Research on Robust Optimal Power Flow of VSC-MTDC AC / DC Uncertain System

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Abstract. For the purpose of large-scale wind power accommodation in a voltage source converter based multi-terminal high voltage direct current (VSC-MTDC) AC/DC system, this study proposed a robust optimal power flow strategy for VSC-MTDC AC/DC uncertain systems. The strategy is suitable for long timescale application while considering the uncertainty in the power production of wind farms. The optimization mathematical model was established using cassette uncertainty set, which takes economy and security metrics into account. The economically optimal decision on limit maximum conservative solutions for all uncertain sets is obtained using the Differential Evolution and Predictor-Corrector Primal-Dual Interior Point Method (DE-PCPDIPM) hybrid optimization algorithm. Wind power transmission efficiency and effectiveness in MTDC is improved by the optimization of VSC control command value and cooperative control among converter stations. The study also proposed to alleviate wind power accommodation problem effectively through coordinated optimization with AC power grid dispatching. The effectiveness of the robust optimization strategy was verified on the IEEE 14-bus system, and the effect of parameter uncertainty range on the robust optimization decision is analyzed. The results showed the improvement in economy operation of the VSC-MTDC AC/DC uncertain system.

1 Introduction

Due to the inverse distribution of energy resources and load allocation in China, the limit of peak load regulation capacity, transmission capacity and relatively weak grid structure in new energy base, a proportion of the available wind power which cannot be consumed locally, has been abandoned during the operation. Consequently, the efficiency level of the wind power development can be seriously influenced [1]. The multi-terminal flexible direct current transmission based on voltage source converter (VSC-MTDC) is an effective solution for the problem of wind power consumption. However, previous studies on VSC-MTDC mainly focused on the control strategy of VSC, whereas research on the optimal operation of VSC-MTDC AC-DC power system to improve wind power consumption has not gained sufficient attention. For example, instruction value for VSC power or voltage control is determined through operation experience in general, which is lack of certain basis [2].

Traditional study on system optimization in electric power system with wind farms is mostly based on the short-term wind power forecast, but the randomness and volatility of wind energy can cause significant error in the accuracy of wind power prediction. Uncertainty in wind power is ignored when optimization strategy applies predicted wind power as an input, hence the random fluctuations of actual wind power which impact economy and safe operation of system cannot be coped with timely. Robust optimization is an effective method for dealing with uncertain ties, which is committed to solve system optimization problem with uncertain parameters. With the robust optimization strategy the solution of the decision –variable must strictly meets the system operating constrains under the uncertainty set, so results are partly conservative inevitably [3].

By combining the VSC distributed voltage control with AC power grid scheduling optimization, the strategy proposed in this article enhances the capacity of wind power consumption. A robust optimal power flow method of VSC-MTDC AC / DC system based on long time scale is proposed considering the uncertainty of wind farm output. The method is capable of reducing the conservatism of the robust decision, and ensuring the economy level of decision-making plan under the long time scales. An unified iterative hybrid optimization algorithm is used to deal with the robust optimization mathematical model of the VSC-MTDC AC / DC uncertain system, which is based on the Differential Evolution(DE) and Predictor-Corrector Primal-Dual Interior Point Method(PCPDIPM).The algorithm provides suitable reference value for VSC voltage and power control, and thus realizes the economic and safe operation of VSC-MTDC AC/DC system with the uncertain wind power conditions.

2 Key Problem Research on Wind Power Consumption

Factors affecting wind power consumption mainly include: 1) consumption level, e.g., the technology performance of wind power grid connection, grid dispatching operation level, etc; 2) consumption ability, e.g., the system adjustment ability, and power grid transmission capacity [4].

As wind power is clean and renewable energy, and conventional thermal power generating cost is higher, grid scheduling optimization can be used to properly regulate the percentage of two types of energy resources, hence improving the economic operation level of the system.

To sum up, VSC coordination control and optimal instruction value improve the transmission capacity and efficiency of MTDC; combined with scheduling optimization of AC power grid, it also can effectively increase wind power consumption.

Cooperative control among converter stations adopts a distributed voltage control method [5]-[6]. VSC rectifier side connect large-scale wind farms and DC power transmission system, which is controlled in fixed AC power control mode since the wind farm output changes with wind speed fluctuations. Active power instruction value of the VSC rectifier side is consistent with the wind farm output, while power losses of transmission line and converter are ignored, thus all wind farm output can be transferred. VSC inverter side connects DC and AC network. DC power flow characteristics are decided by DC voltage distribution of MTDC, and transmission losses are decided by voltage difference between buses. For MTDC network power balance, VSC rectifier side and inverter side are set on fixed DC voltage control.

In order to secure the stability of AC system voltage, VSC inverter side is set to fixed reactive power control.

