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Efficient Utilization of Auxiliary Information on Estimation of Population Mean Using Exponential Type Estimators in Successive Sampling

Surya K. Pal *

School of Studies in Statistics,
Vikram University, Ujjain, M.P., India-456010
suryakantpal6676 @gmail.com

Housila P. Singh

School of Studies in Statistics,
Vikram University, Ujjain, M.P., India-456010
hpsujn@gmail.com

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Abstract

This paper advocates the problem of estimating the population mean on the current occasion in two occasion successive (rotation) sampling under the transformed auxiliary variable using exponential method of estimation. Four different type estimators are suggested for estimating the current population mean in two occasion successive (rotation) sampling. Optimum replacement policies and performance of suggested estimators have been discussed. Outcomes are interpreted through empirical study.

Keywords: Auxiliary variable; Study variable; Successive sampling; Optimum replacement policy; Efficiency.
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1. Introduction

The information on an auxiliary variable is usually available for all the units of a finite population in sample surveys. In many sample survey studies, when the character under study of finite population changes over time, one time survey carried out on a single occasion provides information about the characteristic of the surveyed population for the given occasion and unable to give information regarding the nature of change over different occasions and the estimates of the population parameters over all occasions or on the most recent occasion. There are many problems of practical interest in social sciences in which various characters opt to change over time with respect to different parameters. Hence, one is often concerned with measuring the characteristics of a population on several occasions to estimate the trend in time of population means as a time series or current value of population mean over several points of time. For example, an investigator, or owner of the industry of cold drinks may be paying attention in the following type of problems:
(i) The average or total sale of cold drink for current season;
(ii) The change in average sale of cold drink for two different seasons;
(iii) Simultaneously to know the both (i) and (ii).

*Corresponding author.

Or in many countries, monthly labor-force surveys are conducted to estimate the number of employed individuals and the rate of unemployment. Other examples are weekly or monthly surveys on prices of goods are conducted to determine the customer price index, and political opinion surveys are conducted at regular intervals to know the voter preference. Surveying in which the sampling is done on successive occasions (over years , seasons, months, or weeks) according to the specified rule , with partial replacement of units is called successive (rotation) sampling. Beginning with the work of Jessen (1942) and followed by Patterson (1950), Eckler (1955), Rao and Graham (1964), Singh et al. (1992) and Singh (2005) among others have developed the theory of successive sampling. Feng and Zou (1997) and Biradar and Singh (2001) used the auxiliary information on both the occasions for estimating the current population mean in the successive sampling. Recently Singh and Vishwakarma (2007, 2009) , Singh and Pal (2015 a, b, c, d) and Singh and Pal (2016 a, b, c) have used the auxiliary information on both the occasion and envisaged several estimators for the estimating the population mean on current occasion in two-occasion successive (rotation) sampling. Motivated with the above work and utilizing the information on an auxiliary variable, readily available on the both occasions, we have proposed transformed estimators for estimating the current population mean in two-occasion successive (rotation) sampling. The procedure discussed in the above studies have used information only on the population mean $\bar{Z}$ of the auxiliary variable *z*, while in various survey situations information on other parameters of the auxiliary variable *z* such as coefficient of variation C_z , population standard deviation S_z, population coefficients of skewness $\beta_1(z)$ and kurtosis $\beta_2(z)$; and the correlation coefficient ρ_{yz} between study variable *y* and auxiliary variable *z*; and the correlation coefficient ρ_{xz} between the auxiliary variables *x* and *z* are known. The objective of the present paper is to propose a more precise estimator for estimating the population mean at current occasion in two occasions successive (rotation) sampling in the presence of auxiliary variable.

2. Formulation And Notation Of The Proposed Estimator

Let *U*= (U_1, U_2, ..., U_N) be the finite population of size N units, which has been sampled over two occasions. Let *x* (*y*) be the variable under study on the first (second) occasion respectively. It is assumed that information on an auxiliary variable z (stable over occasion) is readily available for the both the occasions. It is assumed that the population under investigation is large, and the sample size is constant on each occasion. A simple random sample of *n* units is drawn without replacement (*WOR*) on the first occasion. A random sub sample of *m* (= *nλ*) units is retained (matched) from the sample drawn on the first occasion for its use on the current (second) occasion, while a fresh sample of size *u*= (*n-m*) = $n\,\mu$ units is drawn on the current (second) occasion, from the entire population by simple random sampling without replacement (*SRSWOR*) procedure so that the sample size on the current (second) is also *n*. The fractions of the matched and fresh samples are respectively designated by λ and μ such that $\lambda+\mu=1$.

In what follows, we shall use the following notations throughout this paper.

$\bar{X}, \bar{Y}, \bar{Z}$: The population means of the variables x, y and z respectively.

$\bar{x}_m$, $\bar{x}_n$, $\bar{y}_u$, $\bar{y}_m$, $\bar{z}_u$, $\bar{z}_n$: The sample means of the respective variables based on the sample sizes indicated in suffices.

C_x, C_y, C_z : The coefficients of variation of the variables x, y and z respectively,

ρ_{yx}, ρ_{yz}, ρ_{xz} : The correlation coefficients between the variables shown in suffices .

$$S_x^2 = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \bar{X})^2 ,\; S_y^2 = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \bar{Y})^2 ,\; S_z^2 = (N-1)^{-1}\sum_{i=1}^{N}(z_i - \bar{Z})^2$$

are the population mean squares of x, y and z respectively, $f = n/N$: Sampling fraction.

To obtain the bias and mean square error (*MSE*) of suggested class of estimators we define following quantities

$$\bar{y}_u = \bar{Y}(1+e_{0u}), \bar{y}_m = \bar{Y}(1+e_{0m}), \bar{x}_n = \bar{X}(1+e_{1n}), \bar{x}_m = \bar{X}(1+e_{1m}), \bar{z}_u = \bar{Z}(1+e_{2u}),$$
$$\bar{z}_n = \bar{Z}(1+e_{2n}),\ s_{yx(m)} = S_{yx}(1+e_{3m}) \text{ and } s^2_{x(m)} = S^2_x(1+e_{4m})$$

such that

$$E(e_{0u}) = E(e_{0m}) = E(e_{1n}) = E(e_{1m}) = E(e_{2u}) = E(e_{2n}) = E(e_{3m}) = E(e_{4m}) = 0$$

