + \right. \\ & \frac{3}{2} r^{\frac{1}{2}} (C_{31} \sin \frac{3\theta}{2} - \frac{1}{12} \beta C_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta C_{11} \cos \frac{\theta}{2}) + \\ & 2r C_{41} \cos 2\theta + \frac{5}{2} r^{\frac{3}{2}} (C_{51} \sin \frac{5\theta}{2} + \frac{1}{20} \beta C_{31} \cos \frac{5\theta}{2} + \\ & \frac{1}{32} \beta^2 C_{11} \sin \frac{3\theta}{2} + \frac{1}{48} \beta^2 C_{11} \sin \frac{\theta}{2} - \frac{1}{4} \beta C_{31} \cos \frac{\theta}{2}) + \\ & \left. 3r^2 (C_{61} \cos 3\theta - \frac{1}{12} \beta C_{41} \sin 3\theta + \frac{1}{4} \beta C_{41} \cos 3\theta) \right] \\ \tau_{\theta z} = c_{44} & \left[\frac{1}{2} r^{-\frac{1}{2}} A_{11} \cos \frac{\theta}{2} - A_{21} \sin \theta + r^{\frac{1}{2}} (\frac{3}{2} A_{31} \cos \frac{3\theta}{2} + \right. \\ & \frac{1}{8} \beta A_{11} \sin \frac{3\theta}{2} + \frac{1}{8} \beta A_{11} \sin \frac{\theta}{2}) - r A_{41} \sin 2\theta + r^{\frac{3}{2}} \cdot \\ & (\frac{5}{2} A_{51} \cos \frac{5\theta}{2} - \frac{1}{8} \beta A_{31} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 A_{11} \cos \frac{3\theta}{2} + \\ & \frac{1}{96} \beta^2 A_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta A_{31} \sin \frac{\theta}{2}) - r^2 (3 A_{61} \sin 3\theta + \\ & \frac{1}{4} \beta A_{41} \cos 3\theta + \frac{3}{4} \beta A_{41} \sin 3\theta) \left. \right] + e_{15} \left[\frac{1}{2} r^{-\frac{1}{2}} B_{11} \cdot \right. \\ & \cos \frac{\theta}{2} - B_{21} \sin \theta + r^{\frac{1}{2}} (\frac{3}{2} B_{31} \cos \frac{3\theta}{2} + \frac{1}{8} \beta B_{11} \cdot \\ & \sin \frac{3\theta}{2} + \frac{1}{8} \beta B_{11} \sin \frac{\theta}{2}) - r B_{41} \sin 2\theta + r^{\frac{3}{2}} (\frac{5}{2} B_{51} \cdot \\ & \cos \frac{5\theta}{2} - \frac{1}{8} \beta B_{31} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 B_{11} \cos \frac{3\theta}{2} + \\ & \left. \frac{1}{96} \beta^2 B_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta B_{31} \sin \frac{\theta}{2}) - r^2 (3 B_{61} \sin 3\theta + \right. \end{aligned}$$