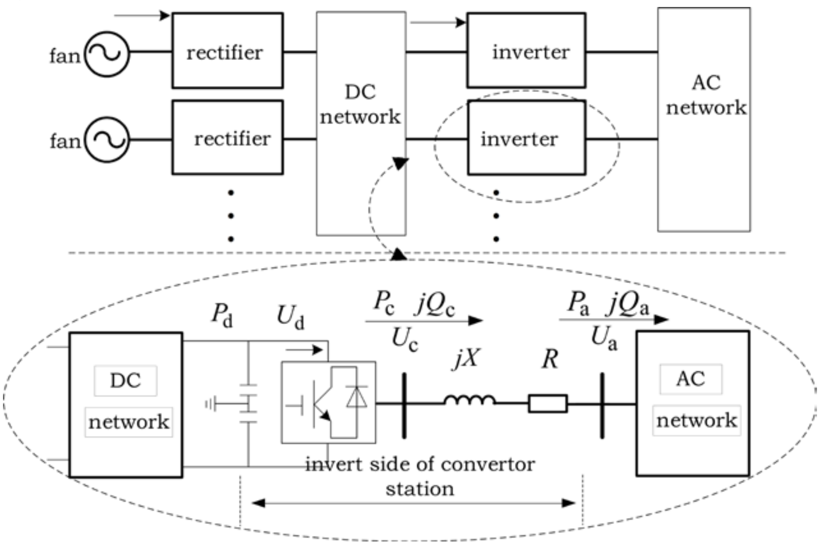

Fig. 1. Structure and local map of VSC-MTDC Grid-connected wind power.

In Fig. 1, VSC power loss is made equivalent change, then overlaid on the power loss P_Δ of converter reactor, represented as equivalent resistance R of converter station loss. Active power flowing out from the inverter is equal to which flowing in when under normal operation, that is:

$$P_{\mathrm{d}} = P_{\mathrm{c}} \tag{1}$$

$$P_{\mathrm{c}} = P_{\mathrm{a}} + P_{\Delta} \tag{2}$$

$$U_{\mathrm{c}} = \mu_{\mathrm{d}} M U_{\mathrm{d}} / \sqrt{2} \tag{3}$$

Voltage utilization rate μ_{d} is related to the modulation ratio M and modulation mode.

3 Fuzzy Multi-Objective Optimization Model of VSC-MTDC AC/DC System

A. Economic optimization model of VSC - MTDC AC/DC system

1) Objective function of transmission loss: Transmission losses of VSC - MTDC AC/DC system include loss of VSC - MTDC and AC power loss, the former mainly includes converter equipment transmission losses, DC transmission line losses, etc. The latter is mainly AC transmission line loss, AC transformer loss, etc.

Optimization model of VSC - MTDC loss is shown as follow, assuming the loss is proportional to transmission capacity [7]:

$$\Delta P_{\mathrm{dc}} = \sum_{i=1}^{m} S_i \kappa\% + \sum_{j=1}^{N_{\mathrm{d}}} \sum_{k=j}^{N_{\mathrm{d}}} \frac{(U_j - U_k)^2}{R_{jk}} \tag{4}$$

Optimization model of AC transmission loss is

$$\Delta P_{\mathrm{ac}} = \sum_{i=1}^{N_{\mathrm{a}}} U_i \sum_{j \in i} U_j G_{ij} \cos\theta_{ij} \tag{5}$$

In conclusion, objective function of VSC - MTDC AC/DC system transmission losses is:

$$\min\ f_1 = \sum_{i=\mathrm{ac,dc}} \Delta P_i \tag{6}$$

2) Objective function of grid scheduling: Power grid scheduling sets minimum generator power cost as objective function. Wind power is renewable clean energy, so generating cost of turbine can be ignored and only AC generator cost is considered. Through the fitting of a quadratic function, the formula can be expressed as follow:

$$\min\ f_2 = \sum_{i=1}^{n_{\mathrm{g}}} (a_{2i} P_{\mathrm{G}i}^2 + a_{1i} P_{\mathrm{G}i} + a_{0i}) \tag{7}$$

B. Security optimization model of VSC - MTDC AC/DC system

Node voltage in electric power network should be stable within a reasonable range, the degree deviating from the rated voltage is set as punishment object:

$$\min\ f_3 = \sum_{i=1}^{N_{\mathrm{d}}} \left| \frac{U_i - U_{\mathrm{dN}}}{U_i} \right| + \sum_{j=1}^{N_{\mathrm{a}}} \left| \frac{U_j - U_{\mathrm{aN}}}{U_j} \right| \tag{8}$$

C. Obfuscation of VSC - MTDC AC/DC system multi-objective optimization model

Dimension of the multiple optimization objective function above is different, and the numerical difference is big. Hence, the simple weighted process will lead the optimization to focus on the target that has larger function value. This article uses fuzzy optimization theory to deal with above function. It transformed each objective function to membership function between 0 and 1, and then weighted method can be used to transform it into single objective optimization model which is easy to solve. The membership functions of objective function f_1、f_2、f_3 are shown as following piecewise linear functions:

$$\eta(f_i) = \begin{cases} 1 & f_i \ge f_i^{\max} \\ 1 - \dfrac{f_i^{\max} - f_i(x)}{f_i^{\max} - f_i^{\min}} & f_i^{\min} \le f_i \le f_i^{\max} \quad i = 1,2,... \\ 0 & f_i < f_i^{\min} \end{cases} \tag{9}$$

In (9), the value of the membership functions reflects the value of the original objective function indirectly. According to the characteristics of membership functions and objective functions in (6)- (8): indexes as small as possible, then the system fuzzy optimal objective function is shown as:

$$\min F = \sum_{i=1}^{3} \alpha_i \eta(f_i) \tag{10}$$

α_i reflects the importance of the membership functions (i.e., the original objective function), and $\sum_i \alpha_i = 1$。

4 Robust Optimization Mathematical Model of VSC-MTDC AC-DC Uncertain Systems Based on The Long Time Scales

A．Cassette modeling of uncertain wind farm output

Conventional wind speed forecasting result is predictive value at a certain time instant, which changes over time. In experiments, we can use Weibull random number generator to simulate forecasted wind speed:

$$v_i = c(-\ln x_i)^{1/k} \tag{11}$$

The relationship between v and $P'(v)$ can be approximately expressed as[8] :

$$P'(v) = \begin{cases} 0 & v < v_{\text{ci}} \ or \ v > v_{\text{co}} \\ k_1 v + k_2 & v_{\text{ci}} \le v \le v_{\text{rate}} \\ P'_{\text{rate}} & v_{\text{rate}} < v < v_{\text{co}} \end{cases} \tag{12}$$

From (12), we can see that turbine output $P'(v)$ and wind speed v are nonlinearly related, if and only if $v_{\text{ci}} \le v \le v_{\text{rate}}$, turbine output is affected by wind speed. That is to say, velocity uncertainty has an influence on turbine output only when the wind speed is in the above range.