and

$$E(e_{0u}^2) = \left(\frac{1}{u}-\frac{1}{N}\right)C_y^2, E(e_{0m}^2) = \left(\frac{1}{m}-\frac{1}{N}\right)C_y^2, E(e_{1m}^2) = \left(\frac{1}{m}-\frac{1}{N}\right)C_x^2, E(e_{1n}^2) = \left(\frac{1}{n}-\frac{1}{N}\right)C_x^2,$$
$$E(e_{2u}^2) = \left(\frac{1}{u}-\frac{1}{N}\right)C_z^2, E(e_{2n}^2) = \left(\frac{1}{n}-\frac{1}{N}\right)C_z^2, E(e_{0u}e_{0m}) = -\frac{1}{N}C_y^2, E(e_{0u}e_{1m}) = -\frac{1}{N}\rho_{yx}C_yC_x,$$
$$E(e_{0u}e_{1n}) = -\frac{1}{N}\rho_{yx}C_yC_x, E(e_{0u}e_{2u}) = \left(\frac{1}{u}-\frac{1}{N}\right)\rho_{yz}C_yC_z, E(e_{0u}e_{2n}) = -\frac{1}{N}\rho_{yz}C_yC_z$$
$$E(e_{0m}e_{1m}) = \left(\frac{1}{m}-\frac{1}{N}\right)\rho_{yx}C_yC_x, E(e_{0m}e_{1n}) = \left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yx}C_yC_x, E(e_{0m}e_{2u}) = -\frac{1}{N}\rho_{yz}C_yC_z,$$
$$E(e_{0m}e_{2n}) = \left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yz}C_yC_z, E(e_{1m}e_{1n}) = \left(\frac{1}{n}-\frac{1}{N}\right)\rho_{yx}C_yC_x, E(e_{1m}e_{2u}) = -\frac{1}{N}\rho_{xz}C_xC_z,$$
$$E(e_{1m}e_{2n}) = \left(\frac{1}{n}-\frac{1}{N}\right)\rho_{xz}C_xC_z, E(e_{1n}e_{2u}) = -\frac{1}{N}\rho_{xz}C_xC_z, E(e_{1n}e_{2n}) = \left(\frac{1}{n}-\frac{1}{N}\right)\rho_{xz}C_xC_z,$$
$$E(e_{2u}e_{2n}) = -\frac{1}{N}C_z^2,\ E(e_{1m}e_{3m}) = \frac{N(N-m)}{(N-1)(N-2)}\frac{\mu_{210}}{m\bar{X}S_{xy}}, E(e_{1n}e_{3m}) = \frac{N(N-n)}{(N-1)(N-2)}\frac{\mu_{210}}{m\bar{X}S_{xy}},$$
$$E(e_{4m}e_{1m}) = \frac{N(N-m)}{(N-1)(N-2)}\frac{\mu_{300}}{m\bar{X}S_x^2}, E(e_{4m}e_{1n}) = \frac{N(N-n)}{(N-1)(N-2)}\frac{\mu_{300}}{n\bar{X}S_x^2},$$
$$\mu_{rst} = E[(x_i-\bar{X})^r(y_i-\bar{Y})^s(z_i-\bar{Z})^t], (r,s,t) \geq 0 \text{ are integers.}$$

3. Estimator Based On Unmatched Portion

To estimate the population mean $\bar{Y}$ on the second (current) occasion an estimator based on a sample of size $u = n\mu$ drawn, afresh on the second occasion is defined by

$$t_u = \bar{y}_u \exp\left\{\frac{\delta(\bar{z}_u^* - \bar{Z}^*)}{\bar{z}_u^* + \bar{Z}^*}\right\}, \tag{3.1}$$
$$= \bar{y}_u \exp\left\{\frac{\delta a(\bar{z}_u - \bar{Z})}{a(\bar{z}_u + \bar{Z}) + 2b}\right\},$$

where $\bar{Z}^* = a\bar{Z} + b$, $\bar{z}_u^* = a\bar{z}_u + b$ and (a,b) are suitably chosen constants. The scalars (a,b) may assume real values as well as parametric values C_z (coefficient of variation of the auxiliary variable *z*), ρ_{xz} (correlation coefficient between *x* and *z*), $\beta_1(z)$ (coefficient of skewness of *z*), $\beta_2(z)$ (coefficient of kurtosis of *z*), $\bar{Z}$ (population mean of *z*) and S_z (standard deviation *z*) etc for instance see Singh and Tailor (2003), Upadhyaya and Singh (1999) and Singh et al.(2004).

3.1 The Bias and MSE of Estimator t_u Based on Unmatched Portion

Expressing (2.1) in term of e's we have

$$t_u = \bar{Y}(1+e_{0u})\exp\left\{\frac{\delta\theta e_{2u}}{1+\theta e_{2u}}\right\},$$

$$= \bar{Y}(1+e_{0u})\exp[\delta\theta e_{2u}(1+\theta e_{2u})^{-1}], \tag{3.2}$$

where

$$\theta = a\bar{Z}/\{2(a\bar{Z}+b)\}.$$

We assume that $|\theta e_{2u}|<1$, so that $(1+\theta e_{2u})^{-1}$ is expandable. Now expanding the right hand side of (3.2), multiplying out and neglecting terms e's having power greater than two we have

$$t_u \cong \bar{Y}\left[1+e_{0u}+\delta\theta e_{2u}-\delta\theta e_{0u}e_{2u}+\frac{\delta\theta^2(\delta-2)}{2}e_{2u}^2\right]$$

or

$$(t_u-\bar{Y}) \cong \bar{Y}\left[e_{0u}+\delta\theta e_{2u}-\delta\theta e_{0u}e_{2u}+\frac{\delta\theta^2(\delta-2)}{2}e_{2u}^2\right]. \tag{3.3}$$

Taking expectation on both sides of (3.3), we get the bias of proposed class of estimators t_u up to first degree of approximation as

$$B(t_u) = \left(\frac{1}{u}-\frac{1}{N}\right)\left(\frac{\bar{Y}\delta\theta}{2}\right)\{2k_{yz}+\theta(\delta-2)\}C_z^2, \tag{3.4}$$

where $k_{yz} = \rho_{yz}(C_y/C_z)$.