$$\begin{aligned} & \frac{1}{4} \beta B_{41} \cos 3\theta + \frac{3}{4} \beta B_{41} \sin 3\theta) \left. \right] + h_{15} \left[\frac{1}{2} r^{-\frac{1}{2}} C_{11} \cdot \right. \\ & \cos \frac{\theta}{2} - C_{21} \sin \theta + r^{\frac{1}{2}} (\frac{3}{2} C_{31} \cos \frac{3\theta}{2} + \frac{1}{8} \beta C_{11} \cdot \\ & \sin \frac{3\theta}{2} + \frac{1}{8} \beta C_{11} \sin \frac{\theta}{2}) - r C_{41} \sin 2\theta + r^{\frac{3}{2}} (\frac{5}{2} C_{51} \cdot \\ & \cos \frac{5\theta}{2} - \frac{1}{8} \beta C_{31} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 C_{11} \cos \frac{3\theta}{2} + \\ & \frac{1}{96} \beta^2 C_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta C_{31} \sin \frac{\theta}{2}) - r^2 (3 C_{61} \sin 3\theta + \\ & \frac{1}{4} \beta C_{41} \cos 3\theta + \frac{3}{4} \beta C_{41} \sin 3\theta) \left. \right] \\ D_r = e_{15} & \left[\frac{1}{2} r^{-\frac{1}{2}} A_{11} \sin \frac{\theta}{2} + A_{21} \cos \theta + \frac{3}{2} r^{\frac{1}{2}} (A_{31} \sin \frac{3\theta}{2} - \right. \\ & \frac{1}{12} \beta A_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta A_{11} \cos \frac{\theta}{2}) + 2r A_{41} \cos 2\theta + \\ & \frac{5}{2} r^{\frac{3}{2}} (A_{51} \sin \frac{5\theta}{2} + \frac{1}{20} \beta A_{31} \cos \frac{5\theta}{2} + \frac{1}{32} \beta^2 A_{11} \sin \frac{3\theta}{2} + \\ & \frac{1}{48} \beta^2 A_{11} \sin \frac{\theta}{2} - \frac{1}{4} \beta A_{31} \cos \frac{\theta}{2}) + 3r^2 (A_{61} \cos 3\theta - \\ & \frac{1}{12} \beta A_{41} \sin 3\theta + \frac{1}{4} \beta A_{41} \cos 3\theta) \left. \right] - e_{11} \left[\frac{1}{2} r^{-\frac{1}{2}} B_{11} \sin \frac{\theta}{2} + \right. \\ & B_{21} \cos \theta + \frac{3}{2} r^{\frac{1}{2}} (B_{31} \sin \frac{3\theta}{2} - \frac{1}{12} \beta B_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta \cdot \\ & B_{11} \cos \frac{\theta}{2}) + 2r B_{41} \cos 2\theta + \frac{5}{2} r^{\frac{3}{2}} (B_{51} \sin \frac{5\theta}{2} + \frac{1}{20} \beta \cdot \\ & B_{31} \cos \frac{5\theta}{2} + \frac{1}{32} \beta^2 B_{11} \sin \frac{3\theta}{2} + \frac{1}{48} \beta^2 B_{11} \sin \frac{\theta}{2} - \\ & \frac{1}{4} \beta B_{31} \cos \frac{\theta}{2}) + 3r^2 (B_{61} \cos 3\theta - \frac{1}{12} \beta B_{41} \sin 3\theta + \\ & \frac{1}{4} \beta B_{41} \cos 3\theta) \left. \right] - d_{11} \left[\frac{1}{2} r^{-\frac{1}{2}} C_{11} \sin \frac{\theta}{2} + C_{21} \cos \theta + \right. \\ & \frac{3}{2} r^{\frac{1}{2}} (C_{31} \sin \frac{3\theta}{2} - \frac{1}{12} \beta C_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta C_{11} \cos \frac{\theta}{2}) + \\ & 2r C_{41} \cos 2\theta + \frac{5}{2} r^{\frac{3}{2}} (C_{51} \sin \frac{5\theta}{2} + \frac{1}{20} \beta C_{31} \cos \frac{5\theta}{2} + \\ & \frac{1}{32} \beta^2 C_{11} \sin \frac{3\theta}{2} + \frac{1}{48} \beta^2 C_{11} \sin \frac{\theta}{2} - \frac{1}{4} \beta C_{31} \cos \frac{\theta}{2}) + \\ & \left. 3r^2 (C_{61} \cos 3\theta - \frac{1}{12} \beta C_{41} \sin 3\theta + \frac{1}{4} \beta C_{41} \cos 3\theta) \right] \\ D_\theta = e_{15} & \left[\frac{1}{2} r^{-\frac{1}{2}} A_{11} \cos \frac{\theta}{2} - A_{21} \sin \theta + r^{\frac{1}{2}} (\frac{3}{2} A_{31} \cos \frac{3\theta}{2} + \right. \\ & \frac{1}{8} \beta A_{11} \sin \frac{3\theta}{2} + \frac{1}{8} \beta A_{11} \sin \frac{\theta}{2}) - r A_{41} \sin 2\theta + r^{\frac{3}{2}} \cdot \end{aligned}$$