For the actual large-scale wind farms, the total wind power fluctuations will greatly reduce when large number of turbines are connected to grid. The large-scale centralized wind farm output model can be represented as:

$$P = \frac{1}{n_{\text{w}}} \sum_{i=1}^{n_{\text{w}}} P'(v_i) \tag{13}$$

The greater the n_{w}, the steadier the wind power that wind farm produces.

The actual wind speed is uncertain, quantitative modeling of uncertainty parameters is an important component of the robust optimization. Cassette model of uncertain parameters is simple, intuitive and easy to handle, can be used to describe the error between the predicted values and the actual value [8]. By adding uncertain factors on the basis of wind speed forecasting data, wind speed under any moment changes in an uncertain neighborhood based on forecast wind speed at that point. Bounded cassette uncertainty set of the actual wind speed is shown in the following formula:

$$V = \left\{ v \in R^{N_T}, \sum_{i \in N_T} \frac{|v_i - v_i^{fix}|}{\hat{v}_i} \le \Delta, v_i \in [v_i - \hat{v}_i, v_i + \hat{v}_i], \forall i \in N_T \right\} \tag{14}$$

Mathematical model of uncertain wind farm output can be expressed as:

$$W = \{P \in R^{N_T} : P_i(v_i^{fix} D_i^-) \le P_i \le P_i(v_i^{fix} D_i^+), \forall i \in N_T\} \tag{15}$$

B．Robust optimal mathematical model of VSC-MTDC AC / DC uncertain system based on long time scale

In previous research work[9]-[11], robust optimization was generally summarized as min-max optimal idea in mathematics, that is, optimal solution of objective function value satisfies limiting case of system under uncertain parameter set. Without considering effect of uncertainty parameters on the decision of the system under long time scales, and in order to meet the limiting case within uncertain set, the decision results are quite conservative.

Cost of MTDC system will increase because of the uncertainty of wind power output for a long time scale, in other words, the economy of system is reduced and the safe operation of the system is impacted. To reduce the conservatism of the decision quantity, and guarantee decision-making plan to maintain a certain economic level in the long time scales, there are two core points about robust optimization in this article: 1). Seeking an ultimate maximum conservative solution in the range of uncertain wind power output by conservative game theory at any time, so as to make optimal results satisfy system cost under the worst condition; 2). In a long time scale, uncertain wind power output is decided rationally, risk game theory is used to improve the economy of system and to seek economic optimal solution on the premise of safe operation. That is, value of independent variable is economic optimum result which comes from the ultimate maximum conservative solutions in the long time scale within all uncertain sets satisfied optimal objective function. Robust optimization model of VSC-MTDC AC/DC uncertain system based on long time scales can be expressed as (16), it can be summarized as min-max-min optimal theory in mathematics.

$$\begin{aligned} &\min_{t\in T}\max_{P\in W}\min F(\boldsymbol{x}, P) \\ &\text{s.t.}\quad \boldsymbol{H}(\boldsymbol{x}, P)=0 \\ &\qquad \boldsymbol{G}(\boldsymbol{x}, P)\le 0 \end{aligned} \tag{16}$$

$\boldsymbol{x}=[P_{\mathrm{G}},Q_{\mathrm{G}},\theta_{\mathrm{a}},U_{\mathrm{a}},P_{\mathrm{c}},Q_{\mathrm{c}},U_{\mathrm{d}}]^{\mathrm{T}}$ includes active power and reactive power of AC generator, voltage phase and amplitude of AC node, controlled quantity of VSC active and reactive power, and the DC bus voltage in MTDC network.

1) *Equality Constraints:* For pure AC nodes, the power equation is:

$$\begin{aligned} \Delta P_{\mathrm{a}i} &= P_{\mathrm{G}i}-U_{\mathrm{a}i}\sum_{j\in i}U_{\mathrm{a}j}(G_{\mathrm{a}ij}\cos\theta_{\mathrm{a}ij}+B_{\mathrm{a}ij}\sin\theta_{\mathrm{a}ij})-P_{\mathrm{D}i} \\ \Delta Q_{\mathrm{a}i} &= Q_{\mathrm{G}i}-U_{\mathrm{a}i}\sum_{j\in i}U_{\mathrm{a}j}(G_{\mathrm{a}ij}\sin\theta_{\mathrm{a}ij}-B_{\mathrm{a}ij}\cos\theta_{\mathrm{a}ij})-Q_{\mathrm{D}i} \end{aligned} \tag{17a}$$