Squaring both sides of (3.3) and neglecting terms of e's having power greater than two , we have

$$(t_u-\bar{Y})^2 \cong \bar{Y}^2[e_{0u}+\delta\theta e_{2u}]^2$$

$$\cong \bar{Y}^2[e_{0u}^2+\delta^2\theta^2 e_{2u}^2+2\theta e_{0u}e_{2u}]. \tag{3.5}$$

Taking expectation on both sides of (3.5), we get the *MSE* of proposed class of estimators t_u up to first degree of approximation as

$$MSE(t_u) = \bar{Y}^2\left(\frac{1}{u}-\frac{1}{N}\right)[C_y^2+\delta\theta C_z^2(\delta\theta+2k_{yz})]. \tag{3.6}$$

Remark 3.1. Since *x* and *y* denote the same study variable over two occasions and *z* is an auxiliary variable correlated to *x* and *y*, therefore as mentioned in Reddy (1978), Cochran (1977) and Feng and Zou (1997), the coefficients of variation is a stable quantity; it can be assumed that the coefficients of variation of *x*, *y*, *z* are considered to be approximately equal (i.e. $C_y \cong C_x \cong C_z$).

Thus putting $C_y \approx C_z$ in (3.4) and (3.6) we get the bias and *MSE* of t_u to the first degree of approximation, respectively, as

$$B(t_u) = \left(\frac{1}{u}-\frac{1}{N}\right)\left(\frac{\bar{Y}\delta\theta C_y^2}{2}\right)\{2\rho_{yz}+\theta(\delta-2)\} \tag{3.7}$$

and

$$MSE(t_u) = \left(\frac{1}{u}-\frac{1}{N}\right)S_y^2\{1+\delta\theta(\delta\theta+2\rho_{yz})\}$$

$$=\left(\frac{1}{u}-\frac{1}{N}\right)S_y^2\alpha_3, \tag{3.8}$$

where

$$\alpha_3=\{1+\delta\theta(\delta\theta+2\rho_{yz})\}.$$

4. Estimator Based On Matched Portion

Three chain-type estimators based on the sample of size m (= $n\lambda$) common to both the occasions are defined by

$$t_{m1}=\bar{y}_m\left(\frac{\bar{x}_n}{\bar{x}_m}\right)\exp\left(\frac{\delta(\bar{z}_n^*-\bar{Z}^*)}{\bar{Z}^*+\bar{z}_n^*}\right),$$

$$=\bar{y}_m\left(\frac{\bar{x}_n}{\bar{x}_m}\right)\exp\left(\frac{a\delta(\bar{z}_n-\bar{Z})}{a(\bar{Z}+\bar{z}_n)+2b}\right) \tag{4.1}$$

$$t_{m2}=\bar{y}_m\exp\left(\frac{\bar{x}_n-\bar{x}_m}{\bar{x}_n+\bar{x}_m}\right)\exp\left(\frac{\delta(\bar{z}_n^*-\bar{Z}^*)}{\bar{Z}^*+\bar{z}_n^*}\right),$$

$$=\bar{y}_m\exp\left(\frac{\bar{x}_n-\bar{x}_m}{\bar{x}_n+\bar{x}_m}\right)\exp\left(\frac{a\delta(\bar{z}_n-\bar{Z})}{a(\bar{Z}+\bar{z}_n)+2b}\right) \tag{4.2}$$

$$t_{m3}=[\bar{y}_m+b_{yx(m)}(\bar{x}_n-\bar{x}_m)]\exp\left(\frac{\delta(\bar{z}_n^*-\bar{Z}^*)}{\bar{Z}^*+\bar{z}_n^*}\right),$$

$$=[\bar{y}_m+b_{yx(m)}(\bar{x}_n-\bar{x}_m)]\exp\left(\frac{a\delta(\bar{z}_n-\bar{Z})}{a(\bar{Z}+\bar{z}_n)+2b}\right), \tag{4.3}$$

where $b_{yx(m)}$ is the sample regression coefficient of y on x based on matched portion and (a, b) are same as defined earlier.

4.1 The Bias and MSE of Estimators Based on Matched Portion

4.1.1 Derivation of the bias and MSE of the class of estimators t_{m1}

Expressing (4.1) in term of e's we have

$$t_{m1}=\bar{Y}(1+e_{0m})(1+e_{1n})(1+e_{1m})^{-1}\exp\left\{\frac{\delta\theta e_{2n}}{(1+\theta e_{2n})}\right\},$$

$$=\bar{Y}(1+e_{0m})(1+e_{1n})(1+e_{1m})^{-1}\exp(\delta\theta e_{2n})(1+\theta e_{2n})^{-1}, \tag{4.4}$$

We assume that $|e_{1m}|<1$ and $|\theta e_{2n}|<1$, so that $(1+e_{1m})^{-1}$ and $(1+\theta e_{2u})^{-1}$ are expandable. Now, expanding the right hand side of (4.4), multiplying out and neglecting terms of e's having power greater than two, we have

$$t_{m1}\cong\bar{Y}[1+e_{0m}-(e_{1m}-e_{1n})-(e_{0m}e_{1m}-e_{0m}e_{1n})+(e_{1m}^2-e_{1m}e_{1n})$$
$$+\delta\theta e_{2n}+\delta\theta e_{0m}e_{2n}-\delta\theta(e_{1m}e_{2n}-e_{1n}e_{2n})+\{\theta^2\delta(\delta-2)/2\}e_{2n}^2],$$

or

$$(t_{m1}-\bar{Y})\cong\bar{Y}[e_{0m}-(e_{1m}-e_{1n})-(e_{0m}e_{1m}-e_{0m}e_{1n})+(e_{1m}^2-e_{1m}e_{1n})$$

$$+\delta\theta e_{2n}+\delta\theta e_{0m}e_{2n}-\delta\theta(e_{1m}e_{2n}-e_{1n}e_{2n})+\{\theta^2\delta(\delta-2)/2\}e_{2n}^2\,]\,. \tag{4.5}$$

Taking expectation on both sides of (4.5), we get the bias of the proposed class of estimators t_{m1} up to first degree of approximation as

$$B(t_{m1})=\bar{Y}\left[\left(\frac{1}{m}-\frac{1}{n}\right)C_x^2(1-k_{yx})+\left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{\delta\theta C_z^2}{2}\right)(\delta+2k_{yz}-2)\right]. \tag{4.6}$$

Squaring both sides of (4.5) and neglecting terms of e's having power greater than two, we have

$$\begin{aligned}(t_{m1}-\bar{Y})^2 &\cong \bar{Y}^2[e_{0m}-(e_{1m}-e_{1n})+\delta\theta e_{2n}]^2\\ &\cong \bar{Y}^2[e_{0m}^2+(e_{1m}-e_{1n})^2+\delta^2\theta^2e_{2n}^2\\ &\quad -2(e_{0m}e_{1m}-e_{0m}e_{1n})+2\delta\theta e_{0m}e_{2n}-2\delta\theta(e_{1m}e_{2n}-e_{1n}e_{2n})]\,.\end{aligned} \tag{4.7}$$