$$\begin{aligned}
& \left(\frac{5}{2} A_{s1} \cos \frac{5\theta}{2} - \frac{1}{8} \beta A_{s1} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 A_{11} \cos \frac{3\theta}{2} + \right. \\
& \frac{1}{96} \beta^2 A_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta A_{s1} \sin \frac{\theta}{2} \left. - r^2 (3 A_{61} \sin 3\theta + \right. \\
& \left. \frac{1}{4} \beta A_{41} \cos 3\theta + \frac{3}{4} \beta A_{41} \sin 3\theta) \right] - \varepsilon_{11} \left[\frac{1}{2} r^{-\frac{1}{2}} B_{11} \cdot \right. \\
& \cos \frac{\theta}{2} - B_{21} \sin \theta + r^2 \left(\frac{3}{2} B_{31} \cos \frac{3\theta}{2} + \frac{1}{8} \beta B_{11} \cdot \right. \\
& \sin \frac{3\theta}{2} + \frac{1}{8} \beta B_{11} \sin \frac{\theta}{2} \left. - r B_{41} \sin 2\theta + r^2 \left(\frac{5}{2} B_{s1} \cdot \right. \right. \\
& \left. \left. \cos \frac{5\theta}{2} - \frac{1}{8} \beta B_{31} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 B_{11} \cos \frac{3\theta}{2} + \right. \right. \\
& \left. \left. \frac{1}{96} \beta^2 B_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta B_{31} \sin \frac{\theta}{2} \right) - r^2 (3 B_{61} \sin 3\theta + \right. \\
& \left. \frac{1}{4} \beta B_{41} \cos 3\theta + \frac{3}{4} \beta B_{41} \sin 3\theta) \right] - d_{11} \left[\frac{1}{2} r^{-\frac{1}{2}} C_{11} \cdot \right. \\
& \cos \frac{\theta}{2} - C_{21} \sin \theta + r^2 \left(\frac{3}{2} C_{31} \cos \frac{3\theta}{2} + \frac{1}{8} \beta C_{11} \cdot \right. \\
& \sin \frac{3\theta}{2} + \frac{1}{8} \beta C_{11} \sin \frac{\theta}{2} \left. - r C_{41} \sin 2\theta + r^2 \left(\frac{5}{2} C_{s1} \cdot \right. \right. \\
& \left. \left. \cos \frac{5\theta}{2} - \frac{1}{8} \beta C_{31} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 C_{11} \cos \frac{3\theta}{2} + \right. \right. \\
& \left. \left. \frac{1}{96} \beta^2 C_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta C_{31} \sin \frac{\theta}{2} \right) - r^2 (3 C_{61} \sin 3\theta + \right. \\
& \left. \frac{1}{4} \beta C_{41} \cos 3\theta + \frac{3}{4} \beta C_{41} \sin 3\theta) \right]
\end{aligned}$$