Combined with the steady-state power inverter features such as (4), node power correction equation of inverter is:

$$\begin{aligned} \Delta P_{\mathrm{c}i} &= -U_{\mathrm{c}i}U_{\mathrm{a}j}(G_{\mathrm{c}ij}\cos\theta_{\mathrm{c}ij}+B_{\mathrm{c}ij}\sin\theta_{\mathrm{c}ij})+P_{\mathrm{d}i} \\ \Delta Q_{\mathrm{c}i} &= -U_{\mathrm{c}i}U_{\mathrm{a}j}(G_{\mathrm{c}ij}\sin\theta_{\mathrm{c}ij}-B_{\mathrm{c}ij}\cos\theta_{\mathrm{c}ij})+Q_{\mathrm{c}i} \end{aligned} \tag{17b}$$

For symmetric DC grounding MTDC network:

$$\Delta P_{\mathrm{d}i} = P_{\mathrm{d}i}-U_{\mathrm{d}i}\sum_{j\in i}Y_{\mathrm{d}ij}(U_{\mathrm{d}i}-U_{\mathrm{d}i}) \tag{17c}$$

Voltage of AC and DC nodes that VSC connect satisfy modulation relation as following:

$$\Delta U_{\mathrm{c}i} = U_{\mathrm{c}i}-\mu M U_{\mathrm{c}i}/\sqrt{2} \tag{17d}$$

2) *Inequality Constraints:* Turbine output restrictions in DC network:

$$P_{\mathrm{dG}\min}\le P_{\mathrm{dG}}\le P_{\mathrm{dG}\max} \tag{18a}$$

Generator output restrictions in AC network:

$$\begin{aligned} P_{\mathrm{aG}\min}&\le P_{\mathrm{aG}}\le P_{\mathrm{aG}\max} \\ Q_{\mathrm{aG}\min}&\le Q_{\mathrm{aG}}\le Q_{\mathrm{aG}\max} \end{aligned} \tag{18b}$$

Node voltage and branch power restrictions in AC and DC network:

$$\begin{aligned} U_{\min}&\le U\le U_{\max} \\ P_{l\min}&\le P_l\le P_{l\max} \end{aligned} \tag{18c}$$

Converter transformer ratio and converter station capacity restrictions:

$$\begin{aligned} K_{\min}&\le K\le K_{\max} \\ P_{\mathrm{c}\min}&\le P_{\mathrm{c}}\le P_{\mathrm{c}\max} \\ Q_{\mathrm{c}\min}&\le Q_{\mathrm{c}}\le Q_{\mathrm{c}\max} \end{aligned} \tag{18d}$$

5 Solution of Robust Optimization Model of VSC-MTDC AC-DC Uncertain System

According to robust optimization core idea in VSC - MTDC AC-DC uncertain systems, solution of robust optimization model shown in (16) is divided into two steps: first step, max-min solution, called evaluation part of robust structure. In the condition of meeting all restrictions, seeking the biggest pay as well as the control variable of system within wind power uncertain set that point-in-time corresponding through the evaluation of the objective function, to guarantee system control quantity can adapt to system cost in worst case; Step 2, min - max solution, called decision making part of the robust structure. From the point of long time scale, wind power output is formed by a number of wind power uncertain set over time arrangement. Robust decision is selecting economic optimal result in a long time scale, and policymakers must be responsible for the consequences of that decision.

Robust evaluation of VSC - MTDC AC-DC uncertain system is solution of large scale non-linear optimization problem composed of multiple constraint conditions and actual operating variables. This paper combines the global optimization of DE algorithm with strong convergence, fast rate of convergence and strong ability to deal with continuous variables of PCPDIPM algorithm, evaluates and solves robust optimization model shown in (16) by using hybrid optimization algorithm DE - PCPDIPM. DE algorithm makes robust evaluation of uncertain wind field output in uncertain interval as the outer loop framework, seeking system pay for the worst case. In every iteration of DE algorithm, PCPDIPM solves power flow constraint mathematical model of VSC - MTDC AC/DC uncertain system as the inner loop. Setting (10) as objective function to realize optimization of distributed voltage control instruction value of VSC and control variables of AC system.

A. Solution of robust evaluation

1) Robust evaluation by outer loop DE algorithm [12]-[13]: Searching randomly within uncertainty sets of wind farm output through DE algorithm, to seek the wind output in the worst case as the input of the system.

a) Initialization: DE algorithm generates initial population of uncertain wind filed output $\boldsymbol{P}$=[P_1, P_2,...P_k] randomly:

$$\{\boldsymbol{P}_i(0) \mid \boldsymbol{P}_i(0)=[P_{i,1},P_{i,2},...P_{i,k}], i=1,2,...N_P\} \tag{19}$$

b) Mutation: Randomly selecting two distinct individuals $\boldsymbol{P}_{r2}(g)$、$\boldsymbol{P}_{r3}(g)$ from the parent population, then synthesizing differential vector of them after scaled and individuals $\boldsymbol{P}_{r1}(g)$ to be mutant, to produce new mutant individual $\boldsymbol{V}_i(g+1)$.Mutation can be expressed by the following formula:

$$\boldsymbol{V}_i(g+1)=\boldsymbol{P}_{r1}(g)+SF\cdot(\boldsymbol{P}_{r2}(g)-\boldsymbol{P}_{r3}(g)), i\neq r1\neq r2\neq r3 \tag{20}$$

c) Crossover: To gain on optimal objective solution, DE algorithm cross mutant vector $\boldsymbol{V}_i(g+1)$ and target vector $\boldsymbol{P}_i(g)$ at a certain probability, to gain g+1-th generation trial individual vector $\boldsymbol{Q}_i(g+1)$=[$Q_{i,1}$,$Q_{i,2}$,…$Q_{i,k}$], j-th component of it is expressed as:

$$Q_{i,j}(g+1)=\begin{cases}V_{i,j}(g+1) & \text{rand}(j)\leq CR \text{ or } j=j_{\text{rand}}\\ P_{i,j}(g) & \text{otherwise}\end{cases} \tag{21}$$

j_{rand} ensures that at least one dimension variable of new trial vector is contributed by the mutation vector.