Taking expectation on both sides of (4.7), we get the *MSE* of the proposed class of estimators t_{m1} up to first degree of approximation as

$$MSE(t_{m1})=\bar{Y}^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)\left\{C_y^2+C_x^2(1-2k_{yx})\right\}+\left(\frac{1}{n}-\frac{1}{N}\right)\left\{C_y^2+\delta\theta C_z^2(\delta\theta+2k_{yz})\right\}\right]. \tag{4.8}$$

Under the assumption $C_y\cong C_x\cong C_z$, the expressions of bias and *MSE* of t_{m1} in (4.6) and (4.8) respectively reduce to:

$$B(t_{m1})=\bar{Y}C_y^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)(1-\rho_{yx})+\left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{\delta\theta}{2}\right)(\delta+2\rho_{yz}-2)\right] \tag{4.9}$$

$$MSE(t_{m1})=S_y^2\left[\frac{1}{m}\alpha_1+\frac{1}{n}\alpha_2-\frac{1}{N}\alpha_3\right], \tag{4.10}$$

where

$$\alpha_1=2(1-\rho_{yx}),\ \alpha_2=[-1+2\rho_{yx}+\delta\theta(\delta\theta+2\rho_{yz})]\ \text{and}\ \alpha_3=(\alpha_1+\alpha_2)=[1+\delta\theta(\delta\theta+2\rho_{yz})].$$

4.1.2 Derivation of the bias and MSE of the class of estimators t_{m2}

Expressing (4.2) in terms of e's we have

$$\begin{aligned}t_{m2}&=\bar{Y}(1+e_{0m})\left\{-\frac{(e_{1m}-e_{1n})}{2}\left(1+\frac{(e_{1m}+e_{1n})}{2}\right)^{-1}\right\}\exp\left\{\delta\theta e_{2n}(1+\theta e_{2n})^{-1}\right\}\\ &=\bar{Y}(1+e_{0m})\left(1-\frac{d_m}{2}+\frac{A_{mn}}{8}-...\right)\left[1+\delta\theta e_{2n}+\frac{\delta(\delta-2)\theta^2}{2}e_{2n}^2+...\right],\end{aligned} \tag{4.11}$$

where $d_m=(e_{1m}-e_{1n})$ and $A_{mn}=(3e_{1m}^2-2e_{1m}e_{1n}-e_{1n}^2)$.

Expanding the right hand side of (4.19), multiplying out and neglecting terms of e's having power greater than two we have

$$t_{m2}\cong\bar{Y}\left[1+e_{0m}-\frac{d_m}{2}-\frac{e_{0m}d_m}{2}+\frac{A_{mn}}{8}+\delta\theta e_{2n}+\delta\theta e_{0m}e_{2n}-\left(\frac{\delta\theta}{2}\right)d_me_{2n}+\frac{\delta(\delta-2)\theta^2}{2}e_{2n}^2\right]$$

or

$$(t_{m2}-\bar{Y})\cong\bar{Y}\left[e_{0m}-\frac{d_m}{2}-\frac{e_{0m}d_m}{2}+\frac{A_{mn}}{8}+\delta\theta e_{2n}+\delta\theta e_{0m}e_{2n}-\left(\frac{\delta\theta}{2}\right)d_me_{2n}+\frac{\delta(\delta-2)\theta^2}{2}e_{2n}^2\right]. \tag{4.12}$$

Taking expectation on both sides of (4.12), we get the bias of the proposed class of estimators t_{m2} to the first degree of approximation, as

$$B(t_{m2}) = \bar{Y}\left[\left(\frac{1}{m}-\frac{1}{n}\right)\frac{C_x^2}{8}(3-4k_{yx}) + \left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{\delta\theta C_z^2}{2}\right)\left\{\theta(\delta-2)+2k_{yz}\right\}\right]. \tag{4.13}$$

Squaring both sides of (4.12) and neglecting terms of e's having power greater than two, we have

$$(t_{m2}-\bar{Y})^2 \cong \bar{Y}^2[e_{0m}-(d_m/2)+\delta\theta e_{2n}]^2$$
$$\cong \bar{Y}^2[e_{0m}^2+(d_m^2/4)+\delta^2\theta^2 e_{2n}^2 - d_m e_{0m} + 2\delta\theta e_{0m}e_{2n} - \delta\theta d_m e_{2n}]. \tag{4.14}$$

Taking expectation on both sides of (4.14), we get the *MSE* of the proposed class of estimators t_{m2} up to first degree of approximation as

$$MSE(t_{m2}) = \bar{Y}^2\left[\left(\frac{1}{m}-\frac{1}{N}\right)C_y^2 + \left(\frac{1}{m}-\frac{1}{n}\right)\frac{C_x^2}{4}(1-4k_{yx}) + \left(\frac{1}{n}-\frac{1}{N}\right)\delta\theta C_z^2(\delta\theta+2k_{yz})\}\right]$$
$$= \bar{Y}^2\left[\frac{1}{m}\left\{C_y^2+\frac{C_x^2}{4}(1-4k_{yx})\right\} + \frac{1}{n}\left\{\delta\theta C_z^2(\delta\theta+2k_{yz}) - \frac{C_x^2}{4}(1-4k_{yx})\right\}\right.$$
$$\left. - \frac{1}{N}\left\{C_y^2+\delta\theta C_z^2(\delta\theta+2k_{yz})\right\}\right]. \tag{4.15}$$

Under the assumption $C_x \cong C_y \cong C_z$, the bias and *MSE* of t_{m2} in (4.3) and (4.15) respectively reduce to

$$B(t_{m2}) = \bar{Y}C_y^2\left[\left(\frac{1}{m}-\frac{1}{n}\right)\frac{1}{8}(3-4\rho_{yx}) + \left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{\delta\theta}{2}\right)\left\{\theta(\delta-2)+2\rho_{yz}\right\}\right] \tag{4.16}$$

$$MSE(t_m) = S_y^2\left[\frac{1}{m}\alpha_1^* + \frac{1}{n}\alpha_2^* - \frac{1}{N}\alpha_3\right], \tag{4.17}$$

where

$$\alpha_1^* = [1+(1/4)(1-4\rho_{yx})], \alpha_2^* = [\delta\theta(\delta\theta+2\rho_{yz})-(1/4)(1-4\rho_{yx})]$$

and $\alpha_3 = (\alpha_1^*+\alpha_2^*) = [1+\delta\theta(\delta\theta+2\rho_{yz})]$.