$$\begin{aligned}
H_r = h_{15} & \left[\frac{1}{2} r^{-\frac{1}{2}} A_{11} \sin \frac{\theta}{2} + A_{21} \cos \theta + \frac{3}{2} r^{\frac{1}{2}} (A_{s1} \sin \frac{3\theta}{2} - \right. \\
& \frac{1}{12} \beta A_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta A_{11} \cos \frac{\theta}{2} + 2 r A_{41} \cos 2\theta + \\
& \frac{5}{2} r^{\frac{3}{2}} (A_{s1} \sin \frac{5\theta}{2} + \frac{1}{20} \beta A_{s1} \cos \frac{5\theta}{2} + \frac{1}{32} \beta^2 A_{11} \sin \frac{3\theta}{2} + \\
& \frac{1}{48} \beta^2 A_{11} \sin \frac{\theta}{2} - \frac{1}{4} \beta A_{s1} \cos \frac{\theta}{2} \left. + 3 r^2 (A_{61} \cos 3\theta - \right. \\
& \frac{1}{12} \beta A_{41} \sin 3\theta + \frac{1}{4} \beta A_{41} \cos 3\theta) \left. - d_{11} \left[\frac{1}{2} r^{-\frac{1}{2}} B_{11} \sin \frac{\theta}{2} + \right. \right. \\
& B_{21} \cos \theta + \frac{3}{2} r^{\frac{1}{2}} (B_{s1} \sin \frac{3\theta}{2} - \frac{1}{12} \beta B_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta \cdot \\
& B_{11} \cos \frac{\theta}{2} + 2 r B_{41} \cos 2\theta + \frac{5}{2} r^{\frac{3}{2}} (B_{s1} \sin \frac{5\theta}{2} + \frac{1}{20} \beta \cdot \\
& B_{s1} \cos \frac{5\theta}{2} + \frac{1}{32} \beta^2 B_{11} \sin \frac{3\theta}{2} + \frac{1}{48} \beta^2 B_{11} \sin \frac{\theta}{2} - \\
& \frac{1}{4} \beta B_{s1} \cos \frac{\theta}{2} \left. + 3 r^2 (B_{61} \cos 3\theta - \frac{1}{12} \beta B_{41} \sin 3\theta + \right. \\
& \frac{1}{4} \beta B_{41} \cos 3\theta) \left. - \mu_{11} \left[\frac{1}{2} r^{-\frac{1}{2}} C_{11} \sin \frac{\theta}{2} + C_{21} \cos \theta + \right. \right. \\
& \frac{3}{2} r^{\frac{1}{2}} (C_{s1} \sin \frac{3\theta}{2} - \frac{1}{12} \beta C_{11} \cos \frac{3\theta}{2} - \frac{1}{4} \beta C_{11} \cos \frac{\theta}{2} \left. + \right. \\
& 2 r C_{41} \cos 2\theta + \frac{5}{2} r^{\frac{3}{2}} (C_{s1} \sin \frac{5\theta}{2} + \frac{1}{20} \beta C_{s1} \cos \frac{5\theta}{2} + \\
& \frac{1}{32} \beta^2 C_{11} \sin \frac{3\theta}{2} + \frac{1}{48} \beta^2 C_{11} \sin \frac{\theta}{2} - \frac{1}{4} \beta C_{s1} \cos \frac{\theta}{2} \left. + \right. \\
& \left. 3 r^2 (C_{61} \cos 3\theta - \frac{1}{12} \beta C_{41} \sin 3\theta + \frac{1}{4} \beta C_{41} \cos 3\theta) \right]
\end{aligned}$$