d) Selection: System fuzzy multi-objective function min$F$ shown in (10) is chosen as DE fitness function, individual which fitness function value is bigger will serve as a new generation target vector for calculation.

$$\boldsymbol{P}_i(g+1)=\begin{cases}\boldsymbol{Q}_i(g+1) & F(\boldsymbol{Q}_i(g+1))\leq F(\boldsymbol{P}_i(g))\\ \boldsymbol{P}_i(g) & \text{otherwise}\end{cases} \tag{22}$$

e) Termination conditions: Stopping the search when the number of iterations g exceeds the maximum number G_m or accuracy of solution meets the requirement.

2) Optimization of system power flow variable by inner loop PCPDIPM [14]-[15]:

a) Construct Lagrange function: Inequality constraints transformed into equality constraints through slack variable:

$$\begin{aligned}\boldsymbol{G}(\boldsymbol{x})+\boldsymbol{u}&=\overline{\boldsymbol{G}}\\ \boldsymbol{G}(\boldsymbol{x})-\boldsymbol{l}&=\underline{\boldsymbol{G}}\end{aligned} \tag{23}$$

Slack variables are constrained by barrier function, and Lagrange function of original objective function is constructed by Lagrange multiplier method:

$$L(\boldsymbol{x},\boldsymbol{y},\boldsymbol{l},\boldsymbol{z},\boldsymbol{u},\boldsymbol{w},\mu) = F(\boldsymbol{x}) - \boldsymbol{y}^{\mathrm{T}}\boldsymbol{H}(\boldsymbol{x}) - \boldsymbol{w}^{\mathrm{T}}[\boldsymbol{G}(\boldsymbol{x}) + \boldsymbol{u} - \overline{\boldsymbol{G}}] - \boldsymbol{z}^{\mathrm{T}}[\boldsymbol{G}(\boldsymbol{x}) - \boldsymbol{l} - \underline{\boldsymbol{G}}] - \mu\sum_{i=1}^{r}\ln u_i - \mu\sum_{j=1}^{r}\ln l_j \tag{24}$$

b)Solution of Lagrange function: Necessary condition for the existence of extremum value of Lagrange objective function is its partial derivative formula has a solution.

The difference between PCPDIPM and PDIPM is addition of high order of slack variable, which accelerated the speed of the steepest descent approximate center path.

Assuming newton total direction $\Delta\boldsymbol{X} = [\Delta\boldsymbol{z},\Delta\boldsymbol{l},\Delta\boldsymbol{w},\Delta\boldsymbol{u},\Delta\boldsymbol{x},\Delta\boldsymbol{y}]^{\mathrm{T}}$， partial derivative formula can be expressed as follows：

$$\nabla_{\lambda}^{2}\boldsymbol{L}(\boldsymbol{X})\cdot\Delta\boldsymbol{X} = \boldsymbol{K}_{\mathrm{af}} + \boldsymbol{K}_{\mathrm{co}} \tag{25}$$

Solution of $\Delta\boldsymbol{X}$ is divided into affine direction $\Delta\boldsymbol{X}_{\mathrm{af}}$ and correction direction $\Delta\boldsymbol{X}_{\mathrm{co}}$.

First, calculate affine direction $\Delta\boldsymbol{X}_{\mathrm{af}}$. Order reduction method is used to deal with matrix operation $\nabla_{\lambda}^{2}\boldsymbol{L}(\boldsymbol{X}_{\mathrm{af}})\cdot\Delta\boldsymbol{X}_{\mathrm{af}} = \boldsymbol{K}_{\mathrm{af}}$. Solution of affine direction $\Delta\boldsymbol{X}_{\mathrm{af}}$ is divided into decomposition calculation of LDL$^{\mathrm{T}}$ in formula (26) and backstepping calculation of (27).

$$\begin{bmatrix} \boldsymbol{H}' & \nabla_{x}\boldsymbol{H}(\boldsymbol{x}) \\ \nabla_{x}^{\mathrm{T}}\boldsymbol{H}(\boldsymbol{x}) & 0 \end{bmatrix}\begin{bmatrix} \Delta\boldsymbol{x}_{\mathrm{af}} \\ \Delta\boldsymbol{y}_{\mathrm{af}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{L}'_{x} \\ -\boldsymbol{H}(\boldsymbol{x}) \end{bmatrix} \tag{26}$$

$$\begin{cases} \Delta\boldsymbol{l}_{\mathrm{af}} = \boldsymbol{L}_{z} + \nabla_{x}^{\mathrm{T}}\boldsymbol{G}(\boldsymbol{x})\Delta\boldsymbol{x} \\ \Delta\boldsymbol{z}_{\mathrm{af}} = -\boldsymbol{z} - \boldsymbol{L}^{-1}\boldsymbol{Z}\Delta\boldsymbol{l}_{\mathrm{af}} \\ \Delta\boldsymbol{u}_{\mathrm{af}} = -\boldsymbol{L}_{w} - \nabla_{x}^{\mathrm{T}}\boldsymbol{G}(\boldsymbol{x})\Delta\boldsymbol{x} \\ \Delta\boldsymbol{w}_{\mathrm{af}} = -\boldsymbol{w} - \boldsymbol{U}^{-1}\boldsymbol{W}\Delta\boldsymbol{u}_{\mathrm{af}} \end{cases} \tag{27}$$

Then calculating affine direction step length， dual gap, center parameter and barrier parameter of the k-th iteration.