4.1.3 Derivation of the bias and MSE of the class of estimators t_{m3}

Expressing (4.3) in terms of e's we have

$$t_{m3} = [\bar{Y}(1+e_{0m}) - \beta_{yx}\bar{X}(1+e_{3m})(e_{1m}-e_{1n})(1+e_{4m})^{-1}]\exp\left\{\frac{\delta\theta e_{2n}}{(1+\theta e_{2n})}\right\}, \tag{4.18}$$

where β_{yx} is the population regression coefficient of y on x.

Expanding the right hand side of (4.18), multiplying out and neglecting terms of e's having power greater than two, we have

$$t_{m3} \cong \bar{Y}[1+e_{0m} - k_{yx}(d_m + e_{3m}d_m - e_{4m}d_m)$$
$$+ \delta\theta e_{2n} + \delta\theta e_{0m}e_{2n} - \delta\theta(e_{1m}e_{2n} - e_{1n}e_{2n}) + \{\theta^2\delta(\delta-2)/2\}e_{2n}^2],$$

or

$$(t_{m3}-\bar{Y}) \cong \bar{Y}[e_{0m} - k_{yx}(d_m + e_{3m}d_m - e_{4m}d_m)$$
$$+ \delta\theta e_{2n} + \delta\theta e_{0m}e_{2n} - \delta\theta(e_{1m}e_{2n} - e_{1n}e_{2n}) + \{\theta^2\delta(\delta-2)/2\}e_{2n}^2]. \tag{4.19}$$

Taking expectation on both sides of (4.19), we get the bias of the proposed class of estimators t_{m3} up to first degree of approximation as

$$B(t_{m3}) = -\bar{Y}\left[\left(\frac{1}{m}-\frac{1}{n}\right)\frac{N}{N-2}\frac{k_{yx}}{\bar{X}}\left(\frac{\mu_{210}}{\mu_{110}}-\frac{\mu_{300}}{\mu_{200}}\right)-\left(\frac{1}{n}-\frac{1}{N}\right)\left(\frac{\delta\theta C_z^2}{2}\right)\left\{\theta(\delta-2)+2k_{yz}\right\}\right]. \tag{4.20}$$

Squaring both sides of (4.19) and neglecting terms of e's having power greater than two, we have

$$(t_{m3}-\bar{Y})^2 \cong \bar{Y}^2[e_{0m}-k_{yx}d_m+\delta\theta e_{2n}]^2$$
$$=\bar{Y}^2[e_{0m}^2+k_{yx}^2d_m^2+\delta^2\theta^2e_{2n}^2-2k_{yx}e_{0m}d_m+2\delta\theta e_{0m}e_{2n}-2k_{yx}\delta\theta d_m e_{2n}]. \tag{4.21}$$

Taking expectation on both sides of (4.21), we get the *MSE* of the proposed class of estimators t_{m3} up to first degree of approximation as

$$MSE(t_{m3}) = \bar{Y}^2\left[\left(\frac{1}{m}-\frac{1}{N}\right)C_y^2-\left(\frac{1}{m}-\frac{1}{n}\right)C_x^2k_{yx}^2+\left(\frac{1}{n}-\frac{1}{N}\right)\delta\theta C_z^2\left(\delta\theta+2k_{yz}\right)\right]$$
$$=\bar{Y}^2\left[\frac{1}{m}C_y^2(1-\rho_{yx}^2)+\frac{1}{n}\left\{\rho_{yx}^2C_y^2+\delta\theta C_z^2(\delta\theta+2k_{yz})\right\}-\frac{1}{N}\left\{C_y^2+\delta\theta C_z^2(\delta\theta+2k_{yz})\right\}\right] \tag{4.22}$$

Under the assumption $C_x \cong C_y \cong C_z$, the *MSE* of t_{m3} in (4.22) reduce to:

$$MSE(t_m) = S_y^2\left[\frac{1}{m}\alpha_1^{**}+\frac{1}{n}\alpha_2^{**}-\frac{1}{N}\alpha_3\right], \tag{4.23}$$

where

$$\alpha_1^{**}=(1-\rho_{yx}^2),\ \alpha_2^{**}=[\rho_{yx}^2+\delta\theta(\delta\theta+2\rho_{yz})],$$

and $\alpha_3=(\alpha_1^{**}+\alpha_2^{**})=[1+\delta\theta(\delta\theta+2\rho_{yz})]$.

4.2 The Covariance between The Estimators of Matched Portion and Unmatched Portion

The covariance between the estimator t_u and the estimator t_{m1} is defined by

$$Cov(t_u,t_{m1}) = E[(t_u-E(t_u))(t_{m1}-E(t_{m1}))]$$
$$=E(t_u-\bar{Y})(t_{m1}-\bar{Y}).$$

Expressing t_u and t_{m1} in terms of e's and neglecting terms of e's having power greater than two, we have

$$Cov(t_u,t_{m1}) = \bar{Y}^2E[(e_{0u}+\delta\theta e_{2u})(e_{0m}-e_{1m}+e_{1n}+\delta\theta e_{2n})],$$
$$=\bar{Y}^2E[e_{0u}e_{0m}-e_{0u}e_{1m}+e_{0u}e_{1n}+\delta\theta e_{0u}e_{2n}+\delta\theta e_{0m}e_{2u}-\delta\theta e_{2u}e_{1m}+\delta\theta e_{1n}e_{2u}+\delta^2\theta^2e_{2u}e_{2n}]$$
$$=-(\bar{Y}^2/N)[C_y^2+\delta\theta(\delta\theta+2k_{yz})C_z^2]. \tag{4.24}$$

Similarly, we can find the covariance between (t_u and t_{m2}) and (t_u and t_{m3}) as

$$Cov(t_u,t_{m2}) = -(\bar{Y}^2/N)[C_y^2+\delta\theta(\delta\theta+2k_{yz})C_z^2] \tag{4.25}$$

and

$$Cov(t_u,t_{m3}) = -(\bar{Y}^2/N)[C_y^2+\delta\theta(\delta\theta+2k_{yz})C_z^2]. \tag{4.26}$$

Under the assumption $C_x \cong C_y \cong C_z$, the expressions in (4.24), (4.25) and (4.26) respectively reduce to:

$$Cov(t_u,t_{m1}) = -(S_y^2/N)\alpha_3, \tag{4.27}$$

$$Cov(t_u, t_{m2}) = -(S_y^2 / N)\alpha_3,$$

(4.28) $$Cov(t_u, t_{m3}) = -(S_y^2 / N)\alpha_3, \quad (4.29)$$

where $\alpha_3 = [1 + \delta\theta(\delta\theta + 2\rho_{yz})]$.