$$\begin{aligned}
 H_{\theta} = & h_{15} \left[\frac{1}{2} r^{-\frac{1}{2}} A_{11} \cos \frac{\theta}{2} - A_{21} \sin \theta + r^2 \left(\frac{3}{2} A_{31} \cos \frac{3\theta}{2} + \right. \right. \\
 & \left. \frac{1}{8} \beta A_{11} \sin \frac{3\theta}{2} + \frac{1}{8} \beta A_{11} \sin \frac{\theta}{2} \right) - r A_{41} \sin 2\theta + r^2 \cdot \\
 & \left(\frac{5}{2} A_{51} \cos \frac{5\theta}{2} - \frac{1}{8} \beta A_{31} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 A_{11} \cos \frac{3\theta}{2} + \right. \\
 & \left. \frac{1}{96} \beta^2 A_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta A_{31} \sin \frac{\theta}{2} \right) - r^2 (3 A_{61} \sin 3\theta + \\
 & \left. \frac{1}{4} \beta A_{41} \cos 3\theta + \frac{3}{4} \beta A_{41} \sin 3\theta) \right] - d_{11} \left[\frac{1}{2} r^{-\frac{1}{2}} B_{11} \cdot \right. \\
 & \left. \cos \frac{\theta}{2} - B_{21} \sin \theta + r^2 \left(\frac{3}{2} B_{31} \cos \frac{3\theta}{2} + \frac{1}{8} \beta B_{11} \cdot \right. \right. \\
 & \left. \left. \sin \frac{3\theta}{2} + \frac{1}{8} \beta B_{11} \sin \frac{\theta}{2} \right) - r B_{41} \sin 2\theta + r^2 \left(\frac{5}{2} B_{51} \cdot \right. \right. \\
 & \left. \left. \cos \frac{5\theta}{2} - \frac{1}{8} \beta B_{31} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 B_{11} \cos \frac{3\theta}{2} + \right. \right. \\
 & \left. \left. \frac{1}{96} \beta^2 B_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta B_{31} \sin \frac{\theta}{2} \right) - r^2 (3 B_{61} \sin 3\theta + \right. \\
 & \left. \left. \frac{1}{4} \beta B_{41} \cos 3\theta + \frac{3}{4} \beta B_{41} \sin 3\theta) \right] - \mu_{11} \left[\frac{1}{2} r^{-\frac{1}{2}} C_{11} \cdot \right. \right. \\
 & \left. \left. \cos \frac{\theta}{2} - C_{21} \sin \theta + r^2 \left(\frac{3}{2} C_{31} \cos \frac{3\theta}{2} + \frac{1}{8} \beta C_{11} \cdot \right. \right. \right. \\
 & \left. \left. \sin \frac{3\theta}{2} + \frac{1}{8} \beta C_{11} \sin \frac{\theta}{2} \right) - r C_{41} \sin 2\theta + r^2 \left(\frac{5}{2} C_{51} \cdot \right. \right. \\
 & \left. \left. \cos \frac{5\theta}{2} - \frac{1}{8} \beta C_{31} \sin \frac{5\theta}{2} + \frac{3}{64} \beta^2 C_{11} \cos \frac{3\theta}{2} + \right. \right. \\
 & \left. \left. \frac{1}{96} \beta^2 C_{11} \cos \frac{\theta}{2} + \frac{1}{8} \beta C_{31} \sin \frac{\theta}{2} \right) - r^2 (3 C_{61} \sin 3\theta + \right. \\
 & \left. \left. \frac{1}{4} \beta C_{41} \cos 3\theta + \frac{3}{4} \beta C_{41} \sin 3\theta) \right] \right] \quad (13)
 \end{aligned}$$

The mode III stress intensity factor (SIF) of the crack tip are defined as

$$\begin{cases}
 K^I = \lim_{r \rightarrow 0} \sqrt{2\pi r} \tau_{\theta z}(r, 0) = \frac{\sqrt{2\pi}}{2} (c_{440} A_{11} + e_{150} B_{11} + h_{150} C_{11}) \\
 K^D = \lim_{r \rightarrow 0} \sqrt{2\pi r} D_{\theta}(r, 0) = \frac{\sqrt{2\pi}}{2} (e_{150} A_{11} - \varepsilon_{110} B_{11} - d_{110} C_{11}) \\
 K^B = \lim_{r \rightarrow 0} \sqrt{2\pi r} B(r, 0) = \frac{\sqrt{2\pi}}{2} (h_{150} A_{11} - d_{110} B_{11} - \mu_{110} C_{11})
 \end{cases} \quad (14)$$

IV. CONCLUSION

The crack-tip higher order fields of magneto-electroelastic materials are obtained by the eigen-expansion method in this paper. The first two items of crack-tip higher order fields have the same mathematical form as ones of homogeneous materials. The effect of nonhomogeneity reflects only in the higher order items. Due to coupling effect of magneto-electroelastic material, crack-tip higher order fields are depending on displacement component, the electric potential and magnetic potential. The crack tip higher-order fields are equivalent to the eigen-functions mathematically. Therefore, they provided the theoretical basis for the numerical simulations, engineering applications, experimental analysis of crack problems of magneto-electroelastic materials

ACKNOWLEDGMENT

The research is supported by the National Natural Science Foundation of China (No1172332.).