Calculating correction direction $\Delta\boldsymbol{X}_{\mathrm{co}}$, Order reduction method is also used to deal with matrix operation $\nabla_{\lambda}^{2}\boldsymbol{L}(\boldsymbol{X}_{\mathrm{co}})\cdot\Delta\boldsymbol{X}_{\mathrm{co}} = \boldsymbol{K}_{\mathrm{co}}$, Solution of correction direction $\Delta\boldsymbol{X}_{\mathrm{co}}$ is divided into decomposition and backstepping calculation of LDL$^{\mathrm{T}}$.

Original and dual variables can be updated after newton direction and iteration step length are obtained:

$$\begin{aligned} \boldsymbol{x}^{k+1} &= \boldsymbol{x}^{k} + \lambda_{\mathrm{p}}\Delta\boldsymbol{x} \\ \boldsymbol{y}^{k+1} &= \boldsymbol{y}^{k} + \lambda_{\mathrm{d}}\Delta\boldsymbol{y} \\ \boldsymbol{l}^{k+1} &= \boldsymbol{l}^{k} + \lambda_{\mathrm{p}}\Delta\boldsymbol{l} \\ \boldsymbol{z}^{k+1} &= \boldsymbol{z}^{k} + \lambda_{\mathrm{d}}\Delta\boldsymbol{z} \\ \boldsymbol{u}^{k+1} &= \boldsymbol{u}^{k} + \lambda_{\mathrm{p}}\Delta\boldsymbol{u} \\ \boldsymbol{w}^{k+1} &= \boldsymbol{w}^{k} + \lambda_{\mathrm{d}}\Delta\boldsymbol{w} \end{aligned} \tag{28}$$

In order to decrease the iteration number of algorithm and to improve the efficiency of calculation, update of the original and dual variables take different iteration step length respectively.

Cycling the above calculation iteration process after original and dual variables are updated. When $\mathrm{Gap} \le \varepsilon_{\mathrm{PM}}$, that is, dual gap achieves required precision or iteration time reaches maximum number, iterative process exits.

B．Solution of robust decision

From the point of long time scale, wind power output is formed by a number of wind power uncertain set over time arrangement. Through robust evaluation, system pay and corresponding power flow control variable under worst case in each uncertain set are solved. The second step of robust optimization called robust decision is selecting economic optimal control variables under long time scales as final results.

Robust optimization flow chart of VSC-MTDC AC-DC uncertain system based on the long time scales is shown in Fig. 2.

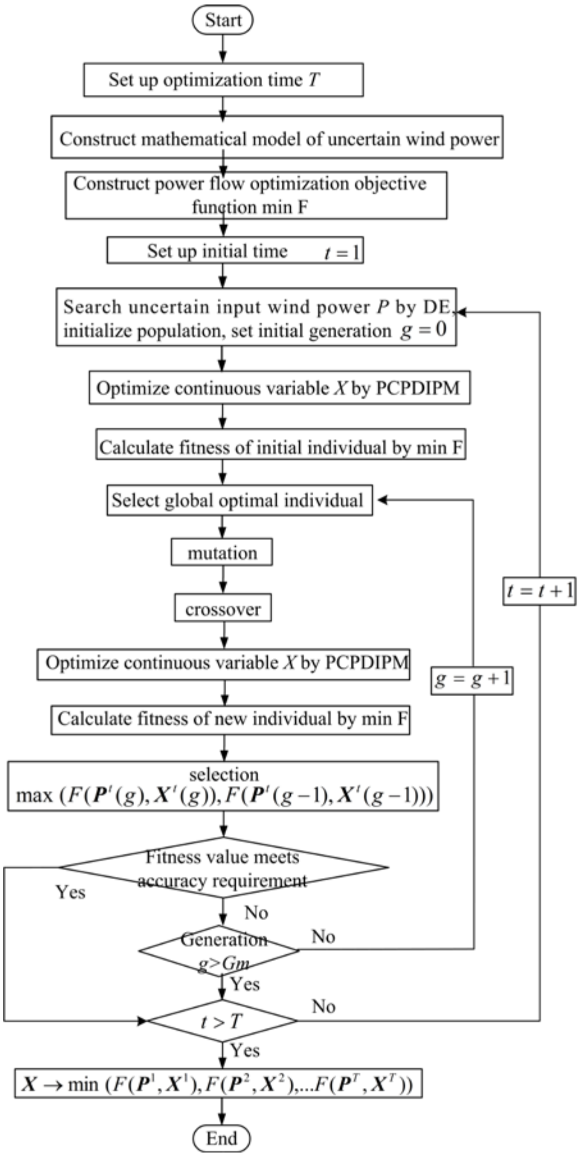

Fig. 2. Flow chart of robust optimization adapt to VSC-MTDC AC/DC uncertain system over a long time scale

6 Examples Analysis

A. Comparison between robust optimization and traditional optimization

The modified IEEE 14 nodes VSC - MTDC wind power grid system is taken as an example for robust optimization. In Fig. 3, G_1、G_2 stand for wind farm, converter station VSC_3、VSC_4 input wind power into AC network, G_3、G_4、G_5 are normal generators for AC network. Parameters of VSC - MTDC AC-DC network: reactor X_4=0.35p.u. , parameters of line, load and transformer in AC network are consistent with these in IEEE14 node system.