5. The Combined Estimator

Combining the estimators t_u and t_{mi}, (i=1, 2, 3) we have the ultimate estimator of the population mean $\bar{Y}$ as

$$t_i = \omega_i t_u + (1 - \omega_i) t_{mi}, \ (i=1, 2, 3); \quad (5.1)$$

where ω_i's (i =1,2,3) are unknown constants to be determined under certain criterion. We note that for estimating the population mean on each occasion the estimator t_u is suitable, which implies that more belief on t_u could be observed by selecting ω_i (i =1, 2, 3) as 1(or near to 1), while for estimating the change over the occasion, the estimators t_{mi} (i =1, 2, 3) could be more suitable and hence ω_i might be selected as 0 (or near to 0). For asserting both the problems simultaneously, the appropriate (optimum) choices of ω_i are needed.

6. Minimum *MSE*s of the Estimators t_i (i=1, 2, 3).

The mean squared errors of the estimators t_i (i =1, 2, 3) to the first degree of approximation are given by

$$MSE(t_i) = [\omega_i^2 MSE(t_u) + (1-\omega_i)^2 MSE(t_{mi}) + 2\omega_i(1-\omega_i)Cov(t_u, t_{mi})], \quad (6.1)$$

where $MSE(t_u)$, $MSE(t_{mi})$, $Cov(t_u, t_{mi})$ (i =1, 2, 3) respectively given by (3.8) , {(4.10), (4.17), (4.23)} and {(4.27), (4.28), (4.29)}. Since the mean squared error of the estimators t_i (i =1, 2, 3) in (6.1) are the functions of unknown constants ω_i (i =1, 2, 3), therefore , we minimize $MSE(t_i)$ (i =1, 2, 3)with respect to ω_i (i =1, 2, 3). Thus minimizing $MSE(t_i)$ with respect ω_i, we get the optimum values of ω_i as

$$\omega_{iopt} = \frac{[MSE(t_{mi}) - Cov(t_u, t_{mi})]}{[MSE(t_u) + MSE(t_{mi}) - 2Cov(t_u, t_{mi})]}, \ (i=1, 2, 3). \quad (6.2)$$

Substitution of (6.2) in (6.1) yields the minimum *MSE* of t_i as

$$Min.MSE(t_i) = \frac{[MSE(t_u)MSE(t_{mi}) - \{Cov(t_u, t_{mi})\}^2]}{[MSE(t_u) + MSE(t_{mi}) - 2Cov(t_u, t_{mi})]}, (i=1, 2, 3). \quad (6.3)$$

Further putting the values from equations (3.8), (4.10), (4.17), (4.23), (4.27), (4.28) and (4.29) in (6.2) and (6.3), the simplified values of ω_{iopt} and $Min.MSE(t_i)$ are obtained as

$$\omega_{1opt} = \frac{\mu_1(\alpha_3 - \mu_1\alpha_2)}{(\alpha_3 - \mu_1^2\alpha_2)}, \quad (6.4)$$

$$Min.MSE(t_1) = \frac{\alpha_3[(1-f)\alpha_3 - \mu_1\alpha_2 + \mu_1^2 f\alpha_2]S_y^2}{n(\alpha_3 - \mu_1^2\alpha_2)}, \quad (6.5)$$

$$\omega_{2opt} = \frac{\mu_2(\alpha_3 - \mu_2\alpha_2^*)}{(\alpha_3 - \mu_2^2\alpha_2^*)}, \quad (6.6)$$

$$Min.MSE(t_2) = \frac{\alpha_3[(1-f)\alpha_3 - \mu_2\alpha_2^* + \mu_2^2 f\alpha_2^*]S_y^2}{n(\alpha_3 - \mu_2^2\alpha_2^*)}, \quad (6.7)$$

$$\omega_{3opt} = \frac{\mu_3(\alpha_3 - \mu_3\alpha_2^{**})}{(\alpha_3 - \mu_3^2\alpha_2^{**})}, \tag{6.8}$$

$$Min.MSE(t_3) = \frac{\alpha_3[(1-f)\alpha_3 - \mu_3\alpha_2^{**} + \mu_3^2 f\alpha_2^{**}]S_y^2}{n(\alpha_3 - \mu_3^2\alpha_2^{**})}, \tag{6.9}$$

where μ_i's (i =1, 2, 3)are the fractions of fresh samples to be drawn afresh on the current (second) occasion.

6.1 Optimum Replacement Policy

To obtain the optimum values of μ_i's (i =1, 2, 3) so that the population mean $\overline{Y}$ may be estimated with maximum precision, we minimize the minimum *MSE*s of the estimator t_i (i =1, 2, 3) given by (6.5), (6.7) and (6.9) respectively with respect to μ_i's (i =1, 2, 3) which result in quadratic equation in μ_i say $\hat{\mu}_i$ (i =1, 2, 3) are given below:

$$\mu_1^2\alpha_2 - 2\mu_1\alpha_3 + \alpha_3 = 0, \tag{6.10}$$

$$\hat{\mu}_1 = \frac{\alpha_3 \pm \sqrt{\alpha_1\alpha_3}}{\alpha_2}, \tag{6.11}$$

$$\mu_2^2\alpha_2^* - 2\mu_2\alpha_3 + \alpha_3 = 0, \tag{6.12}$$

$$\hat{\mu}_2 = \frac{\alpha_3 \pm \sqrt{\alpha_1^*\alpha_3}}{\alpha_2^*}, \tag{6.13}$$

$$\mu_3^2\alpha_2^{**} - 2\mu_3\alpha_3 + \alpha_3 = 0, \tag{6.14}$$

$$\hat{\mu}_3 = \frac{\alpha_3 \pm \sqrt{\alpha_1^{**}\alpha_3}}{\alpha_2^{**}}. \tag{6.15}$$