REFERENCES

- [1] Wang B L, Han J C, Mai Y W, "Mode Iii Fracture of a Magneto-electroelastic Layer: Exact Solution and Discussion of the Crack Face Electromagnetic Boundary Conditions," *International Journal of Fracture*, vol.139, 2006, pp. 27-38, doi:10.1007/s10704-006-6632-1.
- [2] Wan Y, Yue Y, Zhong Z, "A Mode III Crack Crossing the Magneto-electroelastic Bimaterial Interface under Concentrated Magneto-electromechanical Loads," *International Journal of Solids and Structures*, vol.49, 2012, pp. 3008-3021, doi:10.1016/j.ijsolstr.2012.06.001.
- [3] Zhong X-C, Li X-F, "Magneto-electroelastic Analysis for an Opening Crack in a Piezoelectromagnetic Solid," *European Journal of Mechanics - A/Solids*, vol.26, 2007, pp. 405-417, doi:10.1016/j.euromechsol.2006.08.002.
- [4] Huang G Y, Wang B L, Mai Y W, "Effect of Interfacial Cracks on the Effective Properties of Magneto-electroelastic Composites," *Journal of Intelligent Material Systems and Structures*, vol.20, 2009, pp. 963-968, doi:10.1177/1045389x08101564.
- [5] Gao C-F, Kessler H, Balke H, "Crack Problems in Magneto-electroelastic Solids. Part Ii: General Solution of Collinear Cracks," *International Journal of Engineering Science*, vol.41, 2003, pp. 983-994, doi: 10.1016/S0020-7225(02)00324-5.
- [6] Sih G C, Jones R, Song Z F, "Piezomagnetic and Piezoelectric Poling Effects on Mode I and Ii Crack Initiation Behavior of Magneto-electroelastic Materials," *Theoretical and Applied Fracture Mechanics*, vol.40, 2003, pp. 161-186, doi:10.1016/s0167-8442(03)00044-2.
- [7] Li Y-D, Lee K Y, "Fracture Analysis on a Piezoelectric Sensor with a Viscoelastic Interface," *European Journal of Mechanics*, vol.28, 2009, pp. 738-743, doi:10.1016/j.euromechsol.2009.02.003.
- [8] Athanasius L, Ang W T, "Magneto-electroelastodynamic Interaction of Multiple Arbitrarily Oriented Planar Cracks," *Applied Mathematical Modelling* 2013, pp. doi: 10.1016/j.apm.2013.02.013.
- [9] Bhargava R R, Sharma K, "Application of X-Fem to Study Two-Unequal-Collinear Cracks in 2-D Finite Magneto-electroelastic Specimen," *Computational Materials Science*, vol.60, 2012, pp. 75-98, doi: 10.1016/j.commatsci.2012.03.013.
- [10] Topholme G E, "Magneto-electroelastic Media Containing a Row of Moving Shear Cracks," *Mechanics Research Communications*, vol.45, 2012, pp. 48-53, doi: 10.1016/j.mechrescom.2012.07.002.
- [11] Wang B L, Mai Y W, "Crack Tip Field in Piezoelectric/Piezomagnetic Media," *European Journal of Mechanics a-Solids*, vol.22, 2003, pp. 591-602, doi:10.1016/s0997-7538(03)0062-7.
- [12] Zheng J L, Fang Q H, Liu Y W, "A Generalized Screw Dislocation Interacting with Interfacial Cracks Along a Circular Inhomogeneity in Magneto-electroelastic Solids," *Theoretical and Applied Fracture Mechanics*, vol.47, 2007, pp. 205-218, doi:10.1016/j.tafmec.2007.01.005.
- [13] Ma P, Feng W J, Su R K L, "An Electrically Impermeable and Magnetically Permeable Interface Crack with a Contact Zone in a Magneto-electroelastic Bimaterial under Uniform Magneto-electromechanical Loads," *European Journal of Mechanics - A/Solids*, vol.32, 2012, pp. 41-51, doi: 10.1016/j.euromechsol.2011.09.010.
- [14] Li X F, Liu G L, Lee K Y, "Magneto-electroelastic Field Induced by a Crack Terminating at the Interface of a Bi-Magneto-electric Material," *Philosophical Magazine*, vol.89, 2009, pp. 449-463, doi:10.1080/14786430802653428.