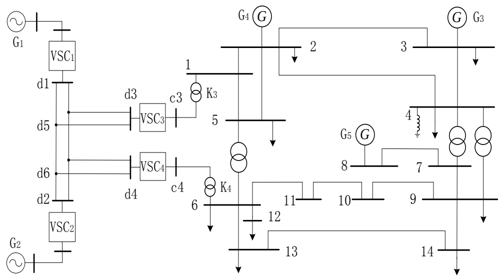

Fig. 3. Modified IEEE 14-bus system with VSC-MTDC Grid-connected wind power

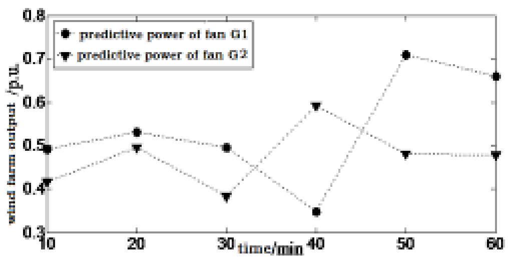

Fig. 4. Short term forecast of wind power output in 1 hour

One hour wind speed forecasting sample with 10 minutes interval is produced according to the formula (11), wind farm G_1: k=2.02、c=17, wind farm G_2: k=1.9、c=15. Prediction of large-scale centralized wind power output in an hour is shown in Fig. 4 according to (12):

At present, in the countries such as Germany and Denmark, the wind speed prediction error is within 10%. Uncertainty factors are added on the basis of one hour wind speed forecasting samples in Fig. 4, and wind speed uncertainty set is established. Scale parameter of uncertain interval set of wind speed is set $D^{\pm}$=±10%, model of uncertain wind power output is constructed according to (15), upper and lower limit of wind power output is shown in Table I.

TABLE I
CASSETTE SET OF UNCERTAIN WIND POWER OUTPUT

Time(min)		10	20	30	40	50	60
G1	upper limit(p.u.)	0.541	0.585	0.546	0.384	0.780	0.727
	lower limit(p.u.)	0.443	0.479	0.446	0.314	0.638	0.595
G2	upper limit(p.u.)	0.458	0.546	0.422	0.652	0.529	0.526
	lower limit(p.u.)	0.374	0.446	0.346	0.534	0.433	0.430

TABLE II
RESULTS OF ROBUST EVALUATION

Time(min)		10	20	30	40	50	60
object(p.u.) / control variable(p.u.)		0.417	0.553	0.409	0.466	0.597	0.581
VCS_1	P_c	0.541	0.585	0.546	0.384	0.780	0.72
	U_d	2.009	2.055	2.005	2.023	2.064	2.062
VSC_2	P_c	0.443	0.546	0.422	0.652	0.529	0.517
	U_d	2.005	2.052	2.000	2.028	2.056	2.055
VSC_3	U_d	1.995	2.039	1.991	2.011	2.044	2.043
	Q_c	0.179	0.196	0.177	0.184	0.207	0.203
VSC_4	U_d	1.99	2.035	1.985	2.009	2.038	2.038
	Q_c	0.133	0.156	0.130	0.146	0.164	0.160
G_3	P_{G3}	0.509	0.462	0.515	0.493	0.402	0.426
	U_a	1.065	1.086	1.062	1.073	1.086	1.086
G_4	P_{G4}	0.575	0.530	0.580	0.559	0.476	0.498
	U_a	1.047	1.067	1.045	1.055	1.066	1.067
G_5	P_{G5}	0.554	0.506	0.559	0.536	0.45	0.473
	U_a	1.100	1.100	1.100	1.100	1.100	1.100

Uncertainty wind power output cassette set is taken as system input. The model in Fig. 3 is performed a power flow optimization calculation by using the proposed robust optimization method based on the long time scales, the robust evaluation results are shown in the Table II below:

Robust decision is the result of meeting the system economics based on long time scales, the results are shown in Table III:

TABLE III
RESULTS OF ROBUST DECISION

facility	VSC_3		VSC_4		G_3		G_4		G_5	
control variable (p.u.)	U_d	Q_c	U_d	Q_c	P_{G3}	U_a	P_{G4}	U_a	P_{G5}	U_a
	1.991	0.177	1.985	0.130	0.515	1.062	0.580	1.045	0.559	1.100

TABLE IV
RESULTS OF TRADITIONAL OPTIMIZATION

facility	VSC_3		VSC_4		G_3		G_4		G_5	
control variable (p.u.)	U_d	Q_c	U_d	Q_c	P_{G3}	U_a	P_{G4}	U_a	P_{G5}	U_a
	2.042	0.199	2.037	0.155	0.459	1.087	0.527	1.068	0.504	1.100

Traditional optimization uses predictive value of wind power output in an hour as the system wind power input for power flow optimization. The model in Fig. 3 is simulated with traditional optimization method, control variables are shown in Table IV:

Actual wind power output and forecast wind power output always have a certain deviation because of the uncertainty of wind speed. Random deviation within ±10% is added on the basis of wind power prediction sample in Fig. 4 to simulate the actual wind power output as shown in Fig. 5. Control quantity of robust optimization and traditional optimization are used to control power flow of model in Fig. 3 respectively, power control instructions of VSC1 and VSC2 are consistent with the actual wind power output.

