

Research on High Speed Toll Plaza

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Abstract. This paper propose a reasonable assumption, and abstract the toll plaza into a model only related to B and L. By using the queuing theory and traffic flow theory, we define the throughput, cost and accident prevent with B and L to acquire the base model, and then calculate the model by using the method of linear weighting in economics.

Introduction

The dramatic increase in traffic burden has highlighted the necessity of rational allocation of toll plaza. At the same time, the need to consider a lot of factors has enhanced the design requirements.

Model Analysis

We decide to establish a total benefit evaluation equation to find relatively better solutions

(1) The scale of the toll plaza and the number of lanes are determined by three factors: traffic volume, traffic capacity and service level. To study the impact of them, we divide the process into the enter, wait and leave stage [1] to describe the time spent in each stage.

(2) Given the rate of accidents and the cost of construction, we look for a design that maximized throughput.

(3) Cost / benefit analysis is a practical way to assess project and policy rationality [2]. We evaluate the performance of toll plazas from rates of incidents, throughput, and construction costs and build an overall benefit evaluation equation to find the optimal solution.

Model Establishment

Definition of variables.

Table 1 Symbol Table-Variables

Symbol	Definition	Units
P	Benefit	dollar
C_h	Charge income	dollar
D_a	Damage of accident	dollar
C_o	Construction cost	dollar
$w_1 w_2 w_3$	Weights	\
T_p	Throughput	\
T_a	Accident times per month	\
S	Square meter	km^2
T	Total average time	s
T_u	Average time at the stage of enter	s
T_w	Average time at the stage of waiting for service	s
T_n	Average time at the stage of leave	h

Throughput. Through traffic flow theory [3], we establish the queuing equation.

Table 2 Symbol Queuing -Variables

Symbol $w(x)$	Definition
q	the traffic flow
q_0	the initial traffic flow
$v(x)$	the traffic flow velocity function
$k(x)$	the traffic flow density function
$w(x)$	the width of the transition region
v_0	the initial traffic flow velocity
λ	the average arrival rate of each window
μ	the average service rate

In general, the length of transition region is directly proportional to the number of the charging window, and we take the coefficient of proportionality as 0.02.

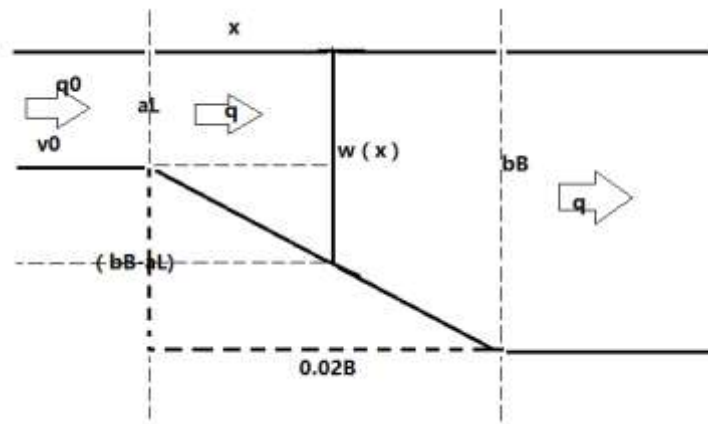


Figure 1. transition stage

$$w(x) = aL + x \frac{bB - aL}{0.02B}$$

$$q = v(x) k(x) = v_0 k_0$$

$$k_0(aL) = k(x) w(x)$$

$$dT_n = \frac{dx}{v(x)}$$

$$T_n = \int_0^{0.02B} dT_n = \frac{0.02aLB}{v_0(bB - aL)} \ln\left(\frac{bB}{aL}\right)$$

Traffic flow does not change at the second stage, And the traffic flow is evenly distributed to each charging window

$$\lambda = q_0/B$$

$$T_w = \frac{1}{\mu - \lambda} = \frac{n}{n\mu - q_0}$$

The third stage is similar to the first stage, so the result of the derivation is:

$$T_u = \frac{0.02bB^2}{v_2(bB - aL)} \ln\left(\frac{bB}{aL}\right)$$

Therefore, the total time for vehicles to pass through the toll plaza is:

$$T = T_n + T_w + T_u = \frac{0.02aLB}{v_0(bB - aL)} \ln\left(\frac{bB}{aL}\right) + \frac{1}{\mu - \lambda} + \frac{0.02bB^2}{v_2(bB - aL)} \ln\left(\frac{bB}{aL}\right)$$

Below we make some constant settings:

Table 3 Symbol Table-Constants

Constant symbols	Definition	Value
a	Highway lane width	0.00375[km]
b	Toll station (Contains toll booths and toll lanes) width	0.005[km]
μ	Average service rate	600[pcu/h]
q_0	Average traffic flow	3400[pcu/h]
v_0	The speed of vehicles entering the toll plaza	40[km/h]
v_2	The speed of vehicles leaving toll plaza	20[km/h]

Take one hour as the unit time, and we can deduce that the number of vehicles passing through the toll plaza per hour (Throughput T_h) is:

$$Th = \frac{1}{T} = \frac{600B - 3400}{B + \frac{B(3L + 8B)(3B - 17)}{45B - 30L} \ln\left(\frac{4B}{3L}\right)}$$

Accident Occurrence Rate. The incidence of accidents is proportional to the gradient.[4]According to the table above, after data regression analysis, we can get the relationship between the gradient degree and the number of traffic accidents [5] :

$$Da = 1.4231e^{8.006471} = 1.4231e^{2.0016 - 1.5012 \frac{L}{B}}$$

Construction Cost. The value of a and b has set.

$$S = 0.02B(2aL + 2bB) + 0.06 * 2bB = 0.0002B2 + 0.00015BL + 0.0006B$$

$$C0 = k3 (0.0002B2 + 0.00015BL + 0.0006B)$$

Benefit. According to the cost / benefit analysis theory , charge income, the damage of accident ,and construction cost affect the comprehensive benefits of toll Plaza.

We make the following reasonable assumptions : The toll plaza charge is directly proportional to the throughput.The accident volume is proportional to the square gradien The construction costs is directly proportional to the area.They each take a different weight

$$P = w_1 C_h - w_1 D_a - w_3 C_o = w_1 k_1 T_p - w_1 k_2 T_a - w_3 k_3 S$$

Calculation and Results

We use genetic algorithms and data fitting to process the data to get the scale factor and weight Data of New Jersey highway bureau (2006) shows that, The average number of vehicles through the toll plaza per month is 2341222,average monthly income is 1049568(\$) and Highway service life is

40(years). Economic loss caused by the accident is 0.76×10^9 (\$)

Through genetic algorithms and data fitting, we get $k_1=157.0841814$ and $k=233000$.

According to the data that US Pasadena highway cost 140 million US dollars per mile in 1993. We set, $k_3 = 10^9$.

Referring to the expert group scoring and the actual situation, we set the weight ratio of the three items is $w_1: w_2: w_3 = 4.7165: 2.999: 1$

So we take: $w_1=4.7165$, $w_2=4.7165$, $w_3=1$

$$C_h = 179.32 * \frac{600B - 3400}{B + \frac{B(3L + 8B)(3B - 17)}{45B - 30L} \ln\left(\frac{4B}{3L}\right)}$$

$$T_a = 1.4231e^{8.0064d_1} = 1.4231e^{2.0016 - 1.5012 \frac{L}{B}}$$

$$D_a = 0.01232 * 10^9 * 1.4231e^{2.0016 - 1.5012 \frac{L}{B}}$$

$$C_o = 10^9 * (0.0002B^2 + 0.00015BL + 0.0006B)$$

$$P = 4.7165 * C_h - 2.9999 * D_a - 1 * C_o$$

Since $B > L$, we only study the area of the right of red marker line, from the image, we can directly see that the range of highest point is $B=6 \sim 9, L=2 \sim 6$.

Now we list the data which is in this range:

Table 4 P (Benefit) (*10 ⁸ dollar)				
$\begin{matrix} L \\ \backslash \\ B \end{matrix}$	6	7	8	9
2	1.655	4.056	4.563	4.57
3	1.691	14.06	4.565	4.582
4	1.719	4.062	4.575	4.559
5	1.748	4.078	4.567	4.563
6	1.791	4.138	4.561	4.533

From the table above and the analysis we have done, considering charge (throughput), damage, and cost, we finalized that the best optimal solution of the B and L is $B=8, L=4$.

Summary

We first use the queuing theory and traffic flow theory to calculate the throughput, and then the data fit to calculate the corresponding costs and the incidence of accidents. Finally, using the linear weighted method in economics to calculate the model, the optimal B and L strategies are obtained: $L = 4, B = 8$.

Reference

- [1] Chunlei Wu and Yuling Chang, "Research on toll lane allocation of toll plaza in expressway", Road construction 2008,10 ;

- [2] <http://www.docin.com/p-806511068.html> ;
- [3] Futian Ren , Xiaoming Liu and Jian Yong ,“Traffic Engineering”[M] , Beijing: People's Communications Press 2003;
- [4] Weiming Liu and Xianyong Gan ,“Research on Optimization Method of Toll System Based on Linear Weighted Model”, Journal of Changsha Communications University ;
- [5] Xiaosong Wu ,“Highway toll station traffic safety research” Chang'an University 2004 ,6;