From equations (6.11), (6.13) and (6.15), it is obvious that the real values of $\hat{\mu}_i$ (i =1, 2, 3) exist. If, the quantities under square roots are greater than or equal to zero (i.e. $\alpha_1\alpha_3 \ge 0$, $\alpha_1^*\alpha_3 \ge 0$ and $\alpha_1^{**}\alpha_3 \ge 0$). For any combination of correlations ρ_{yx} and ρ_{yz}, and the scalars δ, a, b (and hence θ and δ) , which satisfy the condition of real situation, two real values of $\hat{\mu}_i$ (i =1, 2, 3) are possible. Hence, while selecting the values of $\hat{\mu}_i$, it should be observed that $0 \le \hat{\mu}_i \le 1$. Putting the admissible values of $\hat{\mu}_i$ say $\hat{\mu}_i^{(0)}$, from equations (6.11), (6.13) and (6.15) into equations (6.5), (6.7) and (6.9) respectively, we have the following optimum values of minimum mean squared error of the estimators t_i (i =1, 2, 3):

$$Min.MSE(t_1)_{opt} = \frac{\alpha_3[(1-f)\alpha_3 - \hat{\mu}_1^{(0)}\alpha_2 + \hat{\mu}_1^{(0)2} f\alpha_2]S_y^2}{n(\alpha_3 - \hat{\mu}_1^{(0)2}\alpha_2)}, \tag{6.16}$$

$$Min.MSE(t_2)_{opt} = \frac{\alpha_3[(1-f)\alpha_3 - \hat{\mu}_2^{(0)}\alpha_2^* + \hat{\mu}_2^{(0)2} f\alpha_2^*]S_y^2}{n(\alpha_3 - \hat{\mu}_2^{(0)2}\alpha_2^*)}, \tag{6.17}$$

and

$$Min.MSE(t_3)_{opt} = \frac{\alpha_3[(1-f)\alpha_3 - \hat{\mu}_3^{(0)}\alpha_2^{**} + \hat{\mu}_3^{(0)2} f\alpha_2^{**}]S_y^2}{n(\alpha_3 - \hat{\mu}_3^{(0)2}\alpha_2^{**})}. \tag{6.18}$$

7. Efficiency Comparisons

The percent relative efficiencies of the proposed estimators 't_i' (i =1, 2, 3) with respect to (i) usual unbiased estimator $\bar{y}_n$, when there is no matching and (ii) traditional successive sampling estimator $\hat{\hat{Y}} = \varphi\bar{y}_u + (1-\varphi)\bar{y}_{dm}$ when no auxiliary information is used at any occasion, where $\bar{y}_{dm} = \bar{y}_m + \beta_{yx}(\bar{x}_n - \bar{x}_m)$ have been computed for different choices of ρ_{yx}, ρ_{yz} and $\delta = -1$. Since, $\bar{y}_n$ and $\hat{\hat{Y}}$ are unbiased estimators of population mean $\bar{Y}$, therefore, following Sukhatme et al. (1984), the variance of usual unbiased estimator $\bar{y}_n$ and optimum variance of $\bar{y}_\varphi$ are respectively given by

$$Var(\bar{y}_n) = (1-f)\frac{S_y^2}{n}, \tag{7.1}$$

and

$$Var(\hat{\hat{Y}})_{opt} = \left[\left(1+\sqrt{(1-\rho_{yx}^2)}\right) - 2f\right]\frac{S_y^2}{2n}, \tag{7.2}$$

We have computed the percent relative efficiencies of $E_i^{(1)}$ and $E_i^{(2)}$ (i =1, 2, 3) of the proposed estimator 't_i' (i =1, 2, 3) with respect to the usual unbiased estimator $\bar{y}_n$ and $\hat{\hat{Y}}$ by using the following formulae:

$$E_i^{(1)} = \frac{V(\bar{y}_n)}{Min.MSE(t_i)_{opt}} \times 100 \tag{7.3}$$

and

$$E_i^{(2)} = \frac{V(\hat{\hat{Y}})_{opt}}{\min.MSE(t_i)^*_{opt}} \times 100. \tag{7.4}$$

Findings are shown in Table 7.1.

Table7.1. The PREs of t_i (i =1, 2, 3) with respect to $\bar{y}_n$ and $\hat{\hat{Y}}$ for different values of θ, ρ_{yx} and ρ_{yz}.

θ	ρ_{yx}	ρ_{yz}	$\mu_1^{(0)}$	$E_1^{(1)}$	$E_1^{(2)}$	$\mu_2^{(0)}$	$E_2^{(1)}$	$E_2^{(2)}$	$\mu_3^{(0)}$	$E_3^{(1)}$	$E_3^{(2)}$
0.2	0.55	0.50	0.4914	116.77	106.08	0.5228	125.10	113.65	0.5232	125.22	113.76
		0.55	0.4884	118.81	107.93	0.5198	127.33	115.67	0.5202	127.45	115.78
		0.60	0.4853	120.92	109.85	0.5167	129.65	117.78	0.5171	129.78	117.89
		0.65	0.4821	123.13	111.86	0.5135	132.07	119.98	0.5140	132.20	120.09
		0.70	0.4789	125.43	113.95	0.5103	134.59	122.27	0.5107	134.72	122.39
	0.60	0.50	0.5061	120.66	107.26	0.5320	127.58	113.40	0.5339	128.09	113.86
		0.55	0.5031	122.79	109.15	0.5290	129.86	115.43	0.5309	130.39	115.91
		0.60	*	-	-	0.5259	132.24	117.55	0.5279	132.79	118.03
		0.65	0.4968	127.30	113.16	0.5228	134.73	119.76	0.5247	135.28	120.25
		0.70	0.4936	129.71	115.30	0.5195	137.32	122.06	0.5215	137.89	122.57
	0.65	0.50	0.5228	125.10	108.42	0.5420	130.25	112.88	0.5467	131.53	113.99
		0.55	0.5198	127.33	110.35	0.5390	132.60	114.92	0.5437	133.91	116.05
		0.60	0.5167	129.65	112.36	0.5359	135.05	117.04	0.5406	136.39	118.20
		0.65	0.5135	132.07	114.46	0.5327	137.60	119.25	0.5375	138.98	120.44
		0.70	0.5103	134.59	116.64	0.5295	140.27	121.56	0.5343	141.68	122.78
0.3	0.55	0.50	0.4837	122.02	110.84	0.5151	130.85	118.87	0.5156	130.97	118.98
		0.55	0.4789	125.43	113.95	0.5103	134.59	122.27	0.5107	134.72	122.39
		0.60	0.4739	129.07	117.25	0.5052	138.58	125.89	0.5057	138.72	126.02
		0.65	0.4686	132.97	120.79	*	-	-	0.5004	143.00	129.90
		0.70	0.4632	137.14	124.58	0.4945	147.44	133.94	0.4950	147.59	134.07