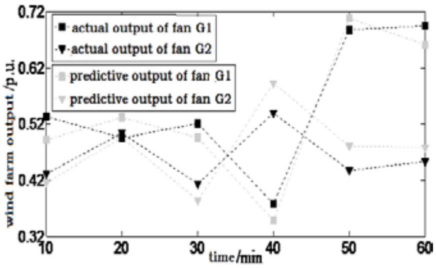

Fig. 5. Simulated actual wind power output in the error of ±10%

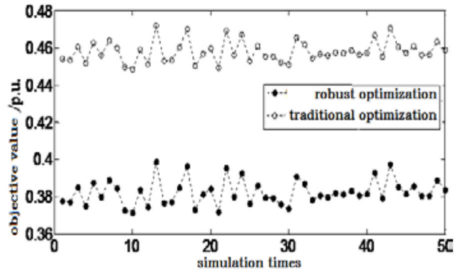

Fig. 6. Comparison of objective optimization function results by robust and traditional optimization methods after 100 times simulation

Taking 100 times actual wind farm output as system input for simulation, comparison between the average objective function in an hour with robust optimization and traditional optimization is shown as below:

As you can see in Fig. 6, robust optimization result is superior to traditional optimization result, average objective function of 100 simulation with traditional optimization is about 0.458, mean value with robust optimization is about 0.383, reduced 16.376% contrast to traditional optimization.

B. Influence of uncertain scope of parameter on robust optimization

The size parameter of the one hour wind speed forecasting uncertain interval set is increased as D±=±15% and D±=±20% respectively. Uncertain wind power output model is established on the basis of wind power output prediction in Fig. 4, power flow of system in Fig. 3 is robust optimized separately. Robust decision results after increasing the size parameter of the wind speed uncertain interval set are shown in the Table V below:

TABLE V
RESULTS OF POWER FLOW BY ROBUST DECISION WHEN $D^{\pm}$=±15% AND $D^{\pm}$=±20%

parameter	control variable (p.u.) / Object(p.u.)	VSC3		VSC4		
		U_d	Q_c	U_d	Q_c	
$D^{\pm}$=±15%	0.442	2.005	0.183	2.000	0.137	
$D^{\pm}$=±20%	0.488	2.019	0.188	2.014	0.144	
	G3		G4		G5	
parameter	P_{G3}	U_a	P_{G4}	U_a	P_{G5}	U_a
$D^{\pm}$=±15%	0.500	1.069	0.567	1.052	0.544	1.100
$D^{\pm}$=±20%	0.486	1.076	0.553	1.058	0.530	1.100

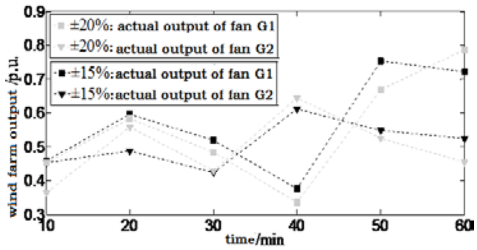

Fig. 7. Simulated actual wind power output in the error of ±15% and 20%

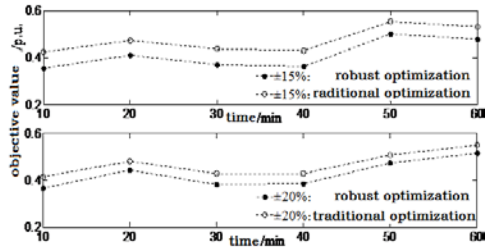

Fig. 8. Comparison of objective optimization function results when $D^{\pm}$=±15% and $D^{\pm}$=±20%

It can be seen from table 5-4, increasing the size of the wind power uncertain interval set leads to the rise of system cost in the worst cases. This is due to fluctuation scope of uncertain wind power expands, new limiting cases of system objective function and constraint conditions are created.

The actual wind power output as shown in Fig. 7 is simulated by increasing random deviation within ±15% and ±20% on the basis of wind power prediction samples in Fig. 4:

Using the robust decision results in Table V to control the corresponding interval set scale parameter system, comparison between objective function optimization results are shown in Fig. 8:

In Fig. 8, when D±=±15%, average objective function value of uncertain wind power output system with traditional optimization in an hour is 0.475 p.u. , average with robust optimization is 0.413 p.u. , reduced by 13.053% contrast to conventional optimization; When D±=±20%, mean value of objective function with traditional optimization is 0.469 p.u. , average with robust optimization is 0.428 p.u. , reduced by 8.742% compared with conventional optimization. While the size of wind power uncertain interval set increases, system operation cost is also raised, which reduce the advantage of robust optimization in this paper over traditional optimization. Therefore enhancing accuracy of wind power prediction, reducing uncertainty range of wind power output, is the foundation to improve economic operation of the system.

7 Conclusion

This article target on the VSC - MTDC AC/DC system, and a long-timescale based power flow robust optimization method is proposed, which considers the uncertainty of wind farm output. The cassette bounded uncertain set is used to present the uncertainty of wind power, and the robust optimization method improves the immunity of the system economic operation to uncertain wind power output. The optimization model is solved by the DE--PCPDIPM hybrid optimization algorithm, which provides VSC and AC scheduling with proper value of power flow control. Economy of the robust optimization proposed in this paper under the long time scales is verified through simulation studies in comparison to the traditional optimization technique. It is concluded that the increase in the scale parameter of uncertainty set will reduce the economy level of robust decision-making significantly.

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