θ	ρ_{yx}	ρ_{yz}	$\mu_1^{(0)}$	$E_1^{(1)}$	$E_1^{(2)}$	$\mu_2^{(0)}$	$E_2^{(1)}$	$E_2^{(2)}$	$\mu_3^{(0)}$	$E_3^{(1)}$	$E_3^{(2)}$
	0.60	0.50	0.4984	126.14	112.12	0.5244	133.47	118.64	0.5263	134.02	119.13
		0.55	0.4936	129.71	115.30	0.5195	137.32	122.06	0.5215	137.89	122.57
		0.60	0.4886	133.51	118.68	0.5145	141.42	125.70	0.5164	142.01	126.23
		0.65	0.4833	137.58	122.29	0.5093	145.80	129.60	0.5112	146.42	130.15
		0.70	0.4778	141.94	126.17	0.5038	150.51	133.79	0.5057	151.16	134.36
	0.65	0.50	0.5151	130.85	113.40	0.5343	136.31	118.13	0.5391	137.67	119.31
		0.55	0.5103	134.59	116.64	0.5295	140.27	121.56	0.5343	141.68	122.78
		0.60	0.5052	138.58	120.10	0.5245	144.48	125.21	0.5293	145.95	126.49
		0.65	*	-	-	0.5193	149.00	129.13	0.5240	150.53	130.45
		0.70	0.4945	147.44	127.78	0.5138	153.84	133.32	0.5186	155.44	134.71
0.4	0.55	0.50	0.4789	125.43	113.95	0.5103	134.59	122.27	0.5107	134.72	122.39
		0.55	0.4721	130.34	118.41	0.5035	139.98	127.16	0.5040	140.11	127.28
		0.60	0.4650	135.72	123.29	0.4964	145.88	132.52	0.4968	146.02	132.65
		0.65	0.4575	141.63	128.66	0.4888	152.37	138.42	0.4892	152.53	138.56
		0.70	0.4495	148.17	134.60	0.4807	159.56	144.95	0.4812	159.73	145.10
	0.60	0.50	0.4936	129.71	115.30	0.5195	137.32	122.06	0.5215	137.89	122.57
		0.55	0.4868	134.84	119.85	0.5128	142.85	126.97	0.5147	143.45	127.51
		0.60	0.4797	140.45	124.85	0.5056	148.90	132.36	0.5076	149.54	132.92
		0.65	0.4721	146.63	130.34	0.4981	155.58	138.29	*	-	-
		0.70	0.4641	153.48	136.42	0.4900	162.97	144.86	0.4919	163.68	145.50
	0.65	0.50	0.5103	134.59	116.64	0.5295	140.27	121.56	0.5343	141.68	122.78
		0.55	0.5035	139.98	121.31	0.5228	145.95	126.49	0.5275	147.44	127.78
		0.60	0.4964	145.88	126.42	0.5156	152.19	131.89	0.5204	153.76	133.25
		0.65	0.4888	152.37	132.05	0.5081	159.06	137.84	0.5128	160.72	139.29
		0.70	0.4807	159.56	138.28	*	-	-	0.5048	168.44	145.97
0.5	0.55	0.50	0.4772	126.62	115.02	0.5086	135.89	123.45	0.5091	136.03	123.57
		0.55	0.4686	132.97	120.79	*	-	-	0.5004	143.00	129.90
		0.60	0.4594	140.09	127.26	0.4907	150.69	136.89	0.4912	150.84	137.03
		0.65	0.4495	148.17	134.60	0.4807	159.56	144.95	0.4812	159.73	145.10
		0.70	0.4387	157.40	142.99	0.4699	169.73	154.19	0.4703	169.91	154.35
	0.60	0.50	0.4919	130.95	116.40	0.5179	138.65	123.25	0.5198	139.23	123.76
		0.55	0.4833	137.58	122.29	0.5093	145.80	129.60	0.5112	146.42	130.15
		0.60	0.4741	145.03	128.92	*	-	-	0.5019	154.51	137.34
		0.65	0.4641	153.48	136.42	0.4900	162.97	144.86	0.4919	163.68	145.50
		0.70	0.4533	163.14	145.02	0.4791	173.42	154.16	0.4811	174.20	154.84
	0.65	0.50	0.5086	135.89	117.77	0.5279	141.64	122.75	0.5326	143.07	123.99
		0.55	*	-	-	0.5193	149.00	129.13	0.5240	150.53	130.45
		0.60	0.4907	150.69	130.59	0.5100	157.27	136.30	0.5148	158.92	137.72
		0.65	0.4807	159.56	138.28	*	-	-	0.5048	168.44	145.97
		0.70	0.4699	169.73	147.10	0.4891	177.43	153.77	0.4939	179.36	155.44

Note: "*" indicates that $\mu_i^{(0)}$ does not exist.

8. Discussions & Conclusions

Table 7.1 exhibits that:

(i) for fixed values of (θ, ρ_{yx}), the values of $\mu_i^{(0)}$ decreases, while the values of $E_i^{(1)}$ and $E_i^{(2)}$ (i =1, 2, 3) increase for increasing value of ρ_{yz}. This implies that if information on highly correlated auxiliary variable is available, increment in the precision of proposed estimators is obtained which reduce the cost of survey. Thus this type of situation is appreciable. Similar trends are obtained for fixed value of (ρ_{yx}, ρ_{yz}) with varying the values of θ.

(ii) for fixed values of (θ, ρ_{yz}), the values of $\mu_i^{(0)}$, $E_i^{(1)}$ and $E_i^{(2)}$ (i =1, 2, 3) increase as the value of ρ_{yx} increase. This behavior is in agreement with Sukhatme at al. (1984), results which explained that more the value of ρ_{yx}, more the fractions of fresh sample is needed at the current (second) occasion.

(iii) minimum values of $\mu_1^{(0)} = 0.4387\ (\cong 0.44)$, $\mu_2^{(0)} = 0.4699\ (\cong 0.47)$ and $\mu_1^{(0)} = 0.4703\ (\cong 0.47)$ which show that the fraction of fresh sample to be replaced is as low as about 44% (in case of estimator t_1) and 47%(in case of estimators t_2 and t_3) of the total sample size .

(iv) $E_1^{(j)} < E_2^{(j)} < E_3^{(j)}$ (j =1, 2) which shows the superiority of the estimator t_3 over the estimators t_1 and t_2. In general the values of $E_i^{(j)}$'s (i=1, 2, 3; j=1, 2) are greater than 100.Thus the proposed classes of estimators t_1, t_2 and t_3 are to be preferred over the usual unbiased estimator $\bar{y}_n$ and the natural estimator $\hat{\bar{Y}}$ in practice.

We conclude from the above discussions that the use of auxiliary information is extremely satisfying in terms of proposed estimators. It is also intelligible that if highly correlated auxiliary variable are used, moderately only a small fraction of sample on current occasion is preferred to be replaced by a fresh sample which reduce the cost of the survey